RaGe Conference 2017 – abstracts

David Cimasoni (Geneva)
Graphs and coverings
Given a finite covering G' of a finite weighted graph G, one can show that any adjacency-type operator on G' (e.g. the discrete Laplace or Kasteleyn operator) is conjugate to the corresponding operator on G twisted by the appropriate representation of the fundamental group of G. When this operator can be used to compute the partition function of a statistical physics model (e.g. the spanning tree or dimer model), this fact implies a number of results, some known, others new. Joint work in progress with Adrien Kassel.

Antoine Dahlqvist (Cambridge)
Yang–Mills measure and the master field on the sphere
We shall discuss about the behavior of the Yang-Mills measure on the sphere for large unitary groups. The observables of this random geometry model are given by traces of some unitary random matrices. As the size of the matrices goes to infinity, we show that the observables concentrate around a limiting field called master field. We shall explain here how a remarkable set of equations — named after the physicists Makeenko and Migdal — allows to obtain this result from the understanding of a particle system on the circle. This study uses a duality of these models with the one dimensional discrete Coulomb gas. All of them undergo a phase transition that we shall describe with these different points of view. This talk is based on a joint work with J. R. Norris.

Adrien Kassel (Lyon)
Some integrable models with gauge symmetry
We define several natural stochastic processes associated to a vector bundle on a graph endowed with a unitary connection. These may serve as a tool to study exactly solvable statistical physics models in the presence of a gauge interaction field. In particular, we present a relation between a covariant Gaussian free field and the occupation time field of a loop soup whose intensity is twisted by the connection. This result generalizes classical isomorphisms of Dynkin, Eisenbaum, Le Jan, and Sznitman. Joint work with Thierry Lévy.

Richard Kenyon (Brown)
Limit shapes beyond dimers
We consider a model of interacting dimers, generalizing the lozenge tiling model. We obtain expressions for the free energy and surface tension, as well as analogs of the "complex Burgers equation" describing limit shapes. The model has some novel phases not found in the dimer model, in particular a phase with anomalous height fluctuations.

Richard Kenyon (Brown)
Tilings with prescribed shapes
In 2007 Oded Schramm extended the Koebe-Andreev-Thurston Circle Packing Theorem to packings with arbitrary smooth convex shapes. We prove an analogous result for tilings with convex polygons. This is based on previous joint work with Scott Sheffield.

Benoît Laslier (Paris 7)

Hydrodynamic limit for a lozenge tiling reversible dynamics"
Lozenge tilings are a natural model of 2+1 dimensional interface that arises for example as the 0 temperature limit of interfaces in 3D Ising model. It is generally expected that microscopic dynamics on these types of models should exhibit a deterministic behaviour at large scale. For reversible dynamics, one expect more precisely to see a kind of mean curvature motion at a diffusive space-time scaling, with two sources of non-linearity : one related to a choice of the appropriate notion of curvature depending only on the equilibrium measure and a kind of diffusion coefficient depending on the choice of dynamics (called the mobility). We prove that this picture indeed holds for a well chosen dynamics on lozenge tilings. The discrete dynamics is chosen in such a way that it displays some form of gradient condition but the mobility is non-trivial. This is joint work with Fabio Toninelli.

Thierry Levy (Paris 6)
Hydrodynamic limit for a lozenge tiling reversible dynamics
(Abstract to follow)

Zhongyang Li (Connecticut)
Phase transitions in the 1−2 model
A configuration in the 1-2 model is a subgraph of the hexagonal lattice, in which each vertex is incident to 1 or 2 edges. By assigning weights to configurations at each vertex, we can define a family of probability measures on the space of these configurations, such that the probability of a configuration is proportional to the product of weights of configurations at vertices. We study the phase transition of the model by investigating the probability measures with varying weights. We explicitly identify the critical weights, in the sense that the edge-edge correlation decays to 0 exponentially in the subcritical case, and converges to a non-zero constant in the supercritical case, under the limit measure obtained from torus approximation. These results are obtained by a novel measure-preserving correspondence between configurations in the 1-2 model and perfect matchings on a decorated graph, which appears to be a more efficient way to solve the model, compared to the holographic algorithm used by computer scientists to study the model. When the weights are uniform, we prove a weak mixing property for the finite-volume measures - this implies the uniqueness of the infinite-volume measure and the fast mixing of a Markov chain Monte Carlo sampling. The major difficulty here is the absence of stochastic monotonicity.

Marcin Lis (Cambridge)
On the double random current nesting field
A configuration of the planar random current model can be viewed as a collection of dual Ising contours together with an independent Bernoulli bond percolation with prescribed success probabilities. The double random current model is simply a superimposition of two iid random current configurations. Its clusters are composed of XOR-Ising contours and of additional components arising from the percolation process or two overlapping single Ising contours. For each such cluster C we toss an independent ±1 symmetric coin X_C. A cluster C is called *odd* around a face u if the contours contained in C assign spin −1 to u under +1 boundary conditions. The double random current nesting field at u is defined to be the sum of X_C over clusters C odd around u. I will provide a measure-preserving mapping between double currents and dimers on a particular bipartite graph. Under this map the nesting field becomes the height function of the dimer model. Using this

connection together with the results of Kenyon, Okounkov and Sheffield on the dimer model, I will prove that the magnetization of the critical Ising model on any biperiodic graph vanishes. This is joint work with Hugo Duminil-Copin.

Jason Miller (Cambridge)
Convergence of percolation on uniform quadrangulations
Let Q be a uniformly random quadrangulation with simple boundary decorated by a critical (p=3/4) face percolation configuration. We prove that the chordal percolation exploration path on Q between two marked boundary edges converges in the scaling limit to SLE(6) on the Brownian disk (equivalently, a Liouville quantum gravity surface). The topology of convergence is the Gromov-Hausdorff-Prokhorov-uniform topology, the natural analog of the Gromov-Hausdorff topology for curve-decorated metric measure spaces. Our method of proof is robust and, up to certain technical steps, extends to any percolation model on a random planar map which can be explored via peeling. Joint work with E. Gwynne.

Gourab Ray (Cambridge)
Universality of height fluctuation of dimer model on Riemann surfaces
We investigate the fluctuation of the height function of the dimer model on a Temperleyan graph embedded on a Riemann surface and satisfying an invariance principle and other mild assumptions. We prove in the case of the torus and the annulus that the fluctuations converge to a limit which is locally a Gaussian free field. Furthermore, this convergence is universal (independent of the underlying graph). This answers a question of Dubedat and Gheissari, and extends work of Dubedat in the case of the torus under an additional assumption of isoradiality. I will also outline ideas and difficulties from a work in progress on how to extend these ideas to general Riemann surfaces. The proof goes through obtaining a universal scaling limit of the cycle rooted spanning forest, which is of independent interest. This part of the argument actually holds on general Riemann surfaces and this extends the work of Kenyon and Kassel.

Marianna Russkikh (Geneva)
Playing dominos in different domains
We will discuss an extension of several results of Richard Kenyon on the dimer model. In 1999 he has shown that fluctuations of the height function of a random dimer tiling on Temperley discretizations of a planar domain converge in the scaling limit to the Gaussian Free Field with Dirichlet boundary conditions. We will discuss an extension of this result to other classes of discretizations. In particular, we will discuss boundary conditions of the coupling function in the so-called "even" domains. Interestingly enough, in this case the coupling function satisfies the same Riemann-type boundary conditions as fermionic observables in the Ising model. The main tool is a factorization of the gradient of the expectation of the height function in the double-dimer model into a product of two discrete holomorphic functions. In particular, we use this factorization to show that, rather surprisingly, the expectation of the double-dimer height function in the Temperley case is exactly discrete harmonic even before taking the scaling limit.

Scott Sheffield (MIT)
Tutte embeddings of the mated-CRT map converge to Liouville quantum gravity

I will argue that the Tutte embeddings (a.k.a.harmonic/barycentric embedding) of certain random planar maps converge to Liouville quantum gravity (LQG). Specifically, I will treat mated-CRT maps, which are discrete versions of the mating of two correlated continuum random trees. They can also be understood as random triangulations "decorated" by labeled space-filling paths of a particular kind. I will argue that the measure induced by the embedding converges to a Liouville quantum gravity measure, that the space-filling path scales to space-filling SLE in the annealed sense, and that simple random walk on the map scales to Brownian motion in the quenched sense. This is joint work with Ewain Gwynne and Jason Miller.

Vittoria Silvestri (Cambridge)
The reset time of Internal DLA
Consider Internal DLA on cylinder graphs of the form GxZ. How does a large cluster typically look like? How long does it take for the process to forget its initial profile? In this talk I will address these questions, explaining how the answer depends on the mixing properties of the base graph G. Joint work with Lionel Levine.

Alan Sola (Stockholm)
Singular scaling limits in Laplacian random growth models
I will report on ongoing joint work with Amanda Turner (Lancaster) and Fredrik Viklund (KTH) on a class of conformal aggregation models where growing structures collapse to slits in a small-particle limit.

Wei Qian (Cambridge)
Coupling the Gaussian Free Fields with free and with zero boundary conditions
(Abstract to follow)