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COALITIONS IN THE INTERNATIONAL NETWORK

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The understanding of the performance and design of national networks with rerouting is now well advanced. In particular, simple and robust dynamic routing strategies, and sympathetic dimensioning procedures, have been developed. However, the extension of this understanding to international networks requires consideration of a number of new issues. An important example concerns the partitioning of benefits. Joint action will lead to a surplus of benefits over costs, relative to the present arrangements. How should these benefits be divided between carriers? In this paper we consider which divisions are likely to be stable against competing (and overlapping) coalitions. In particular we investigate coalitions comprising 3, 4 and 5 members selected from a set of 6 countries: Australia, Canada, France, Japan, UK and the USA. The theoretical tools we use are multi-commodity flow theory and the theory of games.

1 Introduction

Dynamic routing can achieve considerable cost savings in the international network, and can greatly increase its flexibility and robustness. Starting from a trial data set described in Section 2, we use the solution to a standard multi-commodity flow formulation to assess the magnitudes of the savings that are possible from co-operative behaviour within the international network. These savings define the *value* of a coalition between any given subset of countries. The relative strength of members of a coalition can then be assessed. In Section 3 we describe the application of two important game-theoretic techniques: the core and least core; and the Shapley value. These techniques give important insights into the partitioning of the benefits that will follow from joint action. In particular, any agreed scheme for dynamic routing and capacity management in the international network will need to respect the contributions individual members make to the coalition.

A deterministic multi-commodity flow formulation is able to give reasonable estimates of overall cost savings, since busy hour traffic between time zones is large, typically measured in thousands of Erlangs. More refined methods are needed however, to study the behaviour of dynamic routing schemes, the volume of transit flows, and the sensitivity of network performance to errors in traffic forecasts. A particularly useful technique for the analysis of these issues is provided by the bundle dimensioning method. This procedure is a refinement of the multi-commodity flow formulation, and can be much simpler. The procedure is based upon dimensioning the network subject to a number of essential generalized cut constraints. In this paper we illustrate the technique for three and four node networks. For four node networks binding cut constraints often involve traffic between disjoint pairs of countries, and this has interesting consequences for the dimensioning of the international network. For networks with four or more nodes the requirement for efficient network dimensioning will not generally yield a strongly determined solution, and this has important implications for the partitioning of shared benefit between participating members of the coalition.

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2 The trial data set

In this section we present the trial data set used in this paper. The methods described later apply more generally: however a trial data set is important in identifying relevant orders of magnitude, and allows our tentative conclusions to be illustrated in a more immediately comprehensible manner.

The data used in this study consists of a one-way traffic matrix of busy hour traffics between the following countries: Australia (A), Canada (C), France (F), Japan (J), United Kingdom (B) and United States of America (U). The one-way busy hour traffic matrix is given in Table 1. Daily traffic profiles were calculated using the CCITT method [24].

An immediately striking feature of the traffic matrix given in Table 1 is that the larger busy hour traffics between time zones are typically measured in thousands of Erlangs. An important consequence is that errors in traffic forecasts have far greater impact on network performance than do probabilistic variations arising from, for example, the Poisson or other nature of offered traffic. A stream producing a nominal load of 2500 Erlangs will, under Poisson assumptions, have associated a standard error of 50 Erlangs, or 2%. This variation is small compared with that which will arise from day-to-day variation and from errors in traffic forecasts. An implication is that refined methods of calculating implied costs [3,9,13,15,19] simplify greatly: the implied costs calculated from these more sophisticated models will be very nearly equal to the values calculated from deterministic multi-commodity flow models.

In Figure 1 we show the approximate daily profiles of traffic between Japan, UK and USA. Observe the non-coincidence of busy hours, and that UK-USA traffic is over twice that between Japan-USA or Japan-UK.

3 Dominant coalitions

In this section we assess the magnitude of the savings that are possible from co-operative behaviour within the international

	A	B	C	F	J	U
A		966	72	39	187	1177
B	966		896	1426	1116	4323
C	72	896		249	86	0
F	39	1565	249		97	1583
J	187	913	86	97		1522
U	1177	3999	0	1583	2019	

Table 1: One-way busy hour traffic

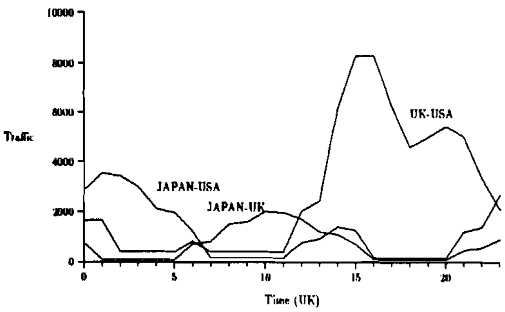

Figure 1: Daily traffic profiles: Japan, UK and USA

network. We study which coalitions are likely to be stable against competing (and overlapping) coalitions. The theoretical tools we use are multi-commodity flow theory [8], and the theory of games [10,17,20,21].

3.1 Potential savings

In this section we dimension the international networks formed by subsets of the countries according to a multi-commodity flow formulation used earlier in [23]. This formulation will be an accurate representation of the network when busy hour loads are of the size indicated by the traffic matrix of Table 1.

In our numerical examples we assume the circuit cost is constant over links. There is no modelling difficulty in using variable costs, but data would, of course, be required. Variable costs will make very little difference to overall network dimensioning in the case of three nodes (see Section 4), but can make a difference for four or more nodes (see Section 5.2). The multi-commodity flow formulation could easily constrain two-link traffic to use just cable, or have at most one hop via satellite, but again data would be required. (On the basis of rough estimates it is not expected that these constraints would substantially alter our tentative conclusions.)

Let $N = \{$ A, B, C, F, J, U $\}$ and for subsets $S \subseteq N$ define the *value* of S by

$$V(S) = (\text{cost separate}) - (\text{cost with coalition}, S).$$

Here 'cost separate' is the cost to the members of the coalition of providing links between themselves if they route all traffic between themselves directly. In contrast 'cost with coalition' is the cost to members of the coalition of providing links between themselves if they route traffic between themselves according to the solution of the corresponding multi-commodity flow problem. Thus $V(S)$ is the potential saving to be had by the formation of the coalition S. Note that for $|S| < 3$, $V(S) = 0$. Table 2 shows $V(S)$ for a number of subsets, S, with $3 \leq |S| \leq 5$.

Observe the large potential savings to be had by a three member coalition between Japan, UK and USA, or between Australia, UK and USA. The savings of these coalitions are about twice as great as those of other three member coalitions. The four member coalition of Australia, Japan, UK and USA achieves a saving (4984) greater than the combined savings achieved by the three member coalitions of Japan, UK and USA *and* Australia, UK and USA (2760 + 2203 = 4963). Thus it is reasonable to view Australia and Japan as complementary rather than as substitutes.

Subset, S	Separate	Coalition	$V(S)$	Saving
B J U	13895	11134	2761	20%
A B U	12610	10406	2204	17%
F J U	6904	5609	1295	19%
A F U	5600	4801	799	14%
B C J	3995	3199	796	20%
A B C	3869	3127	742	19%

Subset, S	Separate	Coalition	$V(S)$	Saving
A B J U	18558	13573	4985	27%
B F J U	20248	16733	3515	17%
A B F U	18847	15982	2865	15%
B C J U	15860	13044	2816	18%
A B C U	14546	12284	2262	16%
A F J U	9713	7990	1723	18%
C F J U	7575	6228	1347	18%
A B C J	6447	5120	1327	21%

Subset, S	Separate	Coalition	$V(S)$	Saving
A B F J U	24990	19188	5802	23%
A B C J U	20667	15570	5097	25%
C B F J U	22712	19142	3570	16%
A B C F U	21283	18359	2924	14%
A C F J U	10529	8707	1822	17%

Table 2: Coalitions, $|S| = 3, 4, 5$

3.2 The core and least core

The previous section concerned itself with the value $V(S)$ of a coalition. In this section we consider how this value, or potential saving, might be divided between the members of the coalition S. Define an *imputation* $(x_i,\ i \in S)$ to be a vector of real numbers such that $\sum_{i \in S} x_i = V(S)$. View $(x_i,\ i \in S)$ as a division between the members of the coalition S of the value $V(S)$ of the coalition. For a set N define the *core* to be the set of imputations $(x_i,\ i \in N)$ such that

$$\sum_{i \in S} x_i \geq V(S) \qquad \forall\, S \subseteq N.$$

It is thus the set of imputations which leave no coalition in a position to improve the payoffs to all of its members.

From Table 2 the core can be calculated and is typically quite large. We therefore devise a means of shrinking it. The *strong ε-core* [20] is the set of imputations $(x_i,\ i \in N)$ such that

$$\sum_{i\in S} x_i \geq V(S) - \varepsilon \qquad \forall S \subseteq N, S \neq \emptyset, N \qquad (1)$$

$$\sum_{i\in N} x_i = V(N). \qquad (2)$$

We may regard ε as an additional cost (or additional gain if ε negative) to the formation of a coalition smaller than N. In our application ε will be negative, and so the strong ε-core will be smaller than the core. How negative must ε be before the strong ε-core disappears? (Define the *least core* to be the smallest strong ε-core [20]). To answer this question we solve the linear program: minimize ε subject to (1) and (2).

Co-operative game theory provides a number of closely related concepts for the analysis of a value function; for example in the above development we might replace the cost ε of forming coalition S by a cost $\varepsilon|S|$ corresponding to a cost *per member* of the coalition, or we might reduce the least core further to obtain the nucleolus [21]. Rather than pursue these possibilities, we present two simple examples illustrating the calculation of the least core.

3.2.1 Examples

First, take $N = \{$ A, B, J, U $\}$. Then the linear program has three optimal basic feasible solutions (x_i) as shown in Table 3, with the optimal $\varepsilon = -1112$. The least core is the set of optimal solutions, given by the convex hull of these three points.

A	B	J	U
1112	1112	1112	1649
1112	1649	1112	1112
1112	1112	1649	1112

Table 3: Extreme points of the least core, $|N| = 4$

A	B	F	J	U
409	2048	409	2528	409
1877	579	409	2528	409
1877	2221	409	886	409
429	3670	409	886	409
409	3670	409	906	409
409	409	409	2528	2048
1877	409	409	2528	579

Table 4: Extreme points of the least core, $|N| = 5$

Observe the symmetry between Japan, UK and USA, and that these three countries are in a favoured position relative to Australia. The convex hull of the above three points is the least core, and has the following interpretation. Any imputation within this convex hull is stable against three members of the coalition deciding to exclude the fourth, even if the three members are together offered an additional inducement of 1112 for doing so. Note that if more than 1112 is offered as an inducement, then Japan, UK and USA will prefer to form a coalition excluding Australia. Alternatively, we could view 1112 as a measure of the additional effort involved for each side in extending the coalition $\{$ B, J, U $\}$ to include Australia that would make it not quite worth the effort.

Our second example has $N = \{$A, B, F, J, U$\}$. This has optimal extreme points as shown in Table 4, with the optimal $\varepsilon = -409$. Observe the rather unfavoured position of France within the least core, the convex hull of these points. Any imputation within this convex hull is stable against a subset deciding to form a smaller coalition, even if the subset is offered an additional inducement of 409 for doing so. Observe that if more than 409 is offered as an inducement then Australia, Japan, UK and USA will prefer to form a coalition excluding France. Alternatively 409 is a measure of the additional effort involved in extending the coalition $\{$ A, B, J, U$\}$ to include France that would make it not quite worth the effort.

In both the above examples the least core, while smaller than the core, is still a fairly large set. There are thus many ways of partitioning the benefits of a coalition which are stable against defection of members of the coalition, and a partition well into the interior of the least core is likely to be very stable. On the other hand, the larger the set of potential members of a coalition, the smaller the set of imputations which will be stable against competing (and overlapping) coalitions. It would be extremely interesting to perform the analysis of this section with a larger set of member countries: how large does the set have to be before the core becomes empty?

3.3 Shapley values

The core and least core identify the claims of groups, but offer no fair or equitable manner for resolving these claims. A completely different approach to a solution is offered by the Shapley value [17,20,10], which is a direct attempt to characterize a concept of fair division. Using essentially four axioms Shapley was able to deduce a unique value for a game. The axioms are: efficiency; a dummy player gets nothing; symmetry; and additivity. The first three axioms are fairly evident: the fourth axiom requires that if we consider two independent games played by the same players, the value calculated by considering the games as one will be the same as that calculated by assigning values to each and then adding them. Under these axioms the *Shapley value* of $V(\cdot)$ for player i is

$$\varphi_i = \sum_{S\subseteq N : i\in S} \frac{|S\setminus\{i\}|!\,|N\setminus S|!}{|N|!} \left(V(S) - V(S\setminus\{i\})\right).$$

There is a simple probabilistic interpretation for this value. Assume the coalition N is built up in a random order; then player i is assigned the expected value of the incremental gain he brings as he joins the coalition.

3.3.1 Examples

For $N = \{$A, B, J, U$\}$ we find Shapley values for the different countries as given in Table 5. Observe that, as in Table 3, Australia is in the least favoured position. For this set of four, UK is in a slightly stronger position than USA, and both are in considerably stronger positions than Japan.

For the second example of $N = \{$ A, B, F, J, U$\}$ the Shapley values are as given in Table 6. Comparing these values with the least core found in Table 4, we see that France remains in the least favoured position. The USA is now in a stronger position than the UK.

A	B	J	U
758	1678	943	1603

Table 5: Shapley values, $|N| = 4$

A	B	F	J	U
826	1479	415	1076	2003

Table 6: Shapley values, $|N| = 5$

There is in general no reason why the Shapley values should lie in the least core. It is interesting, however, that the two approaches rather roughly agree in our two examples, and certainly agree as to who is the weakest coalition member.

4 Bundle dimensioning

This network dimensioning procedure operates by determining link capacities, C_i, such that the blocking across any generalized cut constraint of the network is at most a given level, B. This procedure is a refinement of the multi-commodity flow problem of Section 3.1, and is for three and four node networks much simpler [2,12]. In this section we illustrate the method for the three node network consisting of Japan, UK and USA.

Set

$$\nu_i = \max_t \sum_{j \neq i} \nu_{ij}^t$$

where ν_{ij}^t is the (both-way) traffic between nodes i and j during time period t. From Figure 1 we see that the maxima occur at times 1.00 (Japan), 15.00 (UK) and 15.00 (USA). Let

$$\begin{aligned} N_1 &= C_2 + C_3 \\ N_2 &= C_1 + C_3 \\ N_3 &= C_1 + C_2 \end{aligned}$$

and determine the minimal nodal capacities N_i such that

$$E(\nu_i\,,\, N_i) \leq B\,.$$

(Here $E(\nu, N)$ is Erlang's formula for the loss probability of a single link of capacity N circuits offered Poisson traffic at rate ν.) The link capacities are then determined by

$$\begin{aligned} C_1 &= \frac{1}{2}(N_2 + N_3 - N_1) \\ C_2 &= \frac{1}{2}(N_1 + N_3 - N_2) \qquad (3) \\ C_3 &= \frac{1}{2}(N_1 + N_2 - N_3)\,. \end{aligned}$$

Table 7 presents a comparison of the dimensioning procedures and Table 8 illustrates capacity savings to the three nodes. Note that Table 8 counts capacity in *half* circuits on the assumption that the cost of a circuit is divided equally between the two carriers owning it.

Observe the very close correspondence between the solution to the multi-commodity flow problem and the bundle dimensioning solutions. Changing the bundle dimensioning parameter B from $B = 0.01$ to either 0.005 or 0.05 was observed to have only a small effect on the solution. Finally note that the biggest change in link capacity, compared with that necessary under direct routing, occurs on the Japan-USA link.

Link	Direct routing	MC flow	Bundle $B = 0.01$
B-J	2030	1544	1555
J-U	3542	2100	2108
B-U	8323	7488	7454
TOTAL	13895	11133	11118
Saving (%)		20	20

Table 7: Comparison of dimensioning procedures

Carrier	Direct routing	Bundle $B = 0.01$	Saving (circuits)
B	5176	4505	670
J	2786	1832	954
U	5932	4781	1151

Table 8: Capacity savings

We note in passing that in a network dimensioned according to the direct routing column of Table 7 the excess capacity would allow, with dynamic routing [1,5,6,11], node overloads of 53% for Japan, 24% for USA or 15% for UK (in a node overload all traffics from the designated node are increased by the given percentage) or a general overload of 15% (in a general overload all traffics are increased by the given percentage). The same total capacity, reallocated between links, approximately in proportion to any other of the columns of Table 7, would allow a node overload of 25% for *any* of the three nodes, or a general overload of 25%. All three countries would benefit from the improved general overload performance; the UK would benefit especially from the improved node overload performance.

The various cost estimates of Section 3.1 assumed equal circuit costs on the various links. For three node networks, however, the minimal cost network is markedly insensitive to precise link costs. In particular the multi-commodity flow solution given in Table 7 will remain the optimal solution for *any* link costs, provided only the cost of a circuit on the direct link between two nodes is less than the total cost of a circuit from each of the other two links (the *triangle inequality*). The bundle dimensioning procedure is based on the representation of a good dynamic routing scheme by a repacking strategy [4,7,12].

5 A four node network

In this section we briefly discuss the four node network comprising Australia, Japan, UK and USA. The step from three to four nodes introduces a number of new issues: we no more than touch on some of these. A major new issue is that in four node networks there may be a variety of network optimal dimensioning solutions; recall that for three node networks the network optimal dimensioning solution was essentially unique, and robust even to varying link costs.

5.1 Dimensioning by cut constraints

Let C_{ij} be the capacity of the link connecting nodes i and j and consider the following linear program.

$$\min \sum_{i<j} C_{ij}$$

such that

$$\begin{aligned} \sum_{i\in S, j\notin S} C_{ij} &\geq \max_t \left(\sum_{i\in S, j\notin S} \nu_{ij}^t \right) \ \forall S \subset N,\ S \neq \emptyset, N \\ C_{ij} &\geq 0\,. \end{aligned}$$

Observe that the constraints of this linear program correspond to statements that the capacity across a cut must exceed the maximum flow across that cut.

For a four node network a capacity vector $(C_{ij}, i < j)$ is feasible for the above linear program if and only if it is feasible for the multi-commodity flow problem in Section 3.1 [2]. For networks containing five or more nodes this equivalence no longer holds [2]: a capacity vector $(C_{ij}, i < j)$ may satisfy the various cut constraints, but not be able to support a flow pattern. (In addition to the constraints generated by simple cuts, flow patterns must also satisfy inequalities which may be termed *generalized* cut constraints.) For four node networks the equivalence considerably simplifies network dimensioning procedures.

5.2 Example

Consider the example of $N = \{$ A, B, J, U $\}$ with linear program

$$\min \sum_{i<j} C_{ij}$$

such that

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} C_{AB} \\ C_{AJ} \\ C_{AU} \\ C_{BJ} \\ C_{BU} \\ C_{JU} \end{pmatrix} \geq \begin{pmatrix} 2946 \\ 9709 \\ 4019 \\ 10472 \\ 9954 \\ 6254 \\ 10303 \end{pmatrix}$$

$$C_{ij} \geq 0\,.$$

The extreme point solutions are shown in Table 9. All of these solutions, and any point in their convex hull, achieve the same minimal value for total capacity. In fact solution I is identical to that obtained by the multi-commodity flow problem considered in Section 3.1.

If link costs vary then this can affect the network optimal solution in four node networks. We illustrate this briefly with the above example. Let $c_1, c_2, \ldots, c_6$ be circuit costs on links, labelled in the order of Table 9. Compare $c_1 + c_6$, $c_2 + c_4$ and $c_3 + c_5$. If $c_1 + c_6$ is smallest then solution I is cheaper than solution II or III. If $c_2 + c_4$ is smallest then solution II is favoured, while if $c_3 + c_5$ is smallest then solution III is favoured. Observe that any given country has the same circuit capacity emanating from it under solutions I, II or III. What changes between these solutions is which of the last three constraints is satisfied as an inequality. Observe that these three cut constraints correspond to 2|2 cuts, that is cuts which separate the four nodes into two pairs.

Link	I	II	III
A-J	356	356	37
A-U	1558	1239	1558
A-B	1032	1350	1350
B-J	1713	1394	1713
J-U	1950	2268	2268
B-U	6963	6963	6645

Table 9: Extreme point solutions

5.3 Bundle dimensioning with 4 nodes

Table 10 shows the effect on solutions I, II and III of bundle dimensioning with parameter $B = 0.01$.

Link	I	II	III
A-J	375	375	37
A-U	1569	1231	1569
A-B	1025	1363	1363
B-J	1723	1385	1723
J-U	1937	2275	2275
B-U	6932	6932	6594

Table 10: Extreme point solutions, $B = 0.01$

Table 11 compares the various procedures. To construct this table we have used the average (or centroid) of solutions I, II and III: this is a reasonable procedure, producing some slack on each of the three 2|2 cuts. As in Table 7 the parameter $B = 0.01$ leads to a similar cost saving to the multi-commodity flow solution.

Link	Direct routing	MC flow	Bundle
A-J	375	249	262
A-U	2354	1451	1456
A-B	1932	1244	1250
B-J	2030	1606	1610
J-U	3542	2162	2162
B-U	8323	6857	6819
TOTAL	18558	13572	13563
Saving(%)		27	27

Table 11: Comparison of dimensioning procedures

Carrier	Direct routing	Bundle dimensioning	Saving (circuits)
A	2331	1484	846
B	6142	4840	1302
J	2973	2018	955
U	7110	5220	1889

Table 12: Capacity savings with 4 nodes

Table 12 illustrates capacity savings to the four nodes, using again a centroid solution, and should be compared with

Table 8. Observe that the inclusion of Australia has made almost no difference to the capacity savings for Japan, but has nearly doubled the capacity savings for UK. This provides a further interesting insight into the issues discussed in Section 3: we might expect UK to be particularly enthusiastic about the inclusion of Australia in a coalition.

6 Conclusions

Based on the trial data set, our general conclusions are as follows. If Japan, UK and USA dynamically route traffic between themselves a capacity saving of around 20% is possible. Between Australia, UK and USA the potential saving is around 17%, while between all four countries, Australia, Japan, UK and USA, the potential saving is as high as 27%. It is reasonable to view Australia and Japan as complements rather than substitutes. Capacity savings to the UK are approximately doubled if Australia is included in a coalition with Japan, UK and USA. The USA and UK are the most important members of the potential coalitions studied: their presence or absence makes most difference to the capacity savings achievable.

The potential capacity savings are not shared equally over countries and neither are they shared in proportion to traffic. For a coalition between Japan, UK and USA the potential savings to USA and Japan are greater than to UK. Correspondingly the UK carries a greater peak transit traffic, and also a higher volume of transit traffic.

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