

# Preface

The main topic of this book is the study of the behaviour in equilibrium of vector stochastic processes, or stochastic networks. Such processes have a wide range of applications: to give some examples, the components of the vector may represent queue sizes in a queueing network, gene frequencies in a population, or the condition of fruit trees in an orchard. When a stochastic network is reversible its analysis is greatly simplified, and the first chapter is devoted to a discussion of the concept of reversibility. Two themes emerge from the remainder of the book: first, the various uses of reversibility, in the study of the output from a queue, the flow of current in a conductor, the age of an allele, or the equilibrium distribution of a polymerization process; second, the extent to which the assumption of reversibility can be relaxed without destroying the associated tractability.

The main prerequisite is an understanding of Markov processes at about the level of Feller's *Introduction to Probability Theory and Its Applications*, Volume I. In Section 1.1 the necessary material is very briefly reviewed, primarily to establish terminology and notation.

For their comments and advice I am indebted to many people, particularly Dave Aldous, Andrew Barbour, Dieter Koenig, Rolf Schassberger, and Geoff Watterson. I am especially grateful to Peter Whittle, whose lectures on reversibility first interested me in the subject and without whose encouragement the book would not have been written. Finally, my thanks go to Jackie Kelly for computing the graphs in the book and to Angie Ashton for typing the final draft.

Cambridge, Christmas 1978

FRANK KELLY

# Contents

|                  |   |            |
|------------------|---|------------|
| <b>CHAPTER 1</b> | <b>MARKOV PROCESSES AND REVERSIBILITY</b>       | <b>1</b>   |
| 1.1              | <i>Preliminaries on Markov processes</i>        | 1          |
| 1.2              | <i>Reversibility</i>                            | 5          |
| 1.3              | <i>Birth and death processes</i>                | 10         |
| 1.4              | <i>The Ehrenfest model</i>                      | 17         |
| 1.5              | <i>Kolmogorov's criteria</i>                    | 21         |
| 1.6              | <i>Truncating reversible processes</i>          | 25         |
| 1.7              | <i>Reversed processes</i>                       | 27         |
| <b>CHAPTER 2</b> | <b>MIGRATION PROCESSES</b>                      | <b>34</b>  |
| 2.1              | <i>The output from a simple queue</i>           | 34         |
| 2.2              | <i>A series of simple queues</i>                | 37         |
| 2.3              | <i>Closed migration processes</i>               | 40         |
| 2.4              | <i>Open migration processes</i>                 | 48         |
| <b>CHAPTER 3</b> | <b>QUEUEING NETWORKS</b>                        | <b>57</b>  |
| 3.1              | <i>General customer routes</i>                  | 57         |
| 3.2              | <i>Open networks of quasi-reversible queues</i> | 65         |
| 3.3              | <i>Symmetric queues</i>                         | 72         |
| 3.4              | <i>Closed networks</i>                          | 82         |
| 3.5              | <i>More general arrival rates</i>               | 89         |
| <b>CHAPTER 4</b> | <b>EXAMPLES OF QUEUEING NETWORKS</b>            | <b>95</b>  |
| 4.1              | <i>Communication networks</i>                   | 95         |
| 4.2              | <i>Machine interference</i>                     | 99         |
| 4.3              | <i>Timesharing computers</i>                    | 105        |
| 4.4              | <i>Teletraffic models</i>                       | 108        |
| 4.5              | <i>Compartmental models</i>                     | 113        |
| 4.6              | <i>Miscellaneous applications</i>               | 117        |
| <b>CHAPTER 5</b> | <b>ELECTRICAL ANALOGUES</b>                     | <b>125</b> |
| 5.1              | <i>Random walks</i>                             | 125        |
| 5.2              | <i>Flow models</i>                              | 128        |
| 5.3              | <i>Invasion models</i>                          | 132        |

|           |                                      |     |
|-----------|--------------------------------------|-----|
| CHAPTER 6 | REVERSIBLE MIGRATION PROCESSES       | 135 |
| 6.1       | <i>Migration processes revisited</i> | 135 |
| 6.2       | <i>Social grouping behaviour</i>     | 138 |
| 6.3       | <i>Contrasting flow models</i>       | 140 |
| CHAPTER 7 | POPULATION GENETICS MODELS           | 145 |
| 7.1       | <i>Neutral allele models</i>         | 145 |
| 7.2       | <i>The age of an allele</i>          | 151 |
| 7.3       | <i>Fixation times</i>                | 156 |
| CHAPTER 8 | CLUSTERING PROCESSES                 | 161 |
| 8.1       | <i>Introduction</i>                  | 161 |
| 8.2       | <i>The basic model</i>               | 162 |
| 8.3       | <i>Examples</i>                      | 167 |
| 8.4       | <i>Polymerization processes</i>      | 173 |
| 8.5       | <i>Generalizations</i>               | 180 |
| CHAPTER 9 | SPATIAL PROCESSES                    | 184 |
| 9.1       | <i>Markov fields</i>                 | 184 |
| 9.2       | <i>Reversible spatial processes</i>  | 189 |
| 9.3       | <i>A general spatial process</i>     | 193 |
| 9.4       | <i>Partial balance</i>               | 200 |
|           | <i>References</i>                    | 212 |
|           | <i>Symbol Index</i>                  | 223 |
|           | <i>Subject Index</i>                 | 227 |