Mathematics and Financial Markets

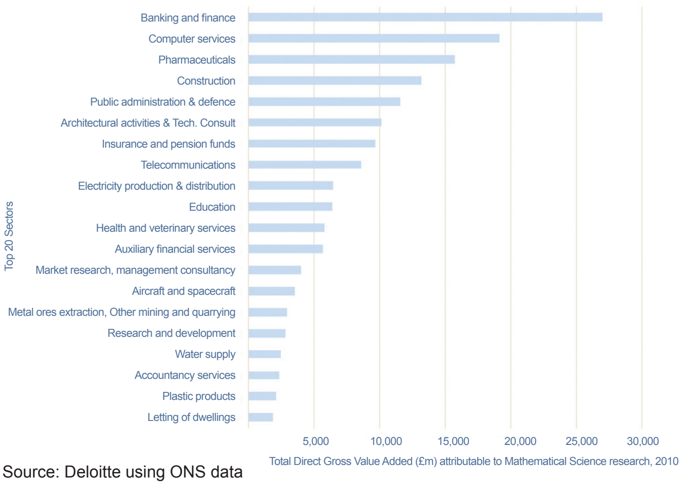
The David Crighton Lecture 2016, was delivered by Frank Kelly on Thursday 12 May 2016 at the Royal Society. The following is a brief outline of the lecture.

A substantial proportion of mathematics graduates, at both first degree and doctoral level, enter the financial services sector (Table 1). This is hardly surprising given the apparent importance of the sector to the economy (Figure 1), and the role of mathematical modelling in the valuation of instruments and the assessment of risk. What is striking is that, with some notable exceptions, few mathematicians have been actively engaged in the design of financial markets. This is undoubtedly a serious challenge with parallels from other large-scale complex networks: to design a distributed system, linking self-interested and intelligent agents, so that the outcome is effective and efficient.

Table 1: STEM graduates entering finance. A much higher proportion of graduates with degrees in mathematics were working in financial services six months after graduating, although these represent much smaller numbers of individuals than other STEM disciplines [1, p. 16].

Degree level	Eng/Tech	Phys Sci	Maths
First degree	5.0%	8.6%	29%
Doctorate	2.4%	2.9%	19%

To illustrate the challenge, consider the question: how might a market for shares in a company operate, to allow liquidity between long-term investors to be provided by short-term traders? In the second part of the talk we describe some preliminary work, joint with Elena Yudovina [2], on this question. We describe a simplified and analytically tractable model of a limit order book where the dynamics are driven by stochastic fluctuations between supply and demand. In a limit order book bids and asks arrive over time, each with an associated price. An arriving bid is either added to the book, if it is lower than all asks present in the book, or it is matched to the lowest ask and both depart. Similarly an arriving ask is either added to the book, if it is higher than all bids present in the book, or it is matched to the highest bid and both depart. Our simplified model of the behaviour of a limit order book has a natural interpretation for a highly traded market on short time scales where there is a separation between the time scale of trading, represented in the model, and a longer time scale on which fundamentals change.



Source: Deloitte using ONS data

Figure 1: Top 20 sectors for direct mathematical sciences Gross Value Added, in £millions. Note that Insurance and Pension Funds, and Auxiliary Financial Services are separate from Banking and Finance [3, p. 31].

There has been considerable discussion recently of the effects of competition between multiple high-frequency traders, and of proposals aimed to slow down markets. A key issue is that traders may wastefully compete on the speed with which they can snipe an order rather than compete on price, and a proposed regulatory response is to use frequent batch auctions, held perhaps several times a second. Our model is a caricature of a real limit order book, but it does provide insight into various high-frequency trading strategies (for example market-making, sniping and mixtures of these) and the impact on Nash equilibria when a market in continuous time is replaced by frequent batch auctions.

… it is important for regulators to assess whether the market's design is well-suited for the social purpose the market is intended to fulfil …

A limit order book is a form of two-sided queue and our analysis is partly motivated by a more general challenge. The study of two-sided queues dates at least to the early paper of Kendall [4], who modelled a taxi-stand with arrivals of both taxis and travellers as a symmetric random walk. Recent theoretical advances involve servers and customers with varying types and constraints on feasible matchings between servers and customers, with applications ranging from large-scale call centres to national waiting lists for organ transplants. Uber is an example of a two-sided queue between travellers and drivers; indeed two-sided queues are becoming pervasive, as technology brings automated matching markets into more and more aspects of our lives. For financial markets certainly, and for some other markets also, it is important for regulators to assess whether the market's design is well-suited for the social purpose the market is intended to fulfil.

As John Kay has written [5], there are good reasons to be sceptical about how much the financial sector contributes to the UK. The UK has competitive strengths, including communications, language and time zone, and hence we should expect the UK to be strong in finance. But private profit without public benefit is a widespread policy challenge in finance. Given the intrinsically mathematical nature of the trading mechanisms used in financial markets, mathematicians have an obligation to help address this challenge.

Frank Kelly CBE FRS Hon FIMA
University of Cambridge

References

1 Royal Society (2009) *Hidden wealth: the contribution of science to service sector innovation*, Royal Society.

2 Kelly, F. and Yudovina, E. (submitted 2015) A Markov model of a limit order book: thresholds, recurrence, and trading strategies, http://arxiv.org/abs/1504.00579

3 Deloitte (2012) Measuring the Economic Benefits of Mathematical Science Research in the UK, EPSRC, www.epsrc.ac.uk/newsevents/news/mathsciresearch/

4 Kendall, D.G. (1951) Some problems in the theory of queues, *Journal of the Royal Statistical Society*, vol. 13, no. 2, pp. 151–185.

5 Kay, J. (2015) *Other People's Money: Masters of the Universe or Servants of the People?*, Profile Books.