European Apportionment via the Cambridge Compromise

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Abstract

Seven mathematicians and one political scientist met at the Cambridge Apportionment Meeting in January 2011. They agreed a unanimous recommendation to the European Parliament for its future apportionments between the EU Member States. This is a short factual account of the reasons that led to the Meeting, of its debates and report, and of some of the ensuing Parliamentary debate.

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1. Background and Brief

1.1. Background

As the European Union has grown and its population has developed, so has the constitution and structure of the European Parliament. In recognition of the need for an orderly allocation of Parliamentary seats between the EU Member States, its Committee on Contitutional Affairs (AFCO) commissioned a Symposium of Mathematicians to "identify a mathematical formula for the distribution of seats which will be durable, transparent and impartial to politics". The purposes of the reform were described thus in [3]:

- The aim of the symposium is to discuss and, if possible, to propose to the Committee on Constitutional Affairs a mathematical formula for the redistribution of the 751 seats in the European Parliament. The formula should be as transparent as possible and capable of being sustained from one Parliamentary mandate to the next.

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- The purpose of the Symposium is to eliminate the political bartering which has characterised the distribution of seats so far by enabling a smooth reallocation of seats once every five years which takes account of migration, demographic shifts and the accession of new Member States.

The current note is more a record of the events surrounding the Cambridge Apportionment Meeting than it is a critical analysis of the politics. An account of the history of the current apportionment of Parliament, and of the associated "political bartering", may be found in [2].

1.2. Cambridge Apportionment Meeting (CAM)

The Symposium took place in the Centre for Mathematical Sciences, Cambridge University, on 28–29 January 2011, under the Directorship of Geoffrey Grimmett (Cambridge) and Friedrich Pukelsheim (Augsburg). The participants were: Jean-François Laslier (Paris), Victoriano Ramírez González (Granada), Richard Rose (Aberdeen, Florence), Wojciech Słomczyński (Kraków), Martin Zachariasen (Copenhagen), Karol Życzkowski (Kraków), advised by Andrew Duff MEP, Rafał Trzaskowski MEP, Guy Deregnaucourt (AFCO), Wolfgang Leonhardt (AFCO), Kevin Wilkins (Cambridge), and in the presence of Thomas Kellermann (Warsaw), and Kai-Friederike Oelbermann (Augsburg).

The formal Report of the Cambridge Apportionment Meeting to the Congressional Affairs Committee may be found at [5]. The discussions and recommendations of CAM are summarized in the current article, together with an account of some of the subsequent debate within the Committee. Opinions expressed here are those of the author alone.

1.3. The constraints

Seat allocations are currently required to adhere to the terms of the Treaty of Lisbon.

- Each Member State is to receive a minimum of 6 seats,
- and a maximum of 96 seats,
- Parliament is constrained to have no more than 751 seats in total (including that of the President),
- allocations are required to satisfy a condition of "degressive proportionality".

CAM was advised by the AFCO representatives that the first three constraints are indeed *inequalities* rather than *equalities*, but nevertheless there existed a general expectation in Parliament that its total size should not be less than 751, and that the smallest States should receive an allocation not greater than 6 seats. The issue of "degressive proportionality" is formulated in more detail in Section 2. In reaching its conclusions, the Symposium took into account the following additional observations concerning the general structure of the European Parliament:

- the EU has currently 27 Member States,
- the smallest population (as published officially by Eurostat[1] is currently 412,970, and the largest 81,802,257,
- future accessions may include a number of States with a spread of sizes,
- there will be migration and demographic changes,
- Member States' population figures will be used as input to the formula.

1.4. The criteria

Participants were sensitive in discussions to the three descriptors provided by the AFCO Committee, namely that the "formula" was required to be *durable*, *transparent* and *impartial to politics*.
Durable: A formula that adapts naturally to possible structural changes in the architecture of the EU, arising for example through accessions by new States, through migration, or through demographic shifts.
Transparent: An apportionment method that is capable of simple and reasonable explanation to EU citizens, irrespective of their backgrounds.
Impartial to politics: A principled and fresh approach, unprejudiced with respect to particular Member States or Political Groups, and free of influence from historical positions beyond the constraints of Section 1.3.

1.5. Summary

A discussion of degressive proportionality is to be found in Section 2. Section 3 contains a discussion of the main recommendations of the Cambridge Apportionment Meeting, which are listed explicitly in Section 4. A brief account of the subsequent debate and resolutions of the Committee on Constitutional Affairs is presented in Section 5. This chapter in the story of European Apportionment ends with the shelving of the mathematical approach.

2. Degressive Proportionality

2.1. Lamassoure–Severin definition

Degressive proportionality has been defined in Paragraph 6 of the Lamassoure–Severin (2007) Motion of [8] as follows.

> 6. [The European Parliament] "Considers that the principle of degressive proportionality means that the ratio between the population and the number of seats of each Member State must vary in relation to their respective populations in such a way that each Member from a more populous Member State represents more citizens than each Member from a less populous Member State and conversely, but also that no less populous Member State has more seats than a more populous Member State."

The principle of degressive proportionality attracted significant debate and a major recommendation at CAM.

[1] http://epp.eurostat.ec.europa.eu/

2.2. CAM recommendation

It was noted that *degressive proportionality* comprises two requirements:

1. no smaller State shall receive more seats than a larger State,
2. the ratio population/seats shall increase as population increases.

Condition 1 is easy to accept. Condition 2, on the other hand, poses a serious practical difficulty, and has in addition been violated in recent Parliamentary apportionments. As noted in [9, 10, 11, 12] and elsewhere, there are hypothetical instances of apportionment for which there exists no solution satisfying both Condition 1 and Condition 2. There was an extensive discussion of this issue at CAM, centred on the following two Options.

A. Adopt a method whose outcomes invariably satisfy Condition 2 but with a possibly reduced Parliament-size.

B. Propose a change to the Lamassoure–Severin definition of degressive proportionality lying within existing law and allowing greater flexibility and transparency.

A method satisfying Option A was presented at CAM (and is summarized in [5, Sect. 6.2]). However, CAM preferred Option B on the grounds of transparency of method, and the desirability of achieving a given Parliament-size.

The recommendation of CAM was to amend Paragraph 6 of the Lamassoure–Severin Motion [8] through the addition of the italicized phrase as follows.

6. [The European Parliament] Considers that the principle of degressive proportionality means that the ratio between the population and the number of seats of each Member State *before rounding to whole numbers* must vary in relation to their respective populations in such a way that each Member from a more populous Member State represents more citizens than each Member from a less populous Member State and conversely, but also that no less populous Member State has more seats than a more populous Member State.

3. Cambridge Compromise

3.1. Base+prop method

The 'Cambridge Compromise' recommendation[2] to the European Parliament is to adopt a base+prop system, formulated in [10] as follows.

The *base+prop* method proceeds in two stages. At the first stage, a fixed *base* number of seats is allocated to each Member State. At the second stage,

[2]The Cambridge Compromise proposal is named in harmony with the so-called Jagiellonian Compromise proposal of [13, 15] for voting within the European Commission.

the remaining seats are allocated to States in *prop*ortion to their population-sizes (subject to rounding, and capping at the maximum). In order to achieve the given Parliament-size, one introduces a further ingredient called the *divisor*.

For given *base* b, *maximum* M, and *divisor* d, define the associated *allocation function* $A_d : [0, \infty) \to [0, \infty)$ by

$$A_d(p) = \min\{b + p/d, M\},$$

The base+prop method is formulated as follows in mathematical terms.

1. Assign to a Member State with population p the *seat share* $A_d(p)$,
2. perform a rounding of the seat share $A_d(p)$ into an integer *seat number* $[A_d(p)]$,
3. adjust the divisor d in such a way that the sum of the seat numbers of all Member States equals the given Parliament-size.

The total house-size with divisor d is

$$T(d) = \sum_i [A_d(p_i)],$$

where the summation is over all Member States. The value of d is chosen in such a way that $T(d)$ equals the prescribed total[3].

The CAM recommendation is to use the base $b = 5$, and to use *rounding upwards*. Outcomes of the Cambridge Compromise are presented in Tables 1 and 2, with 2011 population figures taken from the Eurostat website, and with 27, 28, and 29 Member States.

It was through principled discussion that this recommendation was reached; CAM was instructed to overlook historical apportionments, including the *status quo* as listed in Table 2. Participants recognised the challenges that can be presented by change, and these challenges proved formidable for the AFCO Committee (see Section 5).

3.2. Why base+prop?

The CAM participants considered a variety of apportionment schemes based around several different linear and non-linear apportionment functions[4]. Linear functions were preferred over non-linear functions on grounds of transparency and greater potential for proportionality. The dual constraints of maximum and house-size are obstacles to the search for a *smooth* linear apportionment function (that is, a function that is continuously differentiable, say).

[3] There is normally an interval of such d-values, and there are standard approaches to the question of so-called *ties*. See [1], for example, and also Section 3.4.

[4] Note that every non-decreasing concave apportionment function leads invariably to allocations satisfying the revised form of degressive proportionality of Section 2.2.

Non-linear apportionment functions (following a power[5] or parabolic law, for example) can accommodate numerical constraints in a smoother manner. They can be used to fit curves to plots of data points distributed along (possibly concave) lines of trends, such as the current allocations to Member States. On the other hand, they suffer from arbitrariness, and from lack of transparency. The exercise confronting CAM was not one of fitting a curve to historic data, but rather to form a fresh view of apportionment that is impartial to yesterday's politics.

From amongst linear systems, the *base+prop* method stands out for its transparency. It is degressively proportional in an active way, since the base operates to the profit of Member States at the lower end of the population table. CAM considered that its simplicity outweighed the discontinuity in the first derivative that arises currently through the maximum cap of 96 seats. We noted that this discontinuity will diminish as the EU changes its shape through accessions. The recommendation to adopt the base+prop method was reached through consideration of durability, transparency, impartiality, and degressive proportionality.

CAM noted in passing that the base+prop method can be interpreted as one in which the base is an allocation to Member States, and the remaining seats (prop) are proportional to population (subject to capping at the maximum). This resonates with the founding principles of the EU, enshrined in the Treaty, that the Union is made up both of Member States (enjoying equality in international law) and of citizens (enjoying democratic equality).

3.3. Choice of base and rounding method

The choices of base and rounding methods are intertwined. A smaller base tends to favour larger States; rounding *upwards* is usually viewed as tending to favour smaller States. These choices are informed by the existence of a minimum number m of seats per State, and by degressive proportionality.

Let us write b+R to denote the system with base b and rounding method R, where R may denote one of:

U: upwards rounding,

S: standard rounding to the nearest integer,

D: downwards rounding.

We say that the roundings of a real number x are *well defined* if x is not an integer multiple of $\frac{1}{2}$. It was considered preferable, in the interests of transparency, that the base be an integer.

Recall that $m = 6$, and there is an expectation that the smallest States will indeed receive 6 seats. It was therefore natural to concentrate on the two possibilities:

[5] A power-weighted variant of the Cambridge Compromise is analysed in [4].

6+S: base $b = 6$, *standard rounding* (S),

5+U: base $b = 5$, *upwards rounding* (U).

Each of these two systems allocates at least 6 seats to every State. The minimum allocation is however fragile under the first system (6+S), as illustrated in [5, Sect. 5.3] as follows. The currently smallest Member State is Malta, with a population of 412,970, and it receives an allocation of 6 seats under both the above systems. If, however, its population were to increase by only 8,000 (other populations remaining unchanged), its allocation under 6+S rises to 7. This was considered unacceptable, and for this reason CAM recommended 5+U.

There is an explicit trade-off between base and rounding method (see [6, 7, 14]). Let x be a real number, and let $\lfloor\cdot\rfloor$ (respectively, $\lceil\cdot\rceil$, $[\cdot]$) denote rounding downwards (respectively, upwards, and to the nearest integer). For any 'base' b, we have

$$\lceil b + x\rceil = [b + \tfrac{1}{2} + x] = \lfloor b + 1 + x\rfloor,$$

whenever the roundings are well defined. Subject to the last assumption, the three systems b+U, $(b+\frac{1}{2})$+S, $(b+1)$+D result in the same allocations. In this sense, the systems 5+U, $5\frac{1}{2}$+S, 6+D are equivalent.

3.4. Divisors or D'Hondt?

Democracies have extensive experience of voting systems, and a variety of nomenclature has evolved. The following trans-Atlantic translation chart is included here.

rounding	*Europe*	*USA*
downwards	D'Hondt	Jefferson
standard	Sainte-Laguë	Webster
upwards		Adams

The Cambridge Compromise may be reformulated as a system of any of these three types, and we illustrate this with the case of D'Hondt's method. Allocate to every State the minimum m seats (currently $m = 6$). The remaining seats are allocated according to D'Hondt's method subject to the condition that, when any State attains a total of 96 seats, then it receives no further seats. The ensuing allocation is identical to that of the Cambridge Compromise.

The better to aid the reader, we give a brief explanation of the relevant D'Hondt method in the presence of an integral base and maximum. Write B (respectively, M) for the base (respectively, maximum) allocation, and H for the house-size. Let the population-sizes be $p_1, p_2, \ldots, p_n$.

1. At stage 0, allocate B seats to every State. The remaining $R = H - nB$ seats will be allocated sequentially as follows, until none remain.
2. Suppose, at some stage, that State i has been allocated a_i seats in all. Find a State j such that $p_j/(a_j - B + 1)$ is a maximum, and allocate the next seat to this State.

	Member State	*Population*	*Seats*	*Popn/seats before rounding*	*Popn/seats after rounding*
1	Germany	81,802,257	96	852,106.8	852,106.8
2	France	64,714,074	85	770,259.3	*761,342.0*
3	UK	62,008,048	81	768,264.0	765,531.5
4	Italy	60,340,328	79	766,950.8	763,801.6
5	Spain	45,989,016	62	752,036.4	741,758.3
6	Poland	38,167,329	52	739,643.2	733,987.1
7	Romania	21,462,186	32	687,772.5	670,693.3
8	Netherlands	16,574,989	26	656,745.2	637,499.6
9	Greece	11,305,118	19	601,222.1	595,006.2
10	Belgium	10,839,905	19	594,438.5	*570,521.3*
11	Portugal	10,637,713	18	591,356.6	590,984.1
12	Czech Rep.	10,506,813	18	589,315.9	583,711.8
13	Hungary	10,014,324	18	581,298.7	556,351.3
14	Sweden	9,340,682	17	569,380.7	549,451.9
15	Austria	8,375,290	16	550,056.4	523,455.6
16	Bulgaria	7,563,710	15	531,334.8	504,247.3
17	Denmark	5,534,738	12	470,724.2	461,228.2
18	Slovakia	5,424,925	12	466,706.8	452,077.1
19	Finland	5,351,427	12	463,965.8	445,952.2
20	Ireland	4,467,854	11	427,330.9	406,168.5
21	Lithuania	3,329,039	10	367,250.6	332,903.9
22	Latvia	2,248,374	8	290,290.0	281,046.8
23	Slovenia	2,046,976	8	272,953.4	255,872.0
24	Estonia	1,340,127	7	201,939.0	191,446.7
25	Cyprus	803,147	6	134,291.1	133,857.8
26	Luxembourg	502,066	6	89,446.6	83,677.7
27	Malta	412,970	6	75,027.7	68,828.3
	Total	501,103,425	751		

Table 1: Each State receives one non-base seat for every 819,000 citizens or part thereof. Population/seat ratios are strictly decreasing before rounding, but there are two violations after rounding, namely Belgium and France when reading for the bottom. Data in this and the next table are taken from the Eurostat website http://epp.eurostat.ec.europa.eu/.

	Member State	*Population*	*Now*	*Seats 27 States*	*Seats 28 States*	*Seats 29 States*
1	Germany	81,802,257	99	96	96	96
2	France	64,714,074	74	85	83	82
3	UK	62,008,048	73	81	80	79
4	Italy	60,340,328	73	79	78	77
5	Spain	45,989,016	54	62	61	60
6	Poland	38,167,329	51	52	51	51
7	Romania	21,462,186	33	32	31	31
8	Netherlands	16,574,989	26	26	25	25
9	Greece	11,305,118	22	19	19	19
10	Belgium	10,839,905	22	19	18	18
11	Portugal	10,637,713	22	18	18	18
12	Czech Rep.	10,506,813	22	18	18	18
13	Hungary	10,014,324	22	18	17	17
14	Sweden	9,340,682	20	17	17	17
15	Austria	8,375,290	19	16	16	15
16	Bulgaria	7,563,710	18	15	15	14
17	Denmark	5,534,738	13	12	12	12
18	Slovakia	5,424,925	13	12	12	12
19	Finland	5,351,427	13	12	12	12
20	Ireland	4,467,854	12	11	11	11
21	Croatia	4,425,747	–	–	11	11
22	Lithuania	3,329,039	12	10	9	9
23	Latvia	2,248,374	9	8	8	8
24	Slovenia	2,046,976	8	8	8	8
25	Estonia	1,340,127	6	7	7	7
26	Cyprus	803,147	6	6	6	6
27	Luxembourg	502,066	6	6	6	6
28	Malta	412,970	6	6	6	6
29	Iceland	317,630	–	–	–	6
	Total	505,529,172	751	751	751	754

Table 2: The column labelled '27 States' is the Cambridge Compromise with the present European Union. The next two columns include Croatia and Iceland in that order. The divisors are 819,000 (27 States), 835,000 (28 States), 844,000 (29 States).

3. Repeat the previous step until no seats remain, subject to the condition that any State achieving the maximum number M of seats is removed from the process.

It may be checked that the outcome agrees with the system B+D, which was shown in Section 3.3 to be equivalent to the Cambridge Compromise with base $b = B - 1$. Similar algorithms are of course valid for the Sainte-Laguë and Adams methods.

Ties can occur in the above algorithm, and these correspond to the non-existence of a divisor for some house-size in the formulation of Section 3.1. There are standard ways of breaking ties by casting lots. However, ties are very unlikely to occur in instances of the EU apportionment problem since populations are large and varied. Indeed, subject to a reasonable probabilistic model for population-sizes, the probability of a tie may estimated rigorously.

For further reading, see [6], or perhaps [1, p. 99],

3.5. Choosing the minimum and maximum

The better to understand the role of the minimum, CAM discussed how the minimum and base could be reduced as further States accede to the Union. No final recommendation was reached but two Schemes emerged.

In Scheme A, a cap is introduced on the proportion of seats allocated via the minimum, and the value of the minimum is taken as large as possible subject to this cap. For example, there are currently $27 \times 6 = 162$ seats allocated thus, a proportion of approximately 22%. If, for example, one caps this at 25%, the minimum remains at 6 for a larger Union of 27–31 States, and is reduced to 5 for 32–37 States, and so on. The base b might either be one fewer than the minimum (with rounding upwards), or might follow a rule of the type: b is the smallest fraction such that the smallest State receives exactly the minimum number of seats (with rounding upwards, say).

In Scheme B, one determines the base as a function of the number n of States, and current practice indicates a formula of the type $b = 135/n$. This has the advantage of decreasing steadily as n increases. However, the associated *minimum* decreases in a manner that is sensitive to the smallest population.

Since each State receives by necessity an integral number of seats, one effect of the allocation of seats to new States is a notable lumpiness at the upper end of the population chart. With the minimum held constant, the seats granted to an acceding State are taken from other States in proportion to their populations, and thus mostly from the larger States. Conversely, any adjustment downwards in the minimum allocation releases seats for proportional distribution between the States, of which the largest States gain most.

CAM recommended that consideration be given to the manner in which the minimum allocation should vary in the light of changes to the European Union, and also that the functioning of the maximum allocation be reviewed prior to future apportionments.

3.6. Population statistics

Census data is key to the allocation of seats in the European Parliament. Such population data is usually collected only once a decade. Both the year of the census and the manner of updating can vary between countries. In addition, there can be national variation in the definition of a resident. CAM's final recommendation was that the European Commission be encouraged to ensure that Eurostat review the methods used across the Union.

4. Summary of Recommendations

Principal recommendations

1. Adopt the revised definition of degressive proportionality proposed in Section 2.2 above.
2. For future apportionments of the European Parliament, the method *base+prop* should be employed.
3. The base should be 5, and fractions should be rounded upwards.

Further recommendations

A. Due consideration should be given to the manner in which the minimum, currently 6, and base should vary in the light of future changes in the number of Member States in the European Union.

B. The European Parliament should review the manner of functioning of the maximum constraint on number of seats, currently 96, prior to future apportionments.

C. The Commission should be encouraged to ensure that Eurostat reviews the methods used by Member States in calculating their current populations, in order to ensure accuracy and consistency.

5. Debate in the AFCO Committee

The timetable of discussion in Brussels was as follows. In advance of completion of the final CAM Report, the author was invited (as Director of CAM) to deliver a preview to the Committee on Constitutional Affairs (AFCO) in Brussels on 7 February 2011. There was a Committee discussion on 15 March. The Rapporteur, Andrew Duff, tabled a proposal "for a modification of the Act concerning the election of the Members of the European Parliament by direct universal suffrage of 20 September 1976", and this was the subject of amendments by Committee members, leading in turn to a set of so-called "Compromise Amendments" from the Rapporteur[6]. A vote was taken on 19 April 2011.

[6] Video recordings of the two meetings may be found at http://tinyurl.com/5s63d8r. Versions of the proposals and amendments may be consulted at http://tinyurl.com/6bzedza.

The initial responses of Committee members to the CAM recommendation varied between curiosity verging on support, a desire for clarification, simple misunderstanding, and downright opposition. Several members expressed dismay at the "political" challenges of such a reorganization, and everyone was doubtless sensitive to the needs of Member States, Political Groups, and individual Members of the European Parliament. Amongst the issues that stimulated some MEPs were the changes in allocations to Member States with populations in the 7–11 million range, and the claim by one MEP of unfair treatment of the largest Member State.

The five week intermission between the two Committee meetings permitted a period of reflection and analysis, and contributions at the second meeting were generally more refined. There was some agreement in principle on the desirability of a formulaic approach to apportionment, but only one speaker (apart from the Rapporteur) spoke in support of the Cambridge Compromise. Representatives of several medium-sized countries were particularly implacable.

Committee members tabled 138 amendments to the Rapporteur's Proposal for a modification of the relevant Act. The final three were proposals to employ, respectively, the Cambridge Compromise, a parabolic method, and a power method. These three amendments were not destined to survive the vote, presumably as the consequences of formulaic approaches became clearer to some members of the Committee and of Parliament.

Two of the Rapporteur's twelve "Compromise Amendments" were directly relevant to the Cambridge Compromise. Amendment B proposed a formal definition of degressive proportionality along the lines of Section 2.2, while withdrawing the proposal to adopt a specific mathematical approach. Amendment F compressed the discussion of a "mathematical formula" as follows:

> [The European Parliament] "Proposes to enter into a dialogue with the European Council to explore the possibility of reaching agreement on a *durable*[7] and *transparent* mathematical formula for the apportionment of seats in the Parliament respecting the criteria laid down in the Treaties and the principles of plurality between political parties and solidarity among States."

These Compromise Amendments were agreed by the Committee on 19 April 2011, and the amended Proposal was duly carried.

It is not the purpose of this paper to speculate about the reasons for the unenthusiastic response of the AFCO Committee to this proposal in particular, and to formulaic approaches in general. Change can be tricky to manage and to explain to electorates, especially fundamental change requiring unanimity across EU Member States and affecting the livelihoods and ambitions of individual MEPs. The current allocations give preferential treatment to citizens

[7] Italics by the current author. Recall the *three* criteria of Section 1.4; the criterion of "impartiality" has been omitted.

of medium-sized States at the expense of those of larger States. The tentacles of the Political Groups entangle the EU, and alliances harness power and can frustrate change.

There is also the problem of the largest State. According to the Treaty of Lisbon, no State shall receive more than 96 seats, whereas an uncapped allocation would currently give a greater number to Germany. This feature of Parliamentary structure is illuminated baldly by the Cambridge Compromise using current population figures (the prominence of this cap will fade as the EU is enlarged).

It was argued by some MEPs that, in preferring a linear system, CAM had misunderstood the meaning of "degressive proportionality". Such critics considered that CAM should have designed a formula to reproduce the current profile of Parliament. Not only is this contrary to the terms of reference received from the AFCO Committee, but also the author believes that mathematics is best not used as a tool to legitimize blatantly political deals.

The argument provides, however, a clue as to why formulaic approaches were disfavoured in the vote. Calculations indicate that, as the number of Member States increases, the allocations of many formulaic systems approach the simple linearity of the Cambridge Compromise. For example, with 29 States (including Croatia and Iceland) the allocations of both the parabolic and power methods differ only very slightly from that of the Cambridge formula. It seems that the mid-range bulge can be preserved only through "political bartering", and that the discussion of this paper will resurface in the future.

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References

References

[1] M. Balinski and H. P. Young, *Fair Representation*, Brookings Institution Press, Washington DC, 2001.

[2] A. Duff, *The balance of power in a post-national democracy*, (2011), this volume.

[3] ———, *Mathematical Symposium, Cambridge, 28–29 January 2011*, (2011), European Parliament.

[4] G. R. Grimmett, K.-F. Oelbermann, and F. Pukelsheim, *A power-weighted variant of the EU27 Cambridge Compromise*, (2011), this volume.

[5] G. R. Grimmett, with J.-F. Laslier, F. Pukelsheim, V. Ramírez González, R. Rose, W. Słomczyński, M. Zachariasen, and K. Życzkowski, *The allocation between the EU Member States of the seats in the European Parliament*, European Parliament Studies PE 432.760 (2011), http://tinyurl.com/6f3ff6u.

[6] S. Janson, *Asymptotic bias of some election methods*, (2011).

[7] ———, *Proportional election methods*, (2011), in preparation.

[8] A. Lamassoure and A. Severin, *European Parliament Resolution on "Proposal to amend the Treaty provisions concerning the composition of the European Parliament"*, (2007), adopted on 11 October 2007 (INI/2007/2169).

[9] J. Martínez-Aroza and V. Ramírez-González, *Several methods for degressively proportional allocations. A case study*, Mathematical and Computer Modelling **48** (2008), 1439–1445.

[10] F. Pukelsheim, *Putting citizens first: Representation and power in the European Union*, Institutional Design and Voting Power in the European Union (M. Cichocki and K. Życzkowski, eds.), Ashgate, London, 2010, pp. 235–253.

[11] V. Ramírez González, *Degressive proportionality. Composition of the European Parliament. The parabolic method*, Institutional Design and Voting Power in the European Union (M. Cichocki and K. Życzkowski, eds.), Ashgate, London, 2010, pp. 215–234.

[12] V. Ramírez González, A. Palomares, and M. L. Márquez, *Degressively proportional methods for the allotment of the European Parliament seats amongst the EU Member States*, Mathematics and Democracy, Recent Advances in Voting Systems and Collective Choice (B. Simeone and F. Pukelsheim, eds.), Springer, Berlin, 2006, pp. 205–220.

[13] W. Słomczyński and K. Życzkowski, *Jagiellonian Compromise: an alternative voting system for the Council of the European Union*, Institutional Design and Voting Power in the European Union (M. Cichocki and K. Życzkowski, eds.), Ashgate, London, 2010, pp. 43–58.

[14] M. Zachariasen, *On the relationship between base and rounding-method in divisor-based apportionment*, (2011), Working Note.

[15] K. Życzkowski and W. Słomczyński, *Voting in the European Union: The square-root system of Penrose and a critical point*, (2004), http://arxiv.org/abs/cond-mat/0405396.