GreenYellow GreenYellow
Yellow Yellow
Goldenrod Goldenrod
Dandelion Dandelion
Apricot Apricot
Peach Peach
Melon Melon
YellowOrange YellowOrange
Orange Orange
BurntOrange BurntOrange
Bittersweet Bittersweet
RedOrange RedOrange
Mahogany Mahogany
Maroon Maroon
BrickRed BrickRed
Red Red
OrangeRed OrangeRed
RubineRed RubineRed
WildStrawberry WildStrawberry
Salmon Salmon
CarnationPink CarnationPink
Magenta Magenta

VioletRed VioletRed
Rhodamine Rhodamine
Mulberry Mulberry
RedViolet RedViolet
Fuchsia Fuchsia
Lavender Lavender
Thistle Thistle
Orchid Orchid
DarkOrchid DarkOrchid
Purple Purple
Plum Plum
Violet Violet
RoyalPurple RoyalPurple
BlueViolet BlueViolet
Periwinkle Periwinkle
CadetBlue CadetBlue
CornflowerBlue CornflowerBlue
MidnightBlue MidnightBlue
NavyBlue NavyBlue
RoyalBlue RoyalBlue
Blue Blue
Cerulean Cerulean
Cyan Cyan

ProcessBlue ProcessBlue
SkyBlue SkyBlue
Turquoise Turquoise
TealBlue TealBlue
Aquamarine Aquamarine
BlueGreen BlueGreen
Emerald Emerald
JungleGreen JungleGreen
SeaGreen SeaGreen
Green Green
ForestGreen ForestGreen
PineGreen PineGreen
LimeGreen LimeGreen
YellowGreen YellowGreen
SpringGreen SpringGreen
OliveGreen OliveGreen
RawSienna RawSienna
Sepia Sepia
Brown Brown
Tan Tan
Gray Gray
Black Black
White White

Number Theory in DPMMS

Number Theorists in Cambridge

Arithmetic Algebraic Geometry:

- John Coates
- Tom Fisher
- Tony Scholl
- Nick Shepherd-Barron
- Peter Swinnerton-Dyer

Analytic and Additive Number Theory:

- Alan Baker
- Tim Gowers

Research students: 12 in AAG

Postdocs/visitors

Seminars/Study groups for staff and students; student seminars

Recent highlights in Arithmetic Algebraic Geometry:

- Artin's conjecture (Shepherd-Barron + others)

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II Special values of L-functions of modular forms (Scholl)

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I Artin's conjecture (Shepherd-Barron + others)

II Special values of L-functions of modular forms (Scholl)

III Non-abelian Iwasawa theory (Coates, Fisher + others)

I: Artin's Conjecture for icosahedral representations

$\rho : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to GL_n(\mathbb{C})$ irreducible, non-trivial representation

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$\dim\rho = 1$: classical (Dirichlet L-series)

$\dim\rho = 2$:
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- A_5 (not accessible to base-change: some computational evidence)

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Proved in [1,2] for a wide class of *odd* A_5 representations — uses Wiles's method, study of moduli of abelian surfaces and p-adic modular forms.

[1] N. Shepherd-Barron, R. Taylor: *Mod 2 and mod 5 icosahedral representations.* J.A.M.S. 10 (1997) 283-298

[2] K. Buzzard, M. Dickinson, N. Shepherd-Barron, R. Taylor: *On icosahedral Artin representations.* Duke Math. J. 109 (2001), 283-318

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Deep conjectures relating arithmetic phenomena to special values of L-functions: Birch/Swinnerton-Dyer, Beilinson and Bloch/Kato conjectures (modelled on analytic class number formula)

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- zeta elements also related to p-adic L-functions (Euler systems)

[1] *Motives for modular forms.* Invent. math. 100 (1990), 419–430

[2] *Higher regulators and L-functions of modular forms* (book partly written)

III: Non-commutative Iwasawa Theory of Elliptic Curves

Work in progress: Coates with Kato and others.

$E/\mathbb{Q}$ elliptic curve.

Birch/Swinnerton-Dyer Conjecture: links arithmetic of $E/\mathbb{Q}$ and $L(E,s)$.

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Can now formulate Main Conjecture (p-adic L-function $=$ characteristic element of Selmer group) — numerical evidence! (T & V Dokchitser)

UNIVERSITY OF CAMBRIDGE

NUMBER THEORY IN DPMMS

THE END!