DOMINIC WELSH
1938–2023

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If the principal qualities of an academic are associated with teaching, research, and collegiality, Dominic Welsh was a truly outstanding example to all. He combined excellence as tutor and supervisor over nearly 40 years with a distinguished record in research in probability and discrete mathematics, while creating a community of younger people who remember him with love and respect.

1. Early life

James Anthony Dominic Welsh was born on 29 August 1938 in Port Talbot to parents Teresa O'Callaghan and James Welsh. They addressed him as Dominic, and it was thus that he was known to family, friends, and colleagues.

Dominic was born into an extended family of Irish Catholic origins living in and around Swansea. Numerous members of the family including both his parents were teachers, and his father was Headmaster at St David's School, Swansea[1]. Education was naturally considered a priority for their

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[1]St David's Roman Catholic School is said to have been built to "save the Irish and Belgian children of the district from the risk of being brought up in ignorance of their religion, if not losing it altogether", https://stdavidspriory.wordpress.com/st-davids-school/a-brief-history-2/

four children, though teaching was not the only profession of the family; Dominic's grandfather worked at the Port Talbot steelworks, and his uncle Jim (Bennett), a Dubliner by origin, was the brewer at the historic Buckley's Brewery in Llanelli. All four of Teresa and James' children followed their parents' example in one way or another. Their second son, David (b. 1939), studied in London and Oxford, and became Professor of Classics at the University of Ottawa; Mary (b. 1942) studied in Swansea, became a teacher, and moved to Japan before starting a successful EFL business; Teresa (b. 1952) graduated with a first class degree in mathematics at Swansea, and remained in Port Talbot as a teacher and later a school governor.

Dominic's earliest memories were of the convivial home of his Gran in Port Talbot, where his Aunt May married Jim Bennett to an accompaniment of Irish dancing, music, and songs. With the start of war, his father's school was evacuated from Swansea, and the family moved with the newly born Dave to the nearby Ammanford. Later he was sent back to Port Talbot, where he lived until a nearby bombing raid, probably that of 13 February 1941 on the Morgan family of Corporation Road[2].

As a child Dominic developed a talent for rugby; a love of sports, games, and other competitive activities stayed with him throughout his life. He was invited to trial for the Wales Under-18 XV, but it was not a success. He was poorly served by the age (birthday) cutoff of 1 September; he was assaulted on the pitch by a rival (who later became a prominent international); and he was dispirited by the line of dentures hanging on hooks in the changing room.

For his secondary education, Dominic attended Bishop Gore Grammar School. His mathematical abilities were evidently exceptional, and he was the first boy ever to move from the school to Oxford University. Indeed he was offered places at both Oxford and Cambridge, but the latter would have required him to first spend two years in National Service.

2. Oxford and Merton

Dominic went up in 1957 to Merton College, Oxford, as an Exhibitioner to read Mathematics, and Merton became his intellectual home for ever more. Following his BA in 1960, he visited North America for the first time with a Fulbright award to Carnegie-Mellon University, and thence to Bell Laboratories in Murray Hill. He returned to Oxford in 1961 as a postgraduate, and was awarded a NATO studentship in 1962. He took his DPhil in 1964 under John Hammersley, was appointed Junior Lecturer in the Mathematical Institute in 1963, and was elected to a Tutorship and Fellowship at Merton in 1966. Within the University, he was promoted to a Readership in 1990 and a personal Chair in 1992, while retaining his Fellowship at Merton. He attained the retirement age of 67 in 2005 becoming Emeritus Fellow and

[2] https://www.swansea.gov.uk/article/7112/The-civilian-war-dead-of-Neath-and-Port-Talbot

Professor. He died in Oxford on 30 November 2023 after a period of illness. His funeral and Requiem Mass took place in the College Chapel on 16 December 2023, followed on 1 June 2024 by a memorial service.

His 39 years as a Fellow have been exceeded by only few in recent times, including by his colleague Philip Watson (Fellow, 1950–1993), with whom Dominic shared the privilege of teaching and advising the many mathematics undergraduates at the College during their years in common. No-one has contributed more than he to the marked rise in reputation of the College in mathematics over the last 60 years.

Quite apart from his family life (more soon on that), Dominic was a very busy person indeed. As a CUF lecturer, he had a standard tutorial load of 12 hours per week, plus a lesser lecturing load. Administration had a lighter touch in the 1960s and '70s than now, but Dominic was never a shirker and he undertook his share of College duties, including as Principal of the Postmasters [PoP] (1970–73), Pro-Proctor (1979–80), Sub-Warden (1982–84), and lastly (but not least) Wine Steward (1998–2002).

Within the University, he duly served as Chair of the Faculty (1976–78), of the Board of Mathematical Sciences (1984–86), and of the Mathematical Institute (1996–2001)[3]. His term as Institute Chair was eminently successful, though he was sometimes uncomfortable with the degree of influence and control placed upon his shoulders.

Dominic's strong sense of responsibility was invariably leavened with humour and good sense, and his deepest loves were people and research. He was really very good with young people, and he developed a rare affinity with many of his undergraduate and graduate students.

Dominic was an academic of a now rare breed. He was devoted and generous to his students, both undergraduate and graduate, and he welcomed them into his house and family for nourishment, conversation, and other (usually competitive) activities. He taught up to 15 hours per week during term, and covered a wide range of subjects including the entire first and second year syllabuses. He was active over more than 50 years at a high level in mathematical research; he supervised 28 DPhil students, and he wrote numerous research papers and books with more than 50 collaborators. He undertook a full load of administration in the College and University. All this he achieved with charm and diligence. He probably never held (or applied for) a Research Council grant of substance. (??)

On retirement from Oxford University, Dominic and Bridget enjoyed an extended stay in Barcelona where Dominic visited the Technical University of Catalonia. This was followed by a period based in Bath and an association with the Heilbronn Institute in Bristol. Meanwhile he had exchanged his College accommodation for the house in North Oxford that was his base for the rest of his life.

[3]Who's Who, https://doi.org/10.1093/ww/9780199540884.013.U39334

3. Family life

Dominic and Bridget met on Dominic's 24th birthday in 1962. It was love at first sight. They married on 10 July 1965 and made their first home together in a Woodstock cottage. Their three sons arrived in arithmetic progression: James (b. 1967), Simon (b. 1969), John (b. 1971). The family came to know many of Dominic's students, first through childsitting, and progressing to various activities.

Dominic's family has written[4]: 'Many of his colleagues and students went on to be lifelong friends. Dominic and Bridget would host long laughter-filled Sunday lunches, first at their college home in Kybald Street then later in Rose Lane, where future mathematical geniuses would sit side by side with a trio of grubby-faced mischievous kids.' [Some readers may cavil at the word 'future'.]

Two aspects of Dominic's personality merit mention. He was born into a Catholic family and was committed to Roman Catholicism throughout his life. His religion guided him in moral matters, though he never seemed to judge others. He attended the Catholic Chaplaincy regularly before moving to the Oxford Oratory.

Secondly, he was an avid and competitive sports and games player. Dinner would often be followed by a game, usually designed for children but played with the seriousness of a rugby final at Cardiff Arms Park.

David Stirzaker has written: 'Dominic's pastoral care of students extended to playing both tennis and real tennis with them on the Merton courts. He was fond of describing real tennis as 'chess on the run', ... or was it 'chess on wheels', I forget. He beat me at lawn tennis. He used to invite students round to his house to play group board games. I remember enjoying some very competitive sessions of Diplomacy in the 1960s, which revealed a rich vein of deviousness and cunning in his character. Students and colleagues were invited to watch rugby with him. I well recall watching the Barbarians beat New Zealand in 1973. His joy at Gareth Edwards' try was infectious.[5]' [His family considered him a good loser but a horrible winner.]

Disaster struck in 1990 when John drowned in an accident in Brisbane. To quote again from the Welsh family: 'They were happy times, but no life is without pain. And in 1990, Dominic and Bridget suffered the worst pain imaginable, when their beloved eighteen-year-old son, John, died while travelling in Australia. No one ever fully recovers from the loss of a child,

[4] https://professordominicwelsh.muchloved.com/

[5] David Williams is quoted as saying "[Maths is] like watching the famous match between the Barbarians and the All Blacks. Everyone who saw that game was carried away with the excitement. But it does not compare with the excitement of mathematics." See https://www.walesonline.co.uk/news/wales-news/swansea-professors-maths-victory-2122483.

but eventually Dominic and Bridget somehow found the strength to keep on living, loving and laughing.'

In later life, Bridget and Dominic travelled the world and enjoyed walking holidays.

4. Tributes, reminiscences

James Oxley has written: 'Dominic modelled for me not only how to do research but also how to bring out the best in students. The first time I met him was in the post room at Merton College just after I had arrived in Oxford. Seeing him there, I tentatively approached him with "Dr Welsh?" "Dominic" was his immediate response, quickly followed by an invitation to afternoon tea with his family. His friendly, welcoming, and informal manner made interacting with him great fun.

Dominic's generosity of spirit shone through in our exchanges. After each of us had published books on matroid theory, he apologised to me because he was worried that the 2010 Dover reprint of his 1976 book may hurt the sales of my book. Both Dominic's sage guidance and our deep and enduring friendship enriched my life immensely.'

Neil Loden remembers: 'I was sad to learn of the death of Dominic Welsh whom I remember fondly in his role as the PoP. After a particularly debauched dinner of the 1311 Club in 1969, Alan [Harland], Chris [Hewitt] and I blocked up the door into his rooms by building a wall of bricks (taken from the works going on in Front Quad) from floor to ceiling, which kept him out of his room for the whole of the next morning, while we were nowhere to be seen, being still in bed nursing hangovers. We thought at the time that it was very funny, and even he didn't seem to mind unduly.'

David Stirzaker has written: 'Dominic's pupils generally found him to be one of the most engaging and entertaining of all their lecturers. More than other lecturers, he conveyed the impression that he was enjoying it and found it fun to be talking to us. I recall him remarking of some theorem that, as it stood, it was so hedged about with technical conditions and restrictions as to be about as useful as a barren pear tree. This was quite refreshing after the arid dustiness of most Oxford lectures.'

more to come

5. Work and influence

It may be said that Dominic's mathematical trajectory was influenced most by John Hammersley and Bill Tutte — the former imparted a love of problems associated with counting and chance, and the work of the latter was foundational to Dominic's interests in combinatorics. The human angle of mathematics as a participatory activity played a key role for him. He thrived off discussions with colleagues, usually (ex)-students, throwing ideas and often wild questions around, possibly over tea in his home or in the Oxford Mathematical Institute. One thing would lead to another, and papers, books, and dinners flowed, usually in the company of members of the community that circulated around and were inspired by such meetings.

The 1960s and '70s were a glorious period for combinatorial theory, with Dominic active at the heart of the development in the UK. He organised and edited the proceedings of the 1969 Oxford Conference that came to be viewed as the First British Combinatorial Conference (BCC); the list of 35 speakers included Pál Erdős, Richard Guy, Mark Kac, and Roger Penrose. This gave birth to the continuing series of BCC meetings[6], of which Dominic organised the third, also in Oxford. These were heady days for the subject in the UK.

There follows a brief overview of Dominic's contributions to probability and combinatorics. The author apologises to those whose work is omitted.

Section 6 is centred around his work on percolation and self-avoiding walk, mostly done as a DPhil student; the results summarised there have had great impact over the intervening 60 years. He began taking his own students very soon after his doctoral graduation, and 28 such students are listed on his academic family tree[7], beginning with Adrian Bondy who graduated in 1969. Sections 7 and 8 are directed towards his work, as an individual and often jointly with his students, on matroids and complexity.

Graham Farr, Dillon Mayhew, and James Oxley have written[8]: 'Dominic was a very effective supervisor of research students. [...] He was flexible in his approach and adept at finding a productive mix of patience, firmness, encouragement, plain speaking, and inspiration. [...] Time and again he brought out the best [...], inspiring enduring appreciation and affection.'

In his books, Dominic achieved a high level of communication (and excellent Amazon reviews) through clear exposition and a minimum of mathematical prerequisites. In addition to his more elementary volume [9] on probability (which, wrote one eminent reviewer, reads as though it was written in a punt), he wrote more advanced texts on *Matroid Theory* [25], *Codes and Cryptography* [26], *Complexity: Knots, Colourings and Counting* [27], and *Complexity and Cryptography* (with John Talbot) [23].

A full list of Dominic Welsh's publications is available at https://www.statslab.cam.ac.uk/~grg/papers/welsh-bib.pdf.

sec:perc

6. Percolation and self-avoiding walk

Dominic's DPhil supervisor, John Hammersley, was a pioneering mathematician who contributed some of the very best early work on important topics including self-avoiding walks and percolation. Each of their two joint papers has had substantial impact on a field that has grown in visibility over the decades.

[6]See Norman Biggs' history of the BCC at https://staff.computing.dundee.ac.uk/kedwards/bcc/ancient.html.

[7]Maintained by David Wood at https://users.monash.edu.au/~davidwo/history.html. The tree shows 235 scientific offspring of DJAW at the time of writing.

[8]See their account of Dominic's work at http://matroidunion.org/?p=5304.

6.1. **First-passage percolation.** Percolation is the canonical model for a random spatial medium. It is usually viewed as a *static* model in the sense that there is no dependence on time and no evolutionary aspect. The *first-passage percolation* process, introduced by Hammersley and Welsh in [11] in 1965, includes a time variable, and thus leads to beautiful questions involving growing random sets and their asymptotic behaviour. This work formed the core of Dominic's DPhil thesis [24], and has had enormous influence over the intervening 60 years on the theory of random spatial growth.

Amongst the major advances of this work is the introduction of the concept of a *subadditive* stochastic process, that is, a stationary family $(X_{s,t} : 0 \leq s < t < \infty)$ of random variables satisfying subadditivity: for $s < u < t$, we have $X_{s,t} \leq X_{s,u} + X_{u,t}$. The authors proved an early version of the now famous subadditive ergodic theorem, which states that, subject to a suitable moment condition, the limit $\lim_{t\to\infty} X_{0,t}/t$ exists. This limit theorem has been improved and optimised in various ways since 1964 by Kingman [15, 16], Liggett [18], and others; Kesten and Hammersley weakened the subadditive condition to a distributional assumption, and so on. The subadditive ergodic theorem is one of the major tools of stochastic geometry (and beyond).

6.2. **Self-avoiding walks.** A *self-avoiding walk* (SAW) on a graph G is a path that visits no vertex twice or more. The basic SAW problem is to estimate the number $\sigma_n(G)$ of distinct n-step SAWs starting at a given vertex. This problem is fundamental to the theory of polymers as studied by Flory and co-workers, [5]. For concreteness (and a reason that will soon be clear), let G be the hexagonal lattice, denoted $\mathbb{H}$. It is now standard that σ_n is a (deterministic) subadditive sequence in that $\sigma_{m+n} \leq \sigma_m + \sigma_n$, and it follows that the limit

$$\kappa = \lim_{n\to\infty} \sigma_n^{1/n}$$

exists. The constant $\kappa = \kappa(\mathbb{H})$ is called the *connective constant* of $\mathbb{H}$. One has by subadditivity that $\sigma_n \geq \kappa^n$. In [10], Hammersley and Welsh sought upper bounds for σ_n, and were able to prove that there exists γ such that $\sigma_n \leq \kappa^n \gamma^{\sqrt{n}}$. This they achieved by showing that a SAW can be decomposed as a union of so-called bridges, and by observing that counts of bridges are superadditive rather than subadditive. (Actually they worked with the d-dimensional cubic lattice, but their argument is largely valid in the greater generality of indicable Cayley graphs, see [8]).

Two observations are made about the impact of this work. Firstly, its is believed (but not yet proved) that the true correction term in the two-dimensional case is a power of n rather than an exponential of $\sqrt{n}$. More precisely, it is believed, for a d-dimensional lattice $\mathcal{L}$, that there exist constants $A = A_d$ and $\gamma = \gamma_d$ such that

eq:saw
$$\sigma_n \sim A n^{\gamma-1} \kappa(\mathcal{L})^n, \tag{6.1}$$

(subject to a logarithmic correction when $d = 4$) and moreover $\gamma_2 = \frac{43}{32}$. Equation (6.1) was proved in 1992 by Hara and Slade for the cubic lattice in 5 and more dimensions, in which case we have $\gamma = 1$ (see [19]). The case of two dimensions seems especially hard, and the best rigorous work so far appears to be that of [2] where the $\gamma^{\sqrt{n}}$ is replaced by $\gamma^{n^{\frac{1}{2}-\epsilon}}$ for some $\epsilon > 0$. The full picture in two dimensions may emerge only when we have a proper understanding of the relationship between SAWs and the stochastic Loewner evolution process $\mathrm{SLE}_{8/3}$ (see [7]).

Returning to the case of the hexagonal lattice $\mathbb{H}$, the biggest result on SAWs in recent years is the exact calculation of $\kappa(\mathbb{H})$ by Duminil-Copin and Smirnov [3], namely $\kappa(\mathbb{H}) = \sqrt{2+\sqrt{2}}$, as conjectured by Nienhuis using conformal field theory. The proof is a combination of a deeply original argument combined (in its 'easier' part) with the bridge arguments of Hammersley and Welsh [10].

6.3. **Russo–Seymour–Welsh theory for percolation.** Dominic loved problems, and one of his favourites in the 1970s was to prove that the critical probability of bond percolation on the square lattice $\mathbb{Z}^2$ equals $\frac{1}{2}$. This captivating and beautiful conjecture seemed to defy all attempts. Together with Paul Seymour, Dominic constructed a new tool now named the RSW lemma after Russo [21] and Seymour–Welsh [22].

Bond percolation on $\mathbb{Z}^2$ is given as follows. Let $0 < p < 1$, and declare each edge of $\mathbb{Z}^2$ *open* with probability p and *closed* otherwise (with independence between edges). Let $\theta(p)$ be the probability that the open subgraph possesses an infinite component, and define the *critical probability*

$$p_c = \sup\{p : \theta(p) = 0\}.$$

Hammersley conjectured that $p_c = \frac{1}{2}$, and Ted Harris [12] proved that $p_c \geq \frac{1}{2}$. The conjecture is supported by a symmetry between open edges of $\mathbb{Z}^2$ and closed edges of its dual graph.

Dominic's idea was to study what he called 'sponge percolation', by which he meant the probability $\pi_p(m, n)$ that a $m \times n$ rectangle of $\mathbb{Z}^2$ is crossed from left to right by an open path. By duality, we have $\pi_{\frac{1}{2}}(m+1, m) = \frac{1}{2}$. Together with Seymour, he proved by a complicated geometrical and probabilistic construction that there exists $f : (0,1) \to (0,1)$ such that

$$\pi_p(4m, 2m) \geq f(\pi_p(2m, 2m)).$$

This can be read as saying that, if there is a decent probability of an open crossing of a square, then there is a decent probability of a crossing of a rectangle. This in turn implies a lower bound for the probabilities of still longer paths and cycles.

This RSW lemma is one of the fundamental tools of percolation and related topics (see [17] for a recent result of RSW type). It is the basic tool used by Harry Kesten in his celebrated proof that indeed $p_c = \frac{1}{2}$, [14].

sec:mat

7. Matroid theory

sec:comp

8. Complexity theory

8.1. **Computational complexity.** Students of Hammersley tended to have computational tendencies, and Dominic was no exception. His time at Bell Laboratories was influential, and he soon developed his own ideas for computational methods. His work as a DPhil student inspired two letters about PERT networks to the editor of *Operations Research*, and he followed this with Martin Powell in proposing the so-called Welsh–Powell algorithm [28] for colouring a graph. This is a greedy algorithm: first list the vertices in decreasing order of degree; then construct greedily an independent set containing the first vertex, and colour this set with the first colour; iterate.

Dominic's interests in complexity developed in the 1980s, inspired in part by the volume [6] of Garey and Johnson. He wrote a number of influential review articles, and encouraged his students to work on the classification of graph-theoretic complexity problems including thickness and colouring. The following extract from a review of his survey with his student Tony Mansfield might be written equally about any of his expository work: 'The paper is well written and is eminently readable. [. . .] The authors have artfully employed the best of both intuition and formalism to achieve succinctness and clarity.'

8.2. **Tutte polynomials.** Bill Tutte made explicit his eponymous polynomial in 1947 (see [4, Chap. 34]), and its prominence has much grown since.

The Tutte polynomial of a finite graph $G = (V, E)$ is given by

$$T_G(x, y) = \sum_{A\subseteq E} (x-1)^{r(E)-r(A)}(y-1)^{n(A)},$$

where r denotes rank and n denotes nullity. The study of such polynomials has risen to prominence, and is now a major part of graph theory. By choosing $(x, y) \in \mathbb{R}^2$ suitably, one obtains counts of a number of important features of G such as spanning trees and forests. In addition, T_G includes the chromatic, flow, and knot polynomials as well as the Whitney rank function and the random-cluster/Potts partition functions.

Dominic devoted much of his later years to studying Tutte polynomials. In an important article [13], written with Jaeger and Vertigan, he initiated a systematic study of the complexity of calculating $T_G(x, y)$ for certain classes of graph G and for different values of $(x, y) \in \mathbb{R}^2$ (his book [27] has been influential in this field). Let H be the set of red points of Figure 8.1. They proved that $T_G(x, y)$ is computable in polynomial time on H, whereas its computation is #P-hard on $\mathbb{R}^2 \setminus H$.

Moreover, for planar graphs, T_G may be computed in polynomial time on the hyperbola H_2 of Figure 8.1; this follows by the classic representation by Piet Kasteleyn and Michael Fisher of the Ising partition function in terms of dimer configurations (also known as complete matchings).

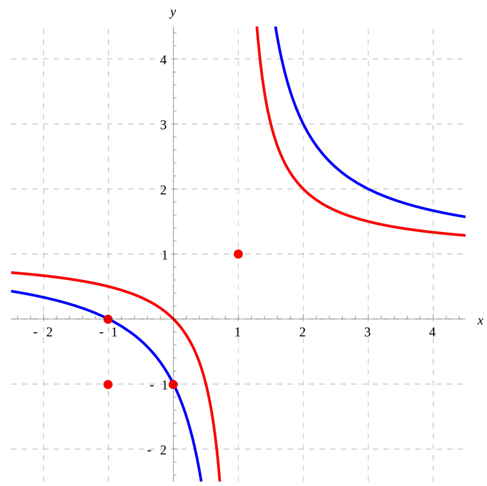

FIGURE 8.1. The parameter plane $\mathbb{R}^2$ of the Tutte polynomial T_G. The red hyperbola H_1 is given by $(x-1)(y-1)=1$. The blue hyperbola H_2, $(x-1)(y-1)=2$, corresponds to the partition function of the Ising model on G. This was probably Dominic's favourite figure during the 1990s. fig:comp

This work was the first of a series of papers by Dominic's students and others on the complexity of the Tutte polynomial and its cousins. In addition, it led to extensions of the work to matroids, and also to the problem of approximating Tutte polynomials.

A *fully polynomial randomised approximation scheme* (FPRAS) for a function $f(x)$ is a randomised algorithm which, for all x and $\epsilon > 0$, outputs a random value $\widehat{f}(x,\epsilon)$ satisfying

$$\mathbb{P}\big(\widehat{f}(x,\epsilon)/f(x) \in (1-\epsilon, 1+\epsilon)\big) \geq \tfrac{3}{4}.$$

For what graphs G and $(x,y) \in \mathbb{R}^2$ does there exist a FPRAS to compute $T_G(x,y)$? Dominic's student James Annan introduced the class of dense graphs: for $\alpha > 0$, $\mathcal{G}_\alpha$ is the class of 'α-*dense* graphs' $G=(V,E)$ satisfying $|E| \geq \alpha|V|$. With Noga Alon and Alan Frieze, Dominic showed in [1] the existence of an FPRAS when $x, y \geq 1$ for the class $\mathcal{G}_\alpha$ with $\alpha > 0$ (with a stronger conclusion when $\alpha > \frac{1}{2}$). Never lacking in bravery, Dominic has conjectured that the condition on α may be removed, but the jury is still out on that (see [4, Chap. 10]).

One further conjecture of Dominic is mentioned. With his student Criel Merino he conjectured in [20] that the number of spanning trees of a graph is no greater than the maximum of the number of acyclic orientations and the number of totally cyclic orientations. This amounts to the inequality

$$T_G(1,1) \leq \max\{T_G(2,0), T_G(0,2)\},$$

and this has been open since 1999.

+ simulation

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have informed this account. Graham Farr has kindly contributed through his very useful account of Dominic's life and work, written jointly with Dillon Mayhew and James Oxley for the Matroid Union; thanks also to Graham for sharing the copy of Dominic's DPhil thesis supplied by the Bodleian Library. The portrait of Dominic was taken by Bridget Welsh in around 2006. Figure 8.1 was published on Wikipedia under the CC-BY-SA 3.0 licence.

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