

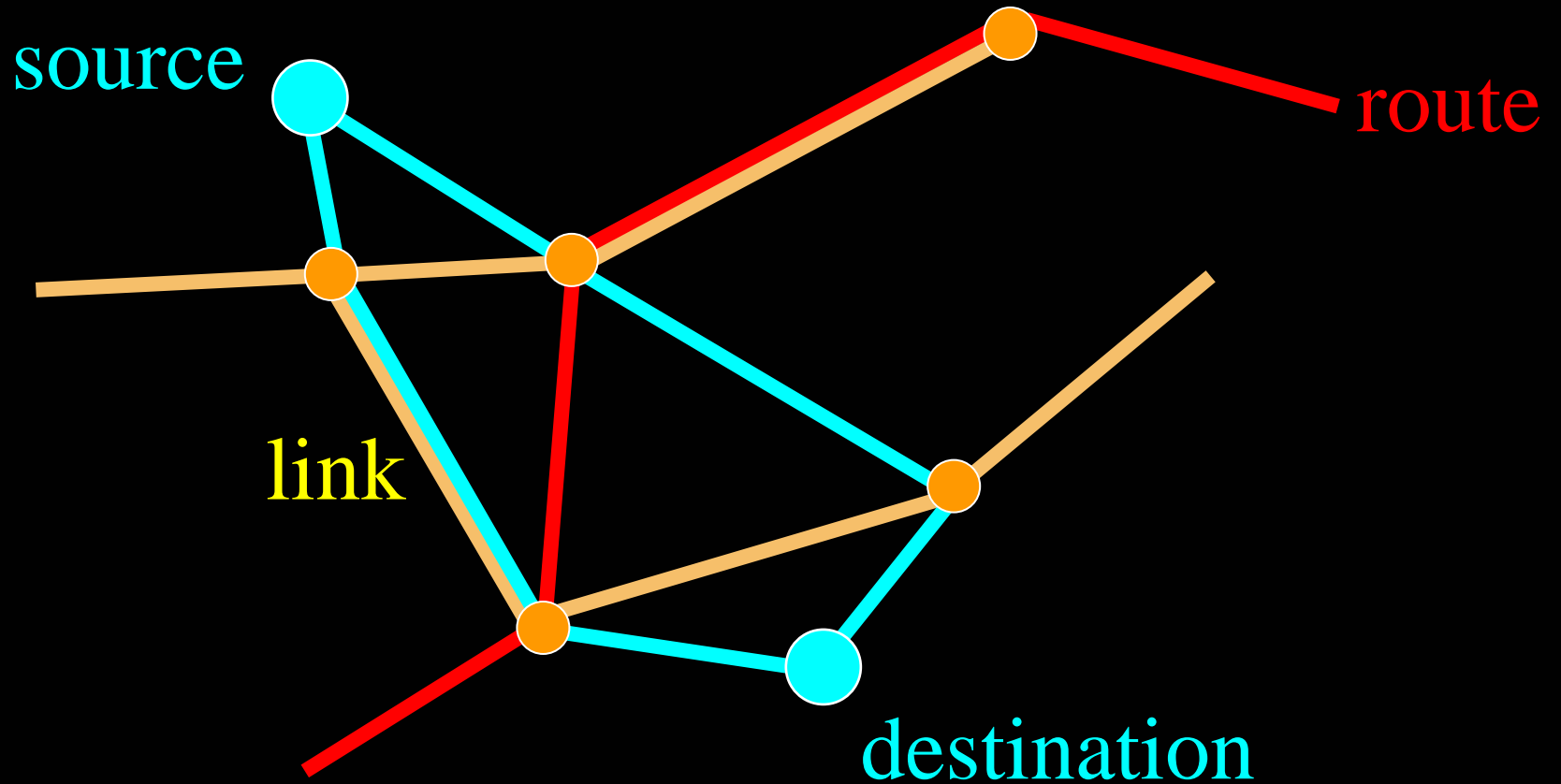
Challenges of road pricing

Frank Kelly

www.statslab.cam.ac.uk/~frank/TALKS

- Basic theory
- History
- Background trends
- Technology
 - components, systems, privacy
- Network modelling issues

Network structure



Notation

J	- set of links
R	- set of routes
$j \in r$	- link j is on route r
S	- set of source-destination pairs
$r \in S$	- route r serves s-d pair s
x_r	- flow rate on route r
y_j	- flow through link j
$D_j(y_j)$	- delay on link j
f_s	- flow between s-d pair s

$$y = Ax \quad f = Hx$$

Wardrop equilibrium

x is a *Wardrop equilibrium* if for every pair of routes, r, r' , serving the same s-d pair

$$x_r > 0 \quad \Rightarrow \quad \sum_{j \in r} D_j(y_j) \leq \sum_{j \in r'} D_j(y_j)$$

If a route r is actively used, then it achieves the minimum delay over all routes serving its s-d pair $s(r)$.
Wardrop, 1952.

Optimization formulation

A Wardrop equilibrium solves the problem:

$$\begin{array}{ll}\text{minimize} & \sum_{j \in J} \int_0^{y_j} D_j(u) du \\ \text{over} & x \geq 0, \quad y \\ \text{subject to} & Hx = f, \quad Ax = y\end{array}$$

Beckmann, McGuire and Winsten, 1956

Let

$$L(x, y; \lambda, \mu) = \sum_{j \in J} \int_0^{y_j} D_j(u) du + \lambda(f - Hx) - \mu(y - Ax)$$

Then

$$\frac{\partial L}{\partial y_j} = D_j(y_j) - \mu_j$$

$$\frac{\partial L}{\partial x_r} = -\lambda_{s(r)} + \sum_{j \in r} \mu_j$$

Thus L minimized when

$$\mu_j = D_j(y_j)$$

$$\lambda_{s(r)} = \sum_{j \in r} \mu_j \quad \text{if} \quad x_r > 0$$

$$\leq \sum_{j \in r} \mu_j \quad \text{if} \quad x_r = 0$$

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minimum delay
over routes
serving $s(r)$

$$\lambda_{s(r)} = \sum_{j \in r} \mu_j \quad \text{if} \quad x_r > 0$$
$$\leq \sum_{j \in r} \mu_j \quad \text{if} \quad x_r = 0$$

Preferred optimization formulation

$$\begin{array}{ll}\text{minimize} & \sum_{j \in J} y_j D_j(y_j) \\ \text{over} & x \geq 0, \quad y \\ \text{subject to} & Hx = f, \quad Ax = y\end{array}$$

Minimum occurs when

$$\begin{aligned}\mu_j &= D_j(y_j) + y_j D'_j(y_j) \\ \lambda_{s(r)} &= \sum_{j \in r} \mu_j \quad \text{if} \quad x_r > 0 \\ &\leq \sum_{j \in r} \mu_j \quad \text{if} \quad x_r = 0\end{aligned}$$

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Minimum occurs when

$$\begin{aligned}\mu_j &= D_j(y_j) + y_j D'(y_j) \\ \lambda_{s(r)} &= \sum_{j \in r} \mu_j \quad \text{if } x_r > 0 \\ &\leq \sum_{j \in r} \mu_j \quad \text{if } x_r = 0\end{aligned}$$

traffic dependent
toll on link j

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History

- Pigou, Knight 1920s
- Wardrop, Beckmann 1950s
- Vickrey, Walter 1960s
- Smeed Report, MoT 1964

Road Pricing: The Economic and Technical Possibilities

History

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- Wardrop, Beckmann 1950s
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Road Pricing: The Economic and Technical Possibilities

- Transport White Paper, 1966: “Road pricing – a metering system to charge directly for the use of congested roads – is from the economic point of view, the most obvious solution to this problem”

Vickrey: Washington (1959) and UK (1960s)

Vickrey not only argued the case for a system of differential prices for roads but provided the details of equipment needed to implement it. ... He described the in-vehicle unit as a “a self-contained, passive response block which will provide a unique signal identifying any object moving on the ground to which it is attached”. The “response blocks” were to be identified by in-pavement equipment he called “electronic interrogators.” The data would be transmitted to a computer which would calculate the charges and make up bills to be sent to vehicle owners. The peak period charges envisioned by Vickrey were about \$1 to \$2 per trip. The total cost of the equipment for doing road pricing in the Washington metro area was estimated to be about \$60 million.

Modes of transport

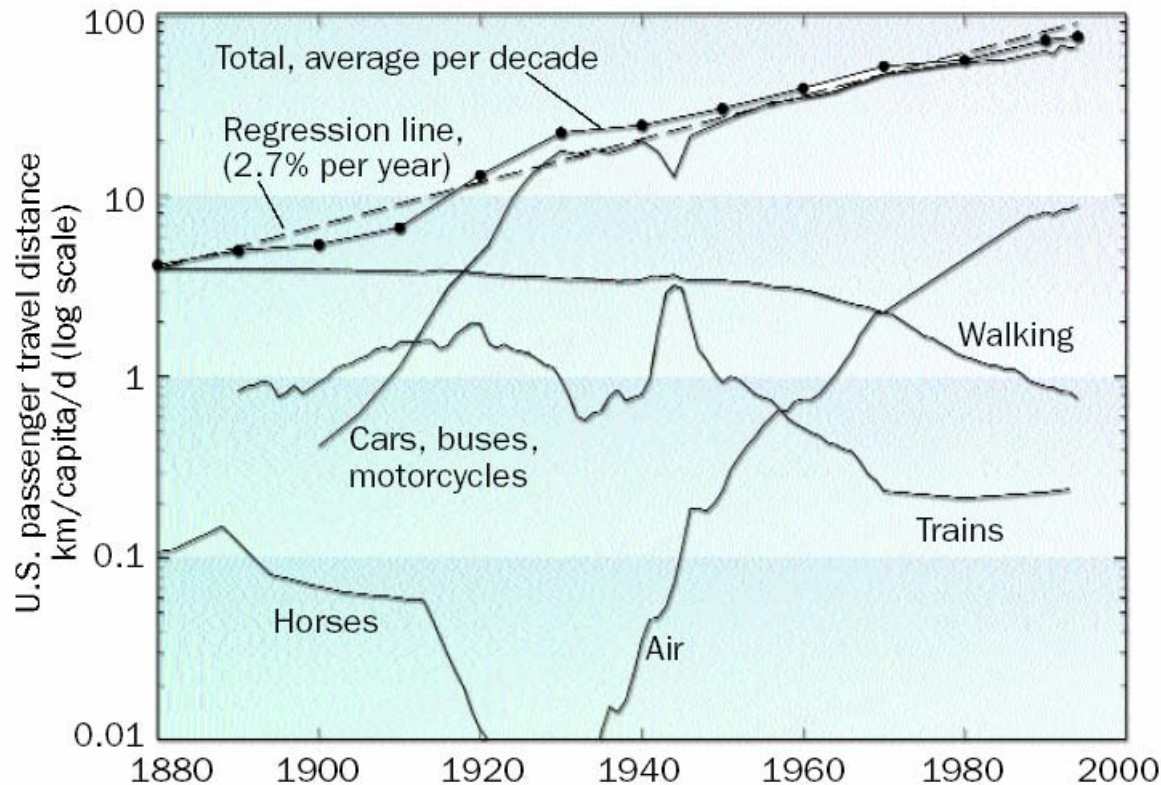
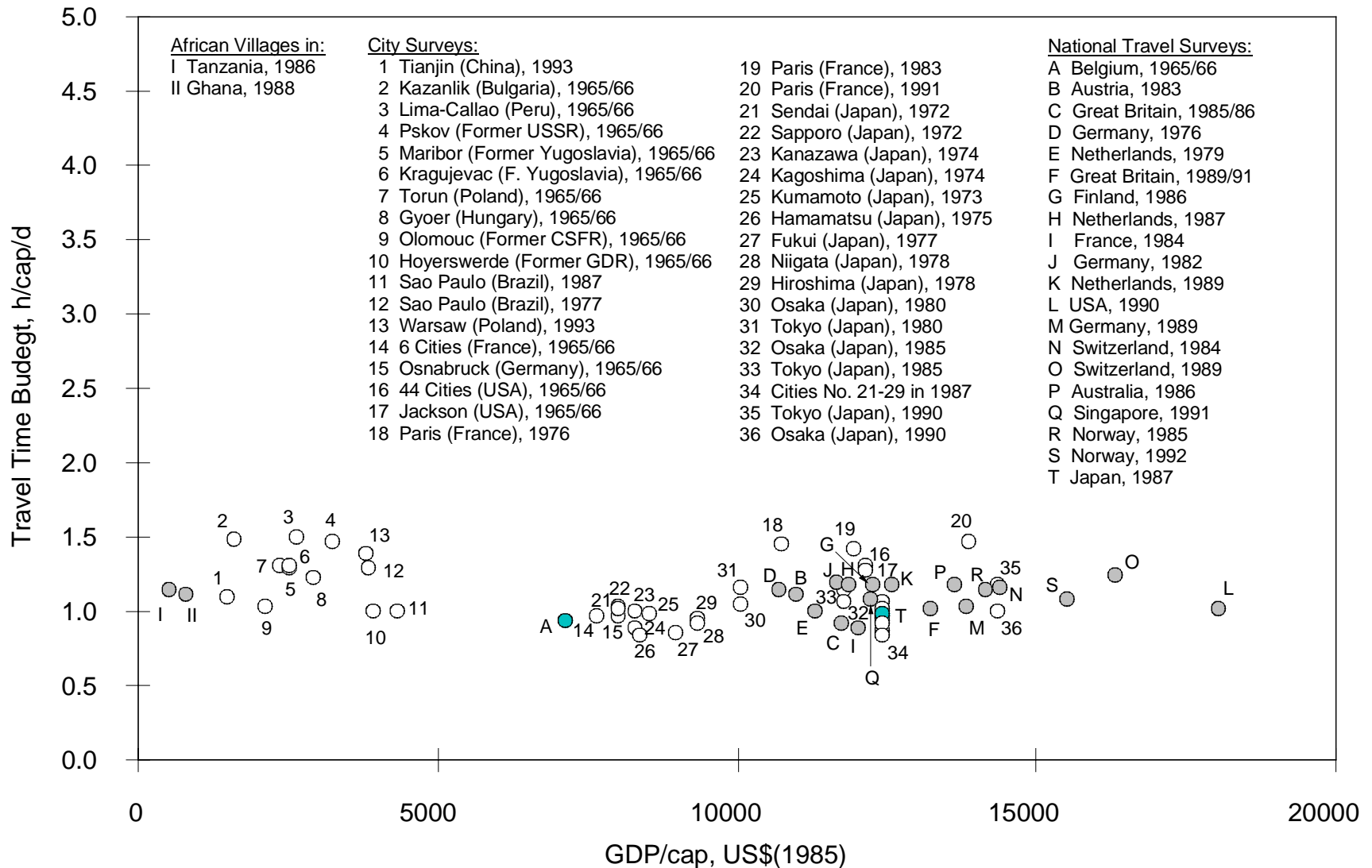


Figure 2. Passenger travel distance per capita per day in the U.S. by all modes shows a decline for horses, walking, and trains; an increase for cars, buses, and motorcycles; and a much more rapid increase for air transportation.

Reference: The evolution of transport
J. H. Ausubel and C. Marchetti
phe.rockefeller.edu/TIP_transport/transport.pdf

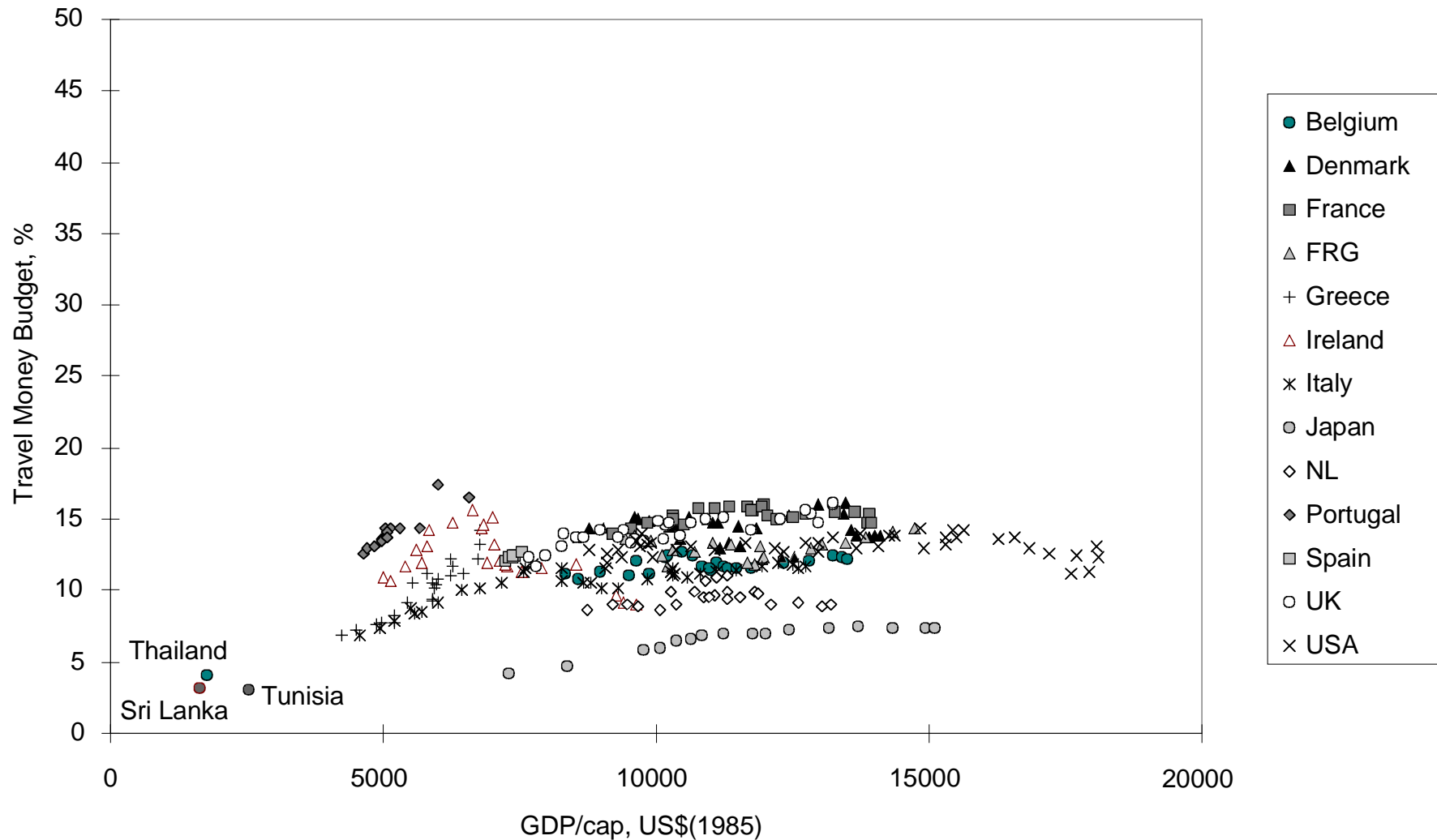
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TRAVEL TIME BUDGET: GLOBAL DATA



Reference: Schafer and Victor (2000) The future mobility of the world population, Transportation Research A34(3), 171-205.

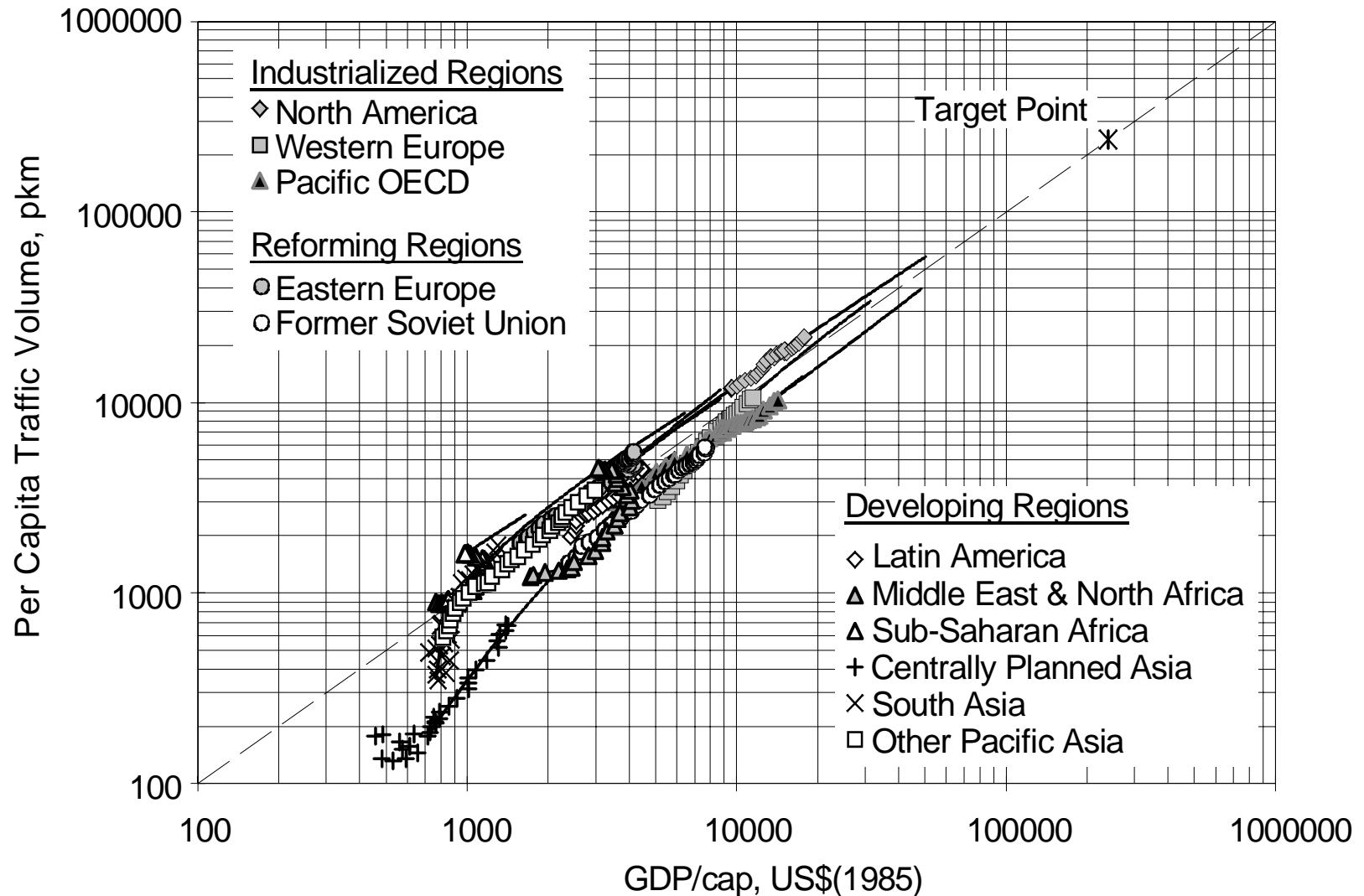
TRAVEL MONEY BUDGET: 16 COUNTRIES



Reference: Schafer and Victor (2000) The future mobility of the world population, Transportation Research A34(3), 171-205.

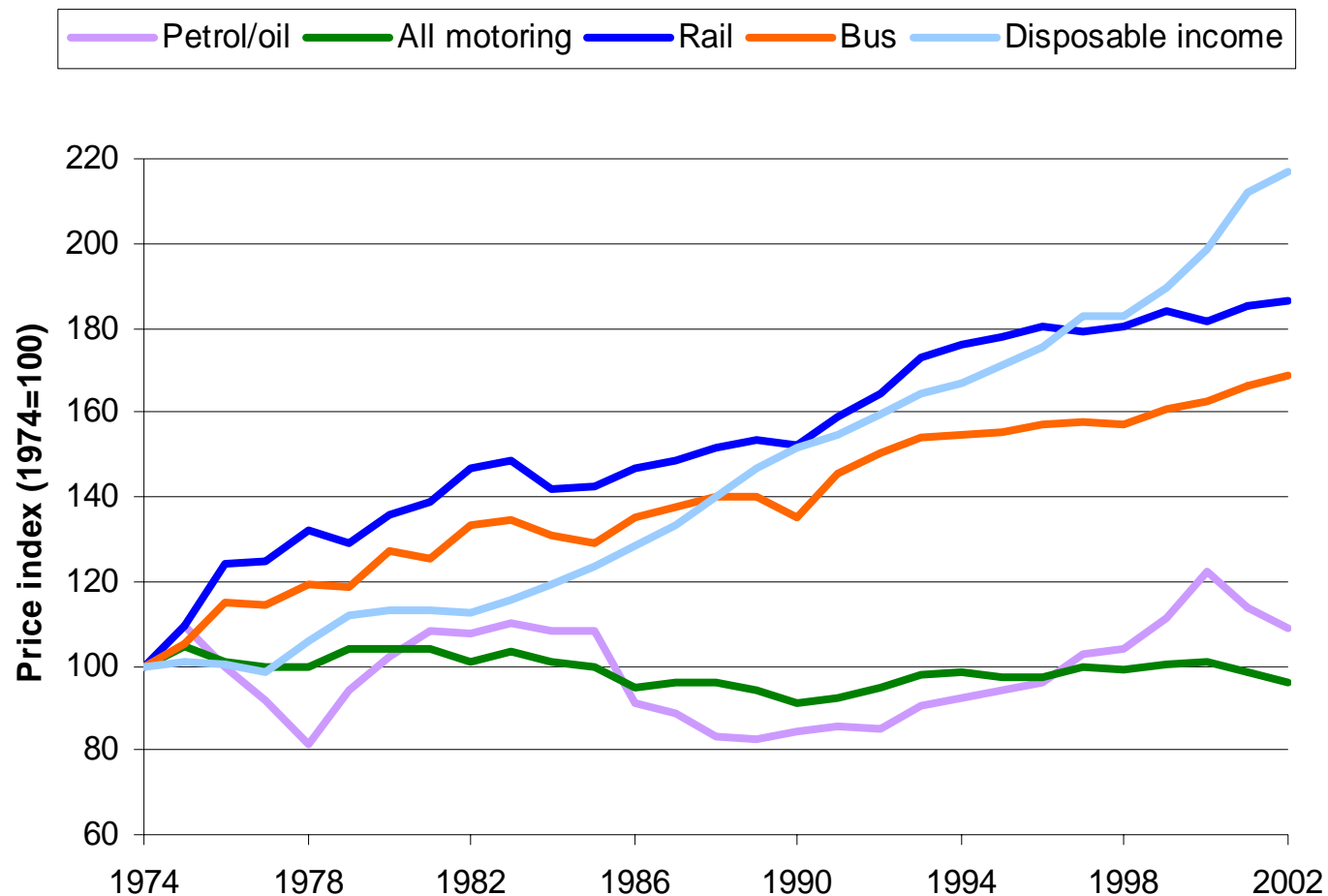
TOTAL MOBILITY

(Data Points: 1960 - 1990; Curves: 1960 - 2050)

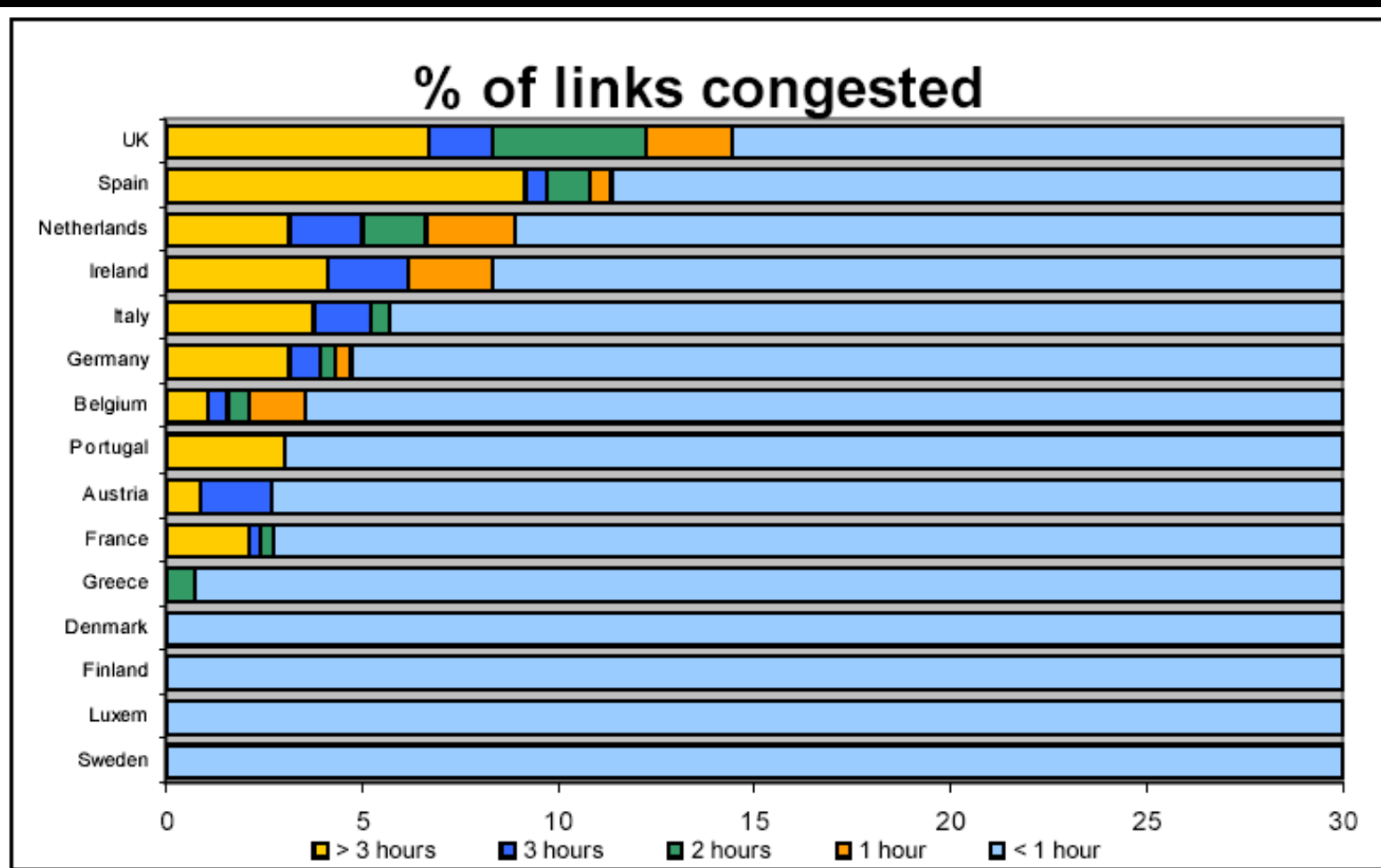


Reference: Schafer and Victor (2000) The future mobility of the world population, Transportation Research A34(3), 171-205.

Real changes in the cost of transport and disposable income: 1974 to 2002



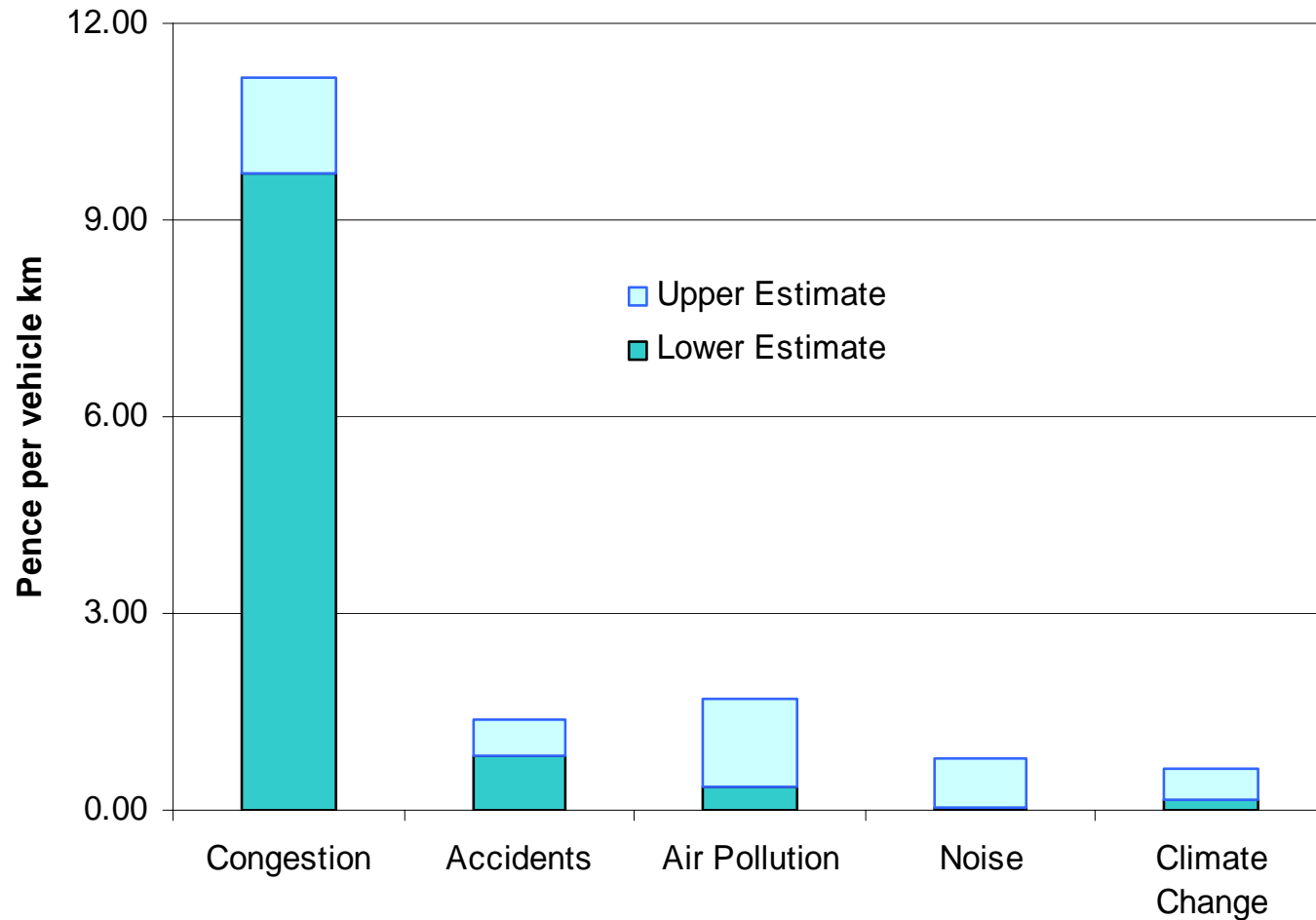
Reference: **Quality of Life Counts: Indicators for a strategy for sustainable development for the United Kingdom, 2004 Update, Indicator T4**
www.sustainabledevelopment.gov.uk/sustainable/quality04/maind/pdf/qolc2004.pdf



Source: European Best Practice in Delivering Integrated Transport, CfIT, 2001

Reference: Commission for Integrated Transport Report: Paying for Road Use, page 20
www.cfit.gov.uk/reports/pfru/pdf/pfru.pdf

1998 Road Sector Marginal Costs



Reference: Institute for Transport Studies, University of Leeds, "Surface Transport Costs and Charges: Great Britain 1998", page 45
www.its.leeds.ac.uk/projects/STCC/downloads/SurfaceTransportCostsReport.pdf

Marginal external costs and tax paid by road users

Pence per km	Marginal external cost of congestion (a)	Environment and safety costs (b)	Fuel duty and VAT on duty (C)	Uncovered externality (a+b)-c
2000	7.3	2.2	5.2	4.3
2010	12.3	1.6	3.9	10.1

Reference: **DfT Road Pricing Feasibility Study, Annex B - Modelling Results and Analysis; Figure B1**

www.dft.gov.uk/stellent/groups/dft_roads/documents/page/dft_roads_029735.pdf

Feasibility study of road pricing

“Forecasts using the National Transport Model suggest that a well-targeted national road pricing scheme has the potential to achieve £10 billion worth of time savings a year (at 2010 traffic levels) in Great Britain alone.”

Reference: **Feasibility study of road pricing in the UK, July 2004, DfT**
www.dft.gov.uk/stellent/groups/dft_roads/documents/divisionhomepage/029709.hcsp

Feasibility study of road pricing

“Forecasts using the National Transport Model suggest that a well-targeted national road pricing scheme has the potential to achieve £10 billion worth of time savings a year (at 2010 traffic levels) in Great Britain alone.”

“It would cost a lot to introduce a national road pricing scheme. As well as the costs of setting it up, there are the costs of enforcement and the back-office systems handling payments and enquiries, as well as compliance costs for industry.... The system could cost as much as £3 billion a year to run.”

Reference: **Feasibility study of road pricing in the UK, July 2004, DfT**
www.dft.gov.uk/stellent/groups/dft_roads/documents/divisionhomepage/029709.hcsp

Cost of Congestion in Wasted Time and Fuel in the Largest Urban Areas

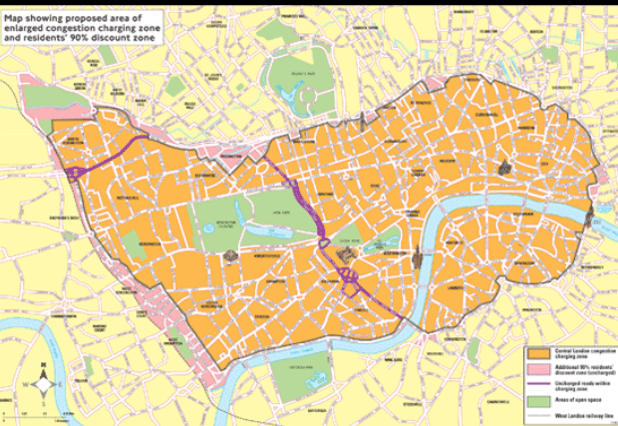
Metro Area	Total Cost (\$ in Millions)	Cost Per Peak Traveler
Los Angeles-Long Beach-Santa Ana CA	\$10,686	\$1,598
San Francisco-Oakland CA	\$2,604	\$1,224
Washington DC-VA-MD	\$2,465	\$1,169
Atlanta GA	\$1,754	\$1,127
Houston TX	\$2,283	\$1,061
Dallas-Fort Worth-Arlington TX	\$2,545	\$1,012
Chicago IL-IN	\$4,274	\$976
Detroit MI	\$2,019	\$955
Miami FL	\$2,485	\$869
Boston MA-NH-RI	\$1,692	\$853
Phoenix AZ	\$1,295	\$931
New York-Newark NY-NJ-CT	\$6,780	\$824
Philadelphia PA-NJ-DE-MD	\$1,885	\$641

Source: Texas Transportation Institute, 2005 Urban Mobility Report

***Reference: National Strategy to Reduce Congestion on
America's Transportation Network, DoT May 2006
isddc.dot.gov/OLPFiles/OST/012988.pdf***

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London Congestion Charge



You can pay your congestion charge:

- Online
- At selected shops, petrol stations and car parks
- By post
- By telephone
- By SMS text message from your mobile phone
- At BT Internet kiosks



Prices:

self-declaration -

- by 10pm £8
- 10pm-midnight £10

camera,... -

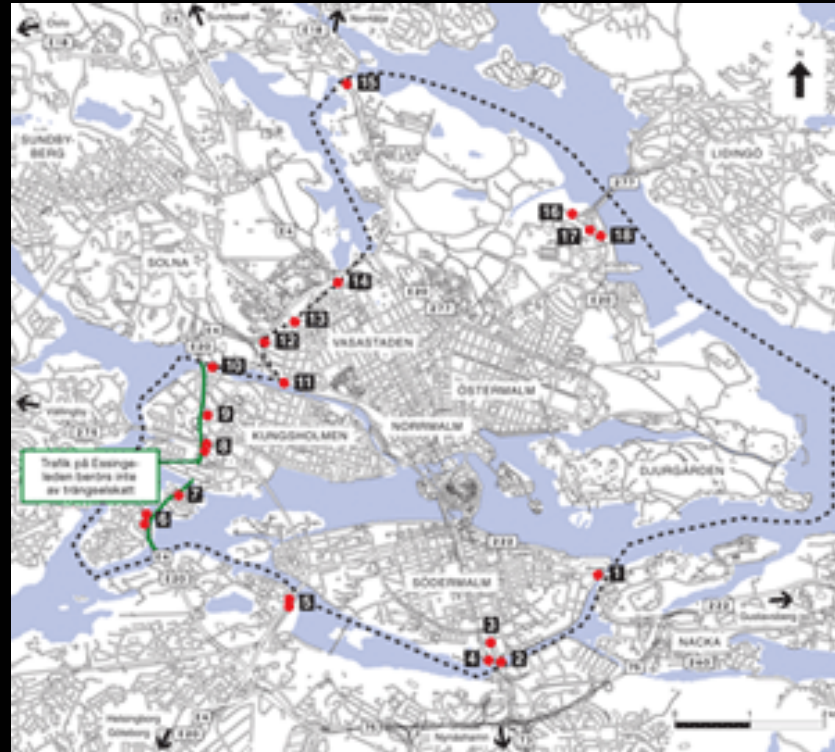
- 14 days £50
- 14-28 days £100
- later £150,



www.cclondon.com

Stockholm Congestion Tax

- 18 control points located at Stockholm city entrances and exits. Vehicles registered automatically by cameras that photograph the number plates. Those vehicles equipped with an electronic onboard unit for direct debit payment are also identified through this means. Vehicles are registered when driving both into and out of the inner city zone.
- Payment must be made within 14 days of passage. Paying the tax is the responsibility of the vehicle owner, and not the driver. No invoice or payment slip is sent out.



Time	(SEK) Amount
06.30-06.59	10 kr
07.00-07.29	15 kr
07.30-08.29	20 kr
08.30-08.59	15 kr
09.00-15.29	10 kr
15.30-15.59	15 kr
16.00-17.29	20 kr
17.30-17.59	15 kr
18.00-18.29	10 kr
18.30-06.29	0 kr

Pay as you drive insurance

Norwich Union's personalised tariffs

Example:

night time (11pm to 5.59am) - £1 per mile

day time (6am to 10.59pm) -4p per mile

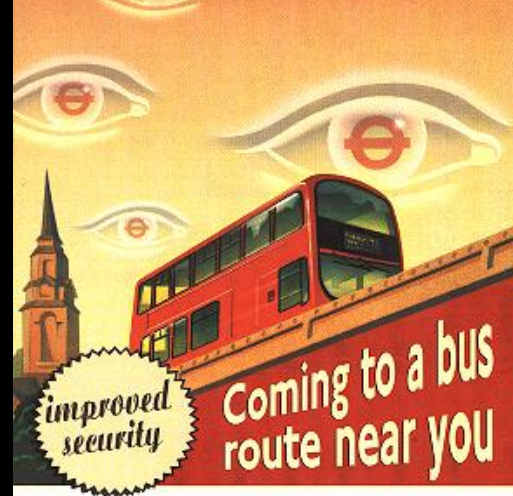


Privacy

Our attitudes are evolving rapidly



FEEL
SECURE BENEATH THE
WATCHFUL
EYES



MAYOR OF LONDON

Buses are getting better



GPS Locator FOR CHILDREN
Comes in two colors:
Galactic Blue and Cosmic Purple



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Parallel roads

The toll on one road is T ; a parallel road is untolled.

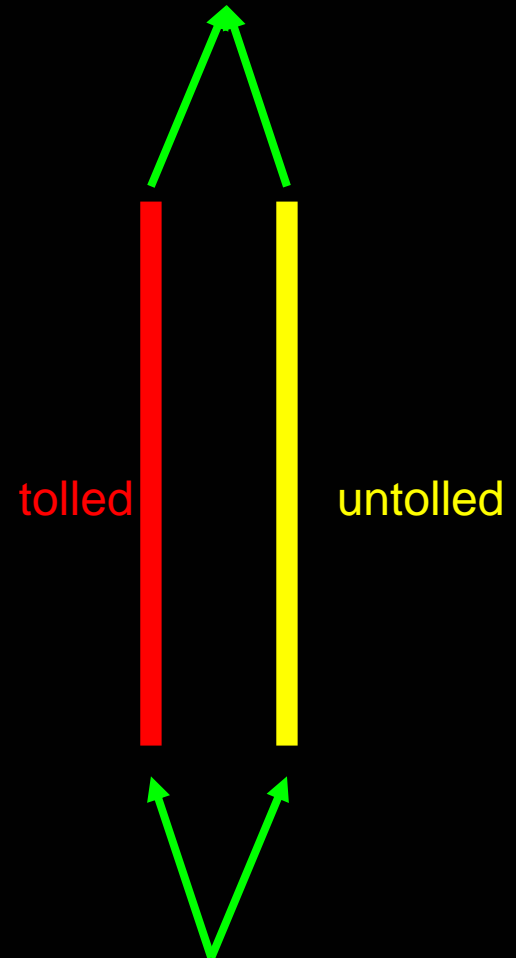
The total traffic on both roads is I (and is unaffected by the toll T).

The delay on each road is a function $D(x)$ of the traffic x on that road, the same function for each road.

Each user has a *value of time*, v , chosen at random from a distribution with density $f(v)$, $v > 0$, and distribution function $F(v)$.

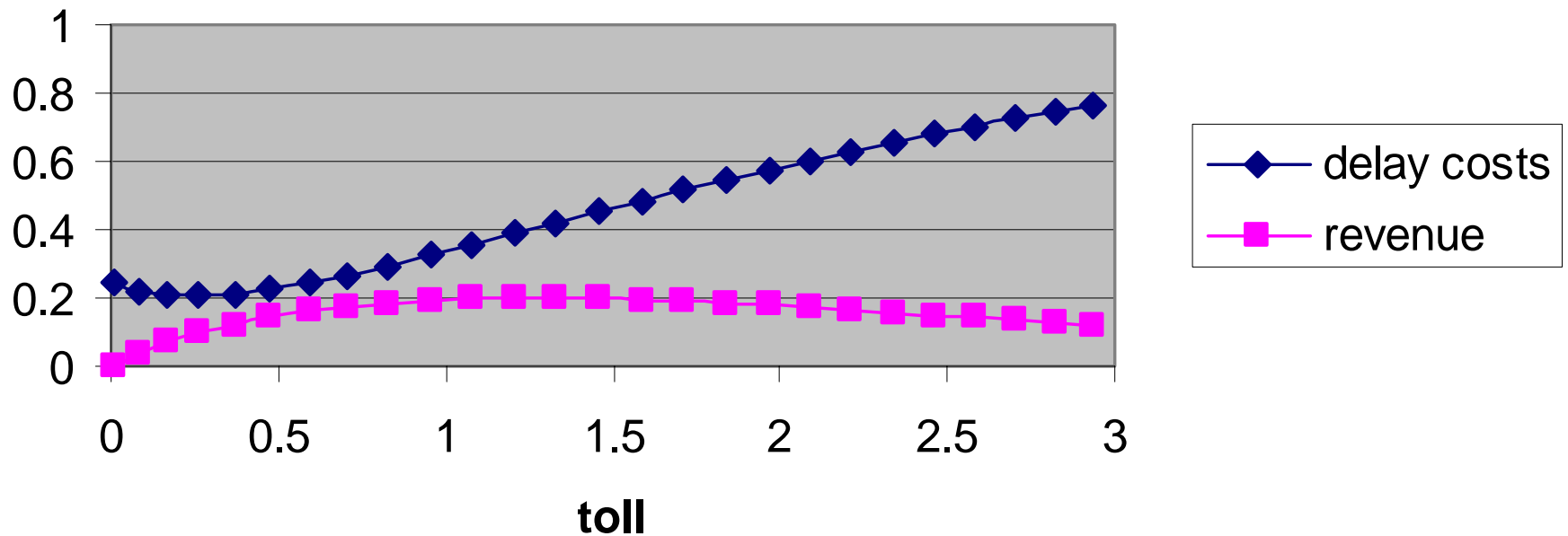
A user will choose the toll road if her value for time is greater than a critical value v^* , the solution to the equation

$$T = [D(F(v^*)) - D(1-F(v^*))] v^*$$



Example 1

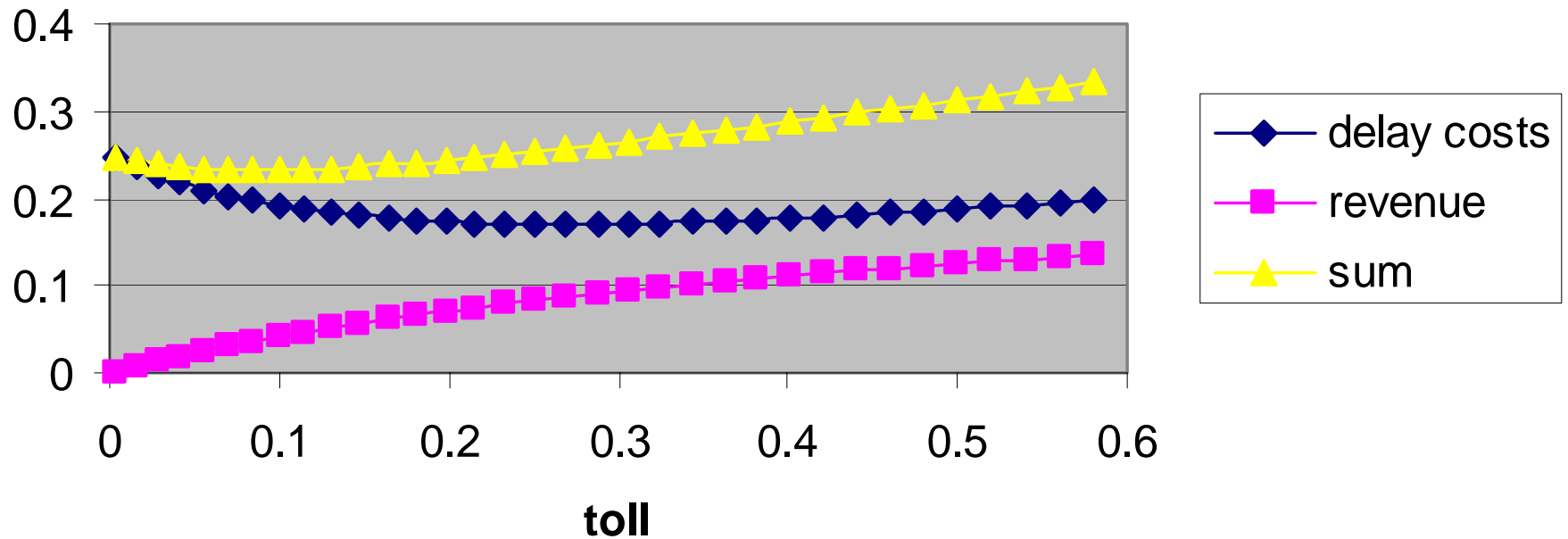
exponential vft



In general, welfare maximized by a positive toll

Example 2

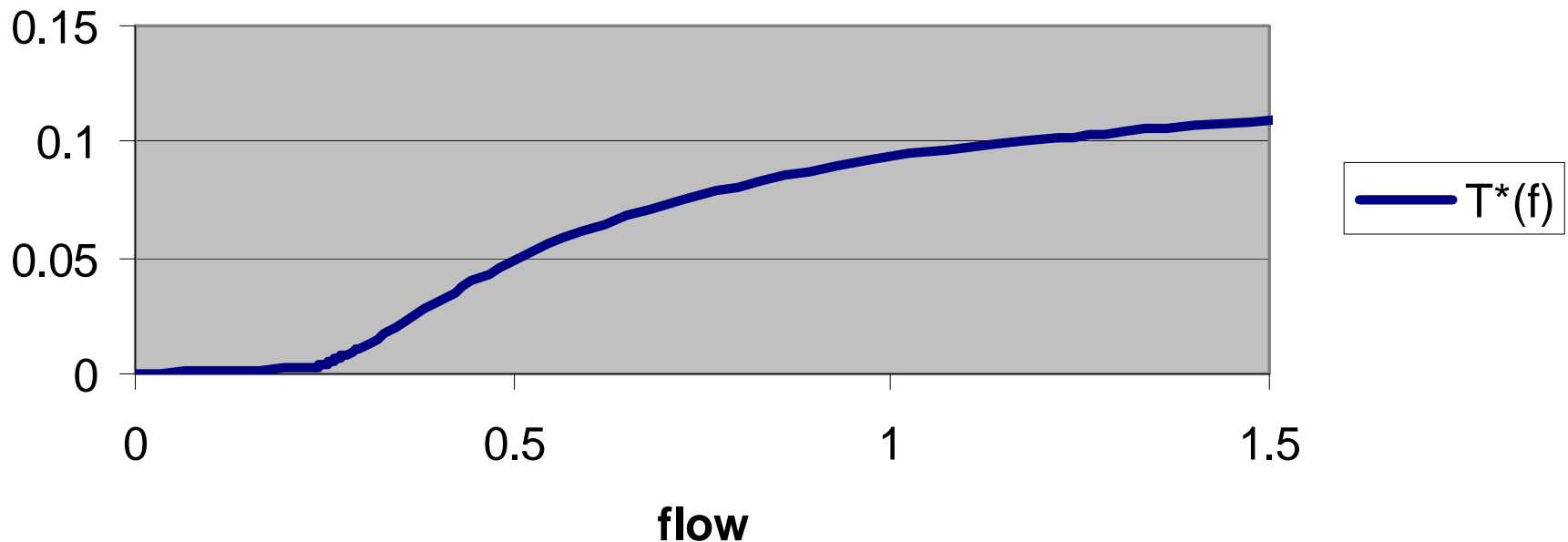
pareto vft



If values of time are sufficiently diverse, welfare may be maximized by a positive toll even if toll revenue lost!

Average toll depends on load

toll for combined system



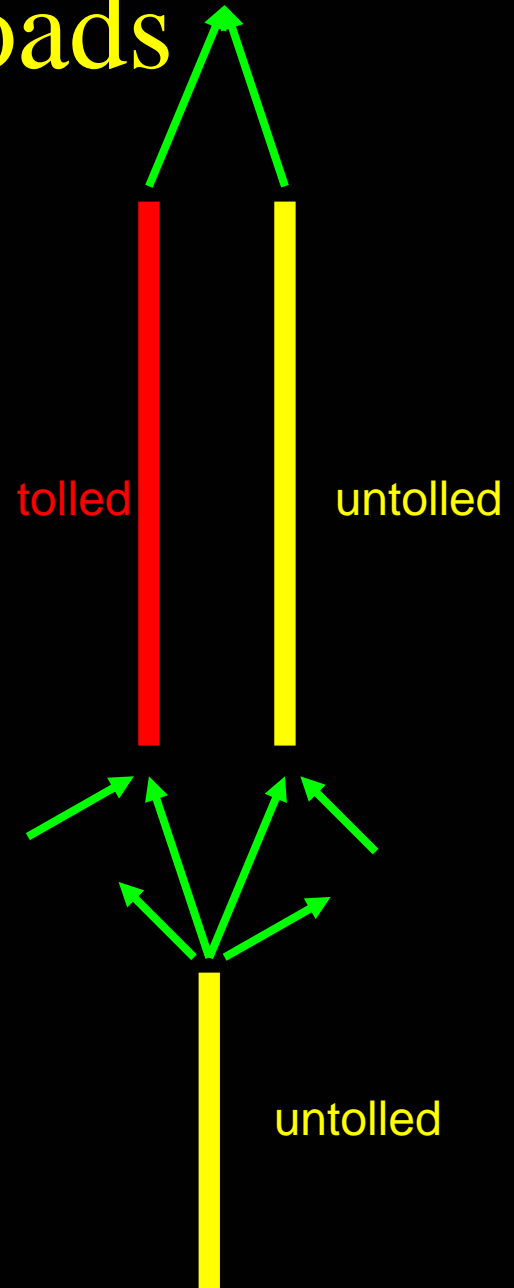
The average toll for the combined system is load dependent. As total flow increases, average toll approaches half the fixed toll. A simple toll on road, together with informed user choices, can have a similar outcome to a complex toll.

Parallel and series roads

Would a revenue maximizing operator of the tolled road charge too much, or too little, in comparison with the system optimum charge?

The answer can go either way.

(Possibility of hierarchical mechanism design.)



Partial tolling in a network

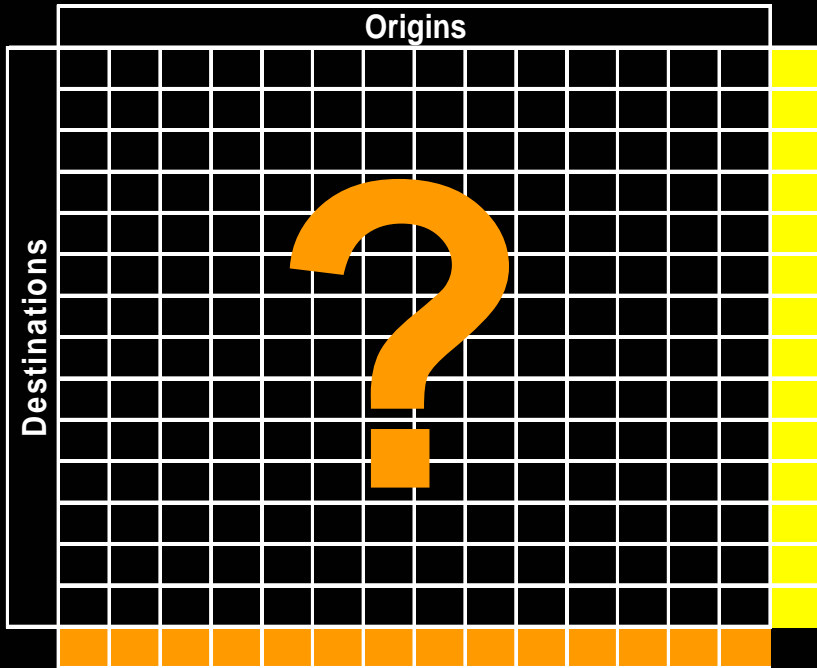
If all values for time are identical, and if all roads can be tolled, then charging the marginal social cost will maximize welfare. The toll on a road is then a local calculation, unaffected by network topology or traffic patterns. But we need more!

In presence of heterogeneity of time values and partial tolling, both network topology and traffic patterns matter.

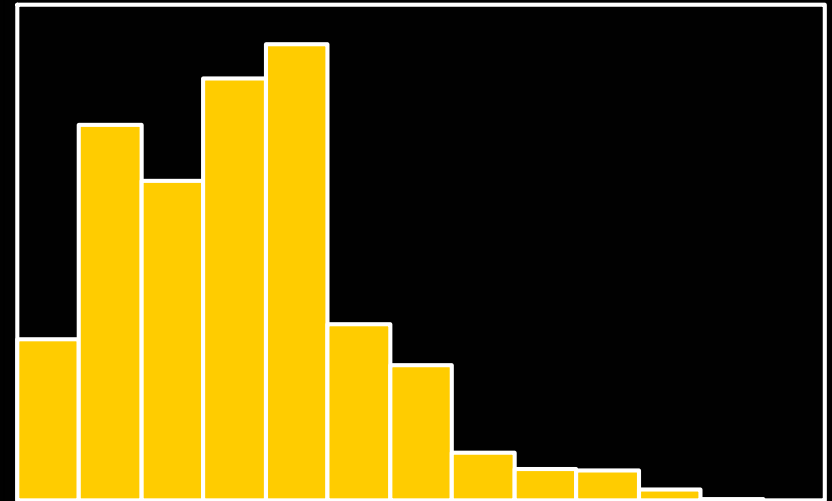


Who travels where?

Origin-Destination matrix



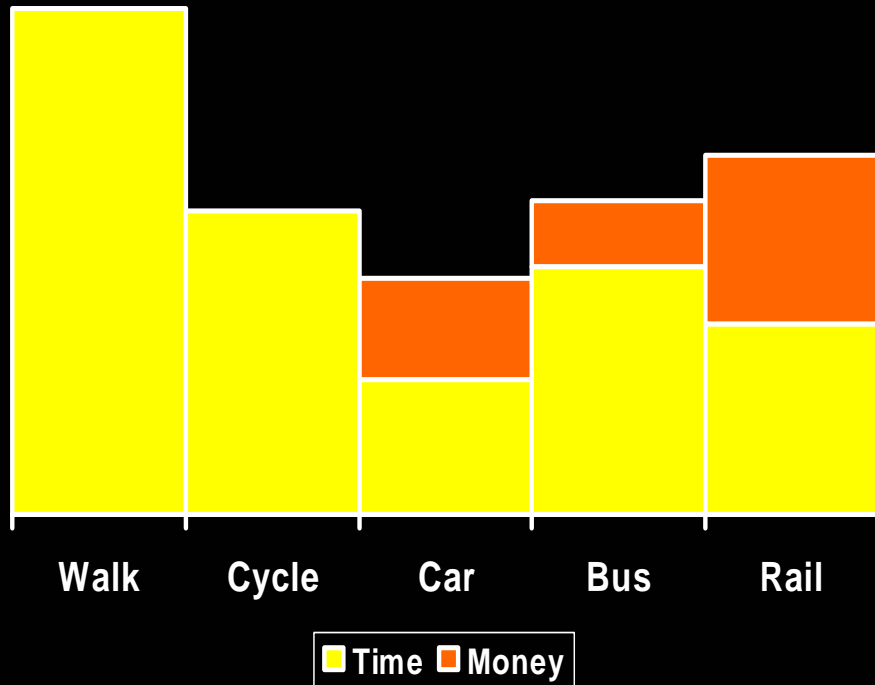
**Known row
and column totals**



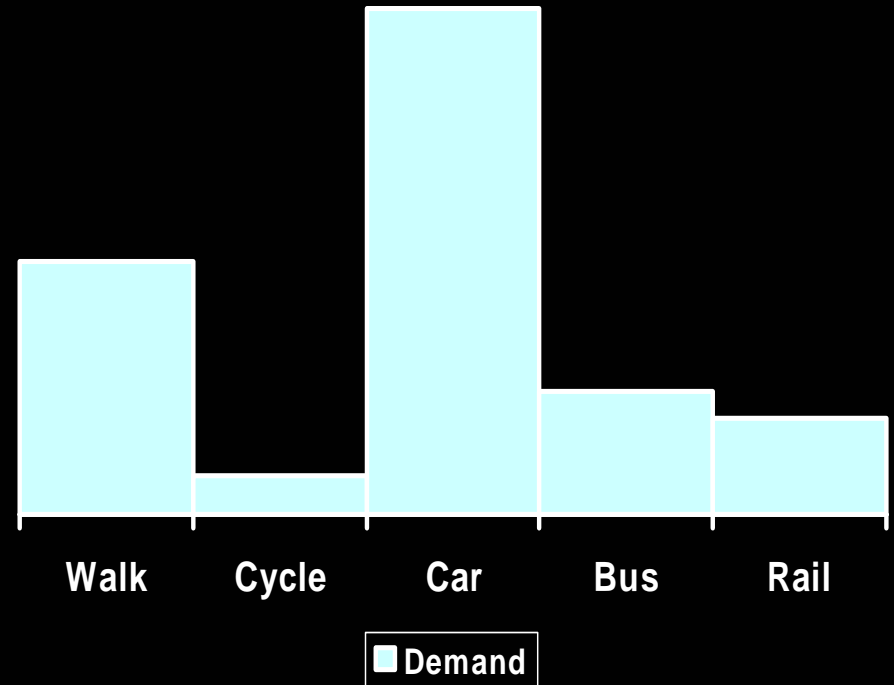
**Known trip
length distributions**

How do people choose modes?

Known time and money costs



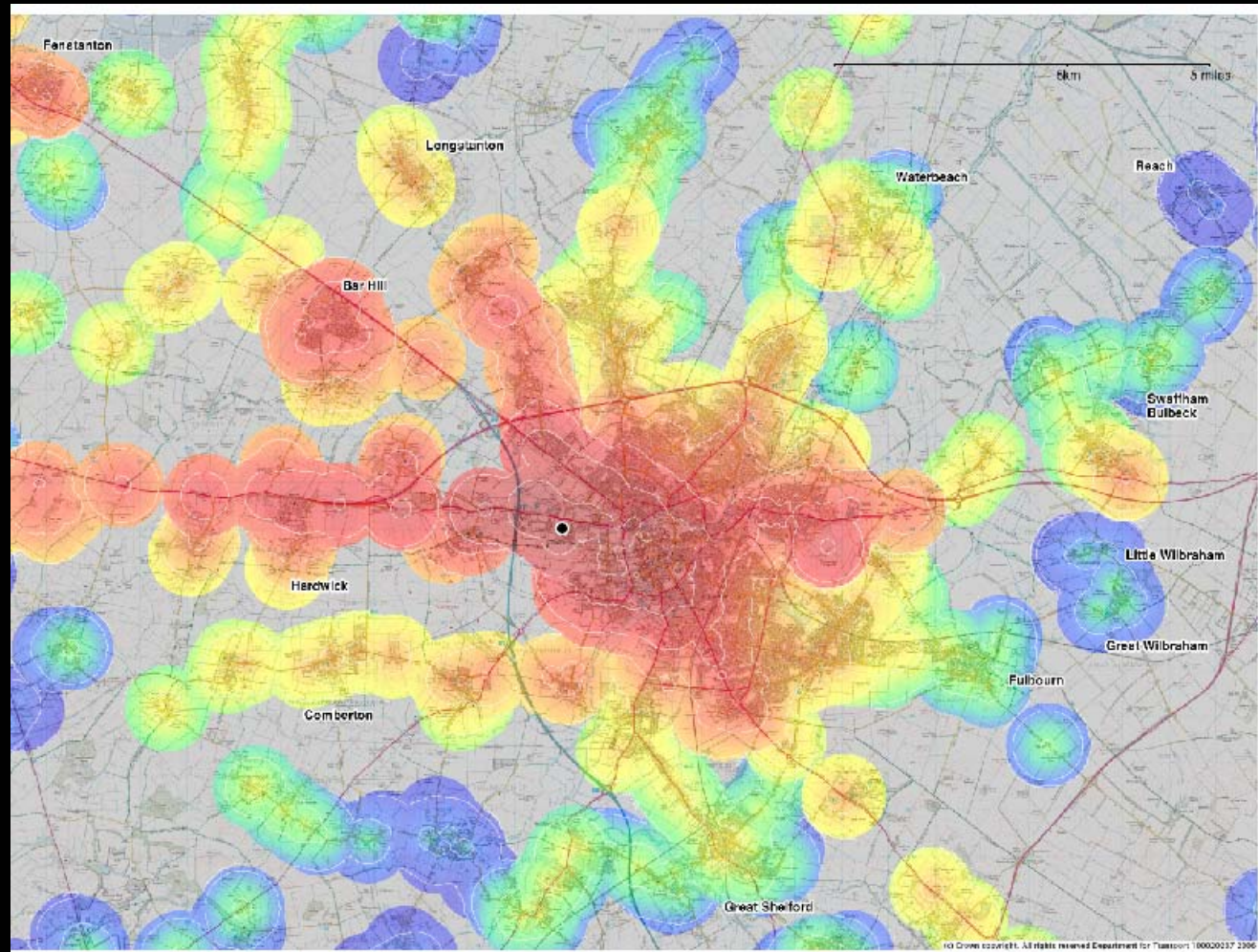
Known demands



BUT un-measurable preferences for comfort, security, and privacy

Land use and transport interaction

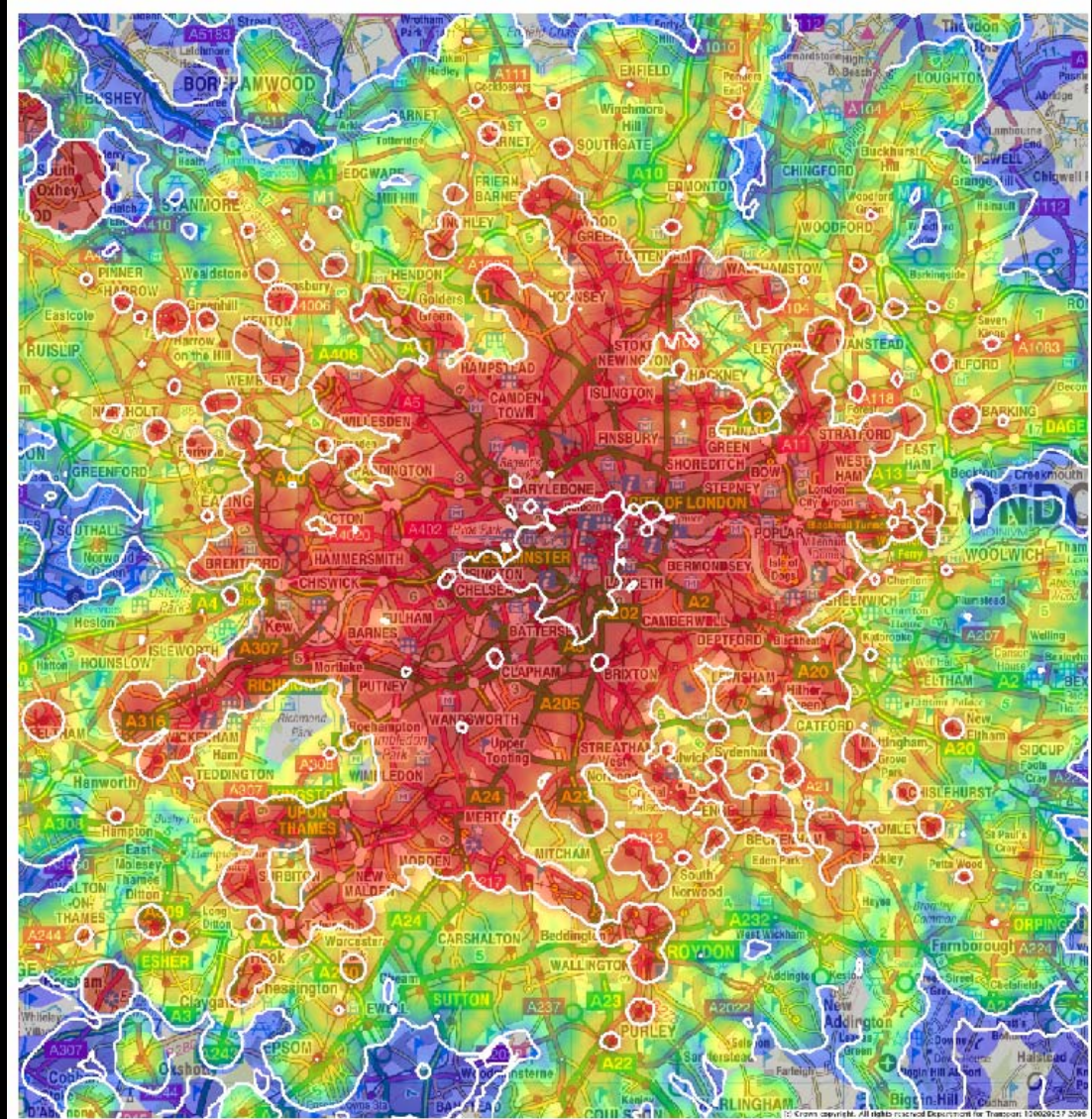
Map of
Cambridge and
surrounds
showing times of
departure to reach
the West
Cambridge site by
9 o'clock on a
weekday morning



Land use and transport interaction

Highly path dependent
and non-convex.

Land values, Disneyland
etc



Discussion