Financial networks and epidemics

Discussion

CFAP meeting – Cambridge

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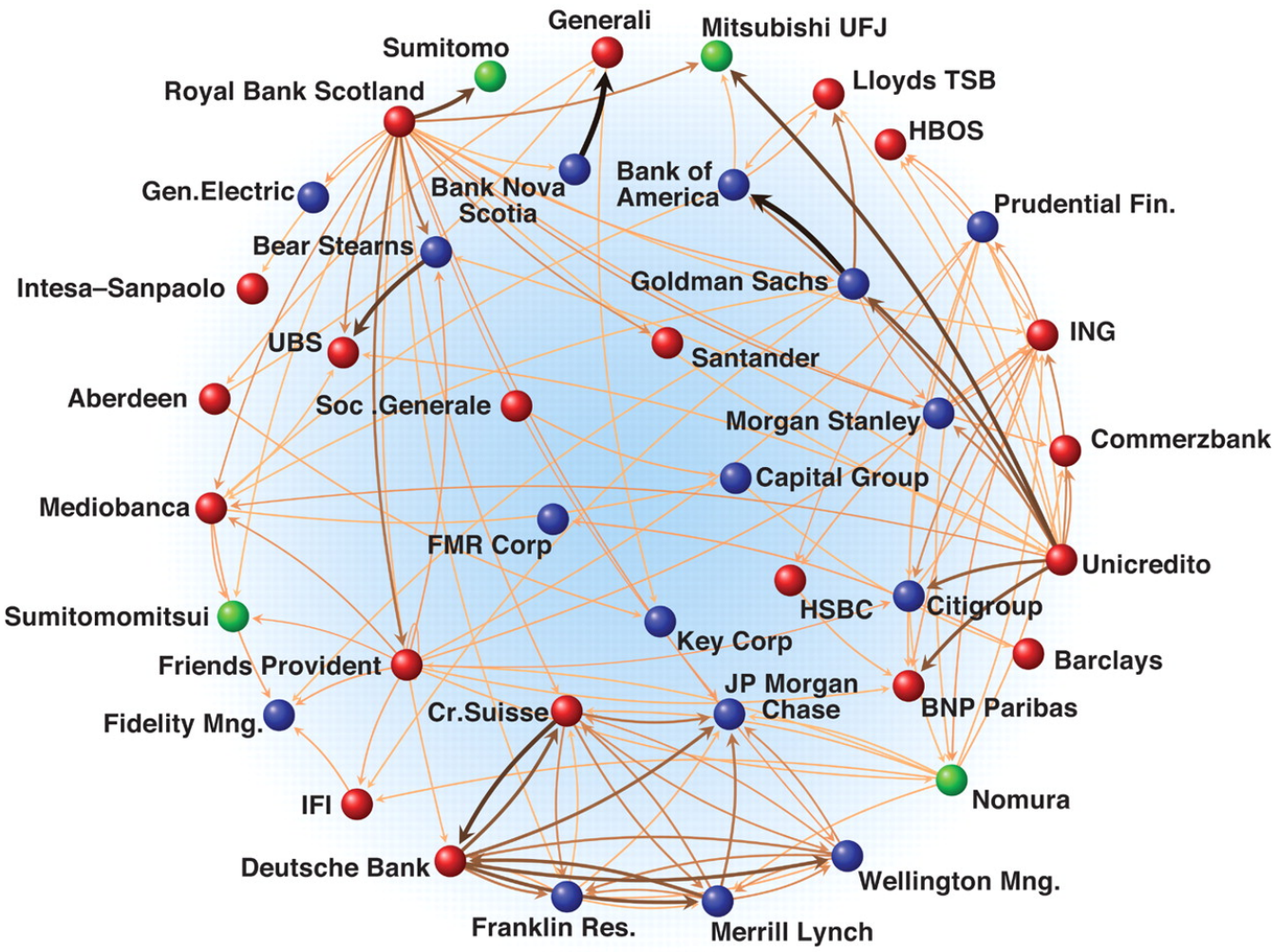
Outline

- Financial networks
- Resource pooling in communication networks
- Open questions on resource pooling

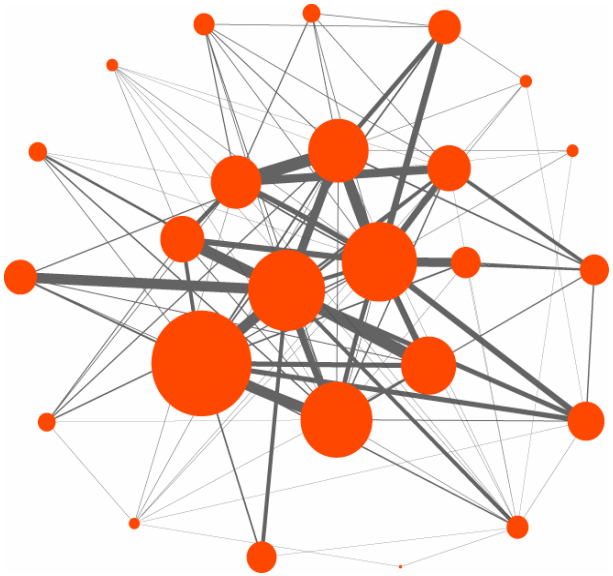
Fig. 2 A sample of the international financial network, where the nodes represent major financial institutions and the links are both directed and weighted and represent the strongest existing relations among them



F. Schweitzer et al., Science 325, 422 -425 (2009)

Published by AAAS

Chart 3.2 Network of large exposures between UK banks(a)(b)(c)



Source: FSA returns.

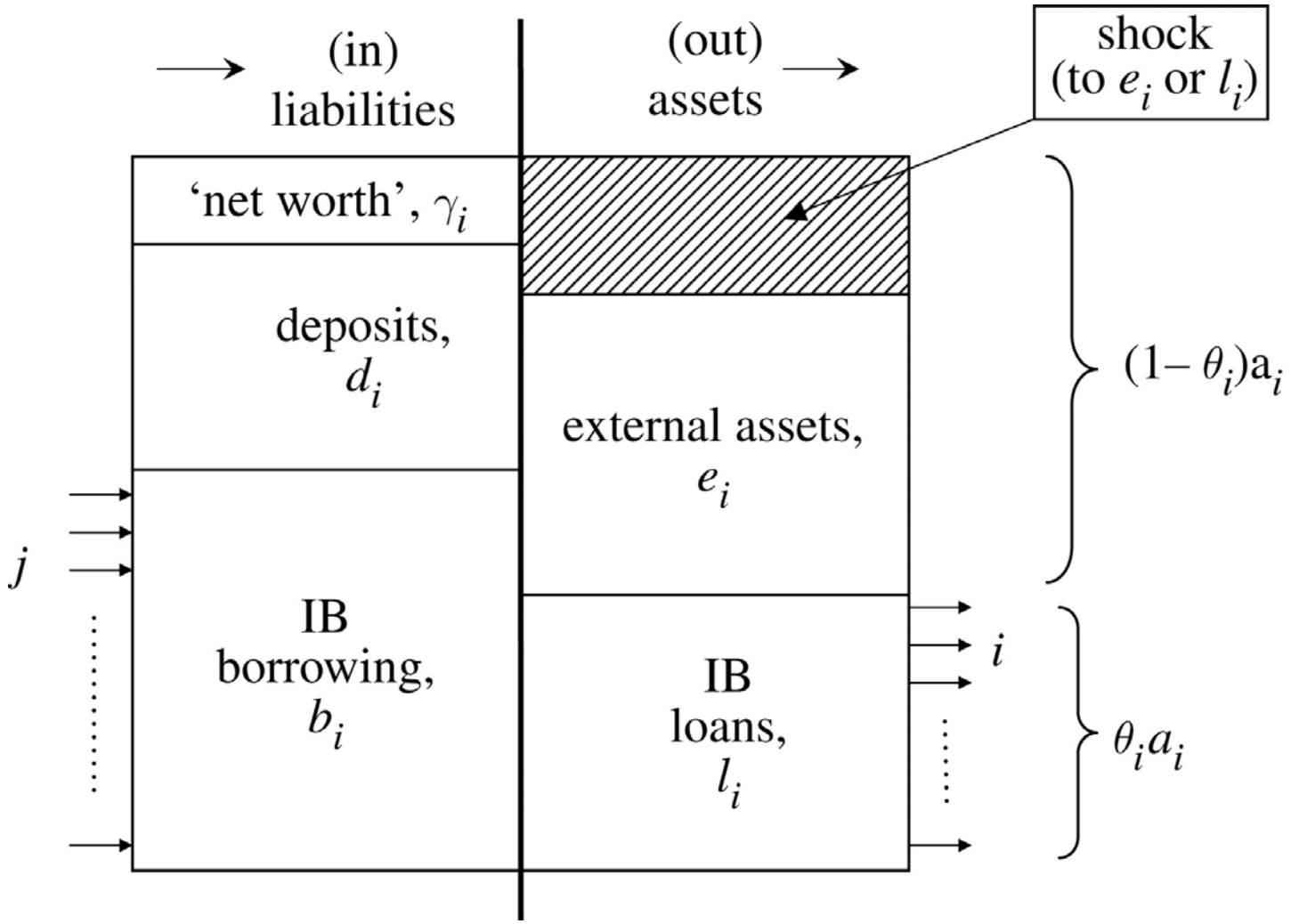
(a) A large exposure is one that exceeds 10% of a lending bank's eligible capital at the end of a period. Eligible capital is defined as Tier 1 plus Tier 2 capital, minus regulatory deductions.

(b) Each node represents a bank in the United Kingdom. The size of each node is scaled in proportion to the sum of (1) the total value of exposures to a bank, and (2) the total value of exposures of the bank to others in the network. The thickness of the line is proportional to the value of a single bilateral exposure.

(c) Based on 2009 Q2 data.

http://www.bankofengland.co.uk/financialstability/

Schematic model for a ‘node’ in the IB network.



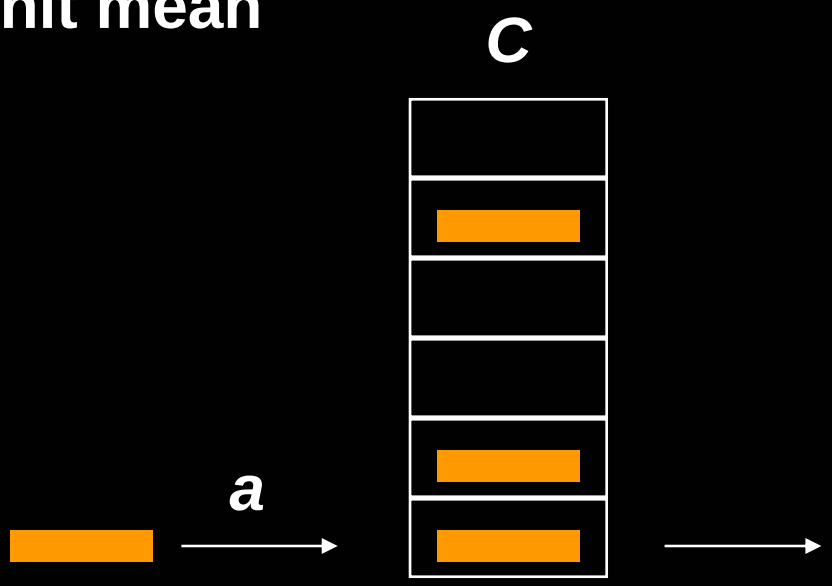
May R M , Arinaminpathy N J. R. Soc. Interface
doi:10.1098/rsif.2009.0359

JOURNAL OF THE ROYAL SOCIETY Interface

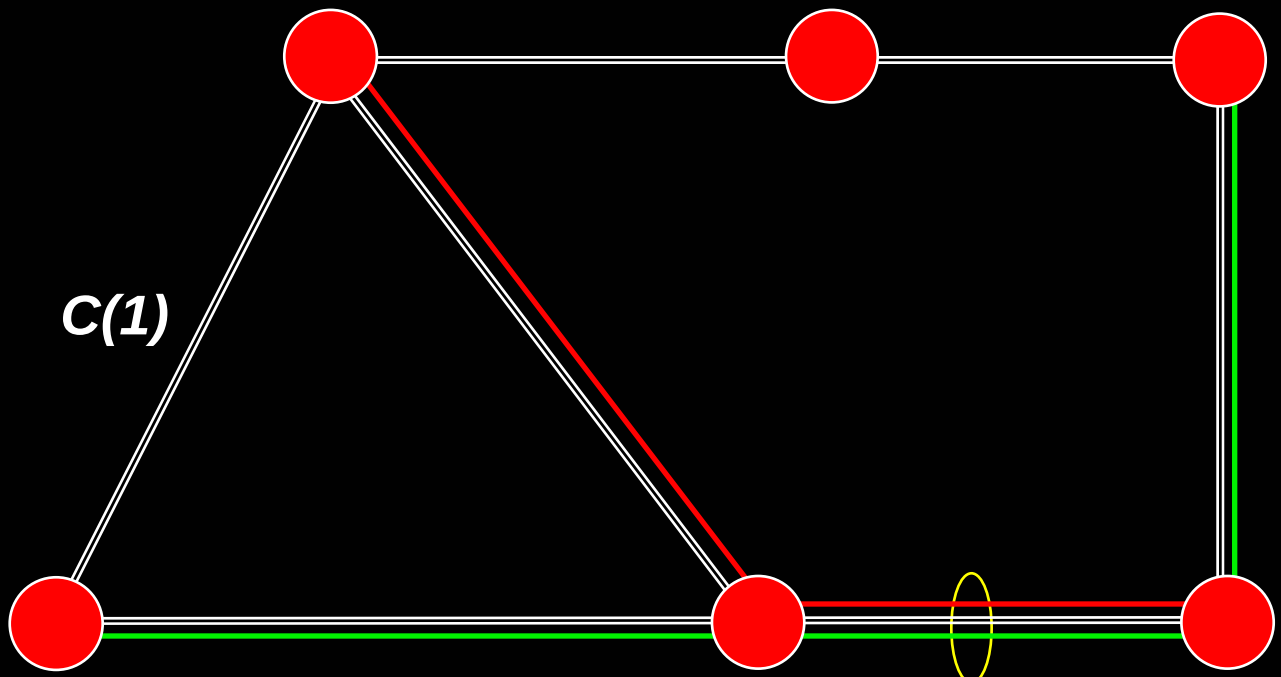
Erlang's formula

- **calls arrive randomly, at rate *a***
- **resource has *C* circuits**
- **accepted calls hold a circuit for a random holding time, with unit mean**
- **blocked calls are lost**
- **proportion of calls lost is:**

$$E(a, C) = \frac{a^C / C!}{\sum_0^C (a^n / n!)}$$



A loss network



Link constraint: $\sum_r A(j,r)\, n(r) \le C(j)$

Resource pooling

Aims:

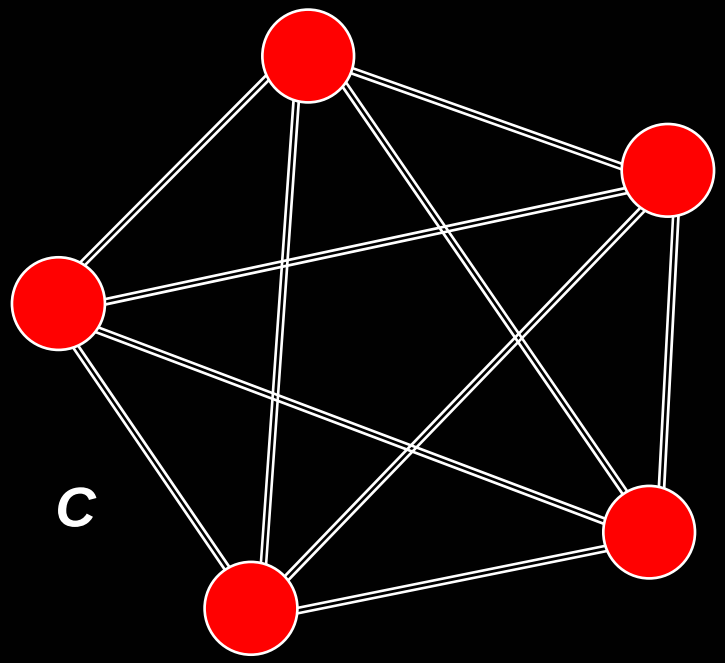
- respond robustly to failures and overloads
- lessen the impact of forecasting errors
- make use of spare capacity in the network
- permit flexible use of network resources

Problems:

- instability
- complexity

Example: alternative routing

- Complete graph
- All links have capacity C
- Call routed directly if possible; otherwise one randomly chosen alternative route may be tried



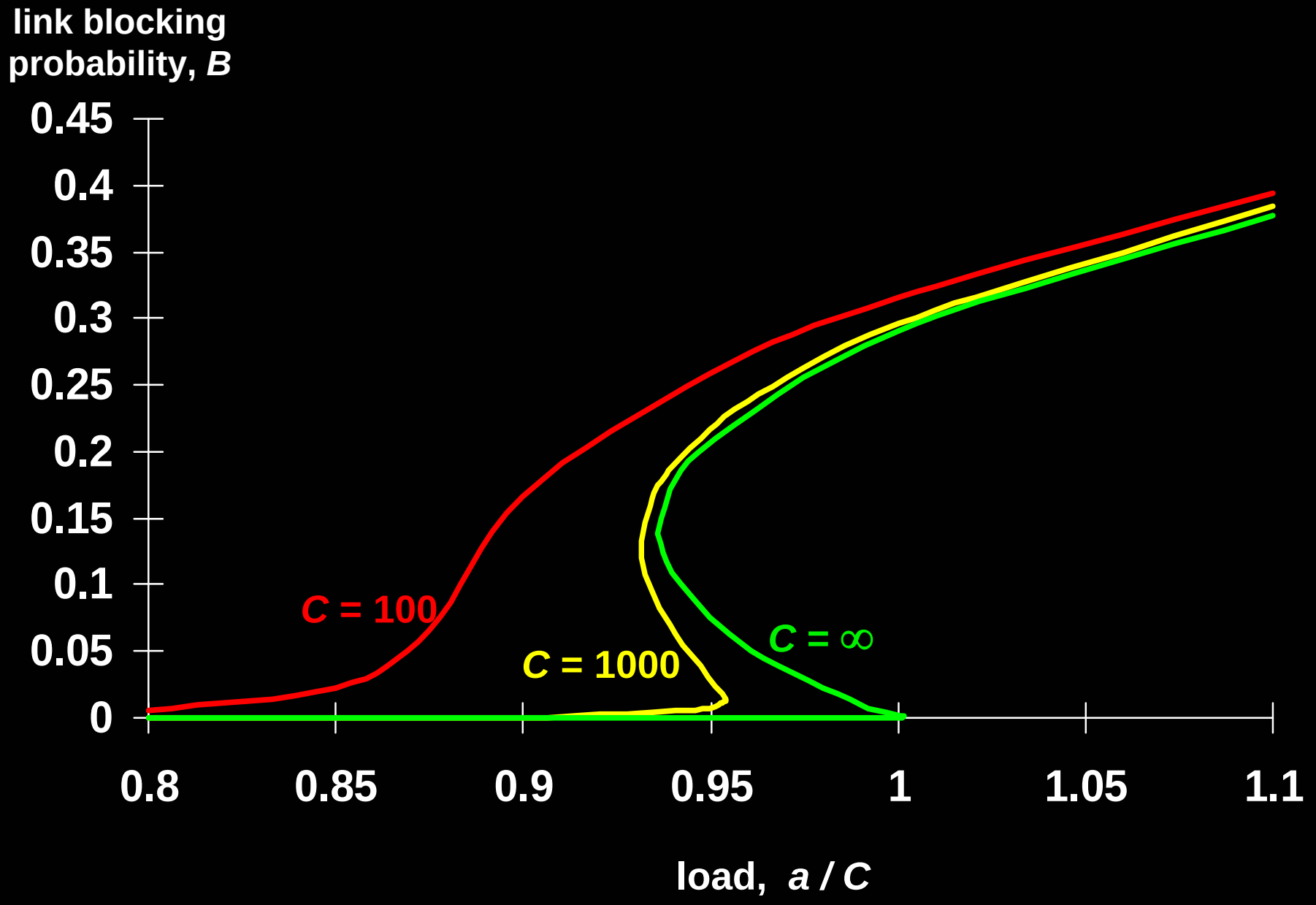
Marbukh 1984, Gibbens, Hunt, K 1990,
Crametz, Hunt 1991, Graham, Méléard 1993, 1994

alternative routing

- Arrival rate per link - a
- Capacity per link - C
- Let B be the link blocking probability
- Then as the number of nodes grows, the blocking probability B approaches a solution of:

$$B = \mathrm{E}(\mathrm{a}[1 + 2B(1 - B)], C)$$

instability, and hysteresis



Open questions on resource pooling

- Resource pooling does indeed
  - respond robustly to failures and overloads
  - lessen the impact of forecasting errors
  - make use of spare capacity in the network
  - permit flexible use of network resources
- But
  - can produce phase transitions if load amplified
  - obscures the approach of capacity overload
- Can decentralised control take account of system-wide risks?