Putting the Hobson-Rogers model to the test.

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Abstract

In this paper, we take the model of Hobson & Rogers (1998) for the movement of an asset, and we fit it to data. The model of Hobson & Rogers is a stochastic volatility model, where the volatility depends on the *offset* of the current log-price from its exponentially-weighted historical value. As such, the model is complete, and there are unique preference-independent prices. In the datasets we study, there is very clear evidence that the volatility does indeed vary with offset; we use PDE methods, and simulation methods, to fit the model to data, and find that the fit is generally better than the Heston model.

Introduction

The striking work of (BLACK and SCHOLES 1973) and the simultaneous opening of the Chicago Board Option Exchange in 1973 started a new era for financial mathematics. The Black-Scholes formula is still widely used among traders to price vanilla options, at least as a metric for the risk implied by actual quotes. However, especially after the market crash of 1987 and the advent of powerful computers and new mathematical technologies, many efforts have been devoted to develop new models and to study their pricing implications.

As a matter of fact, volatility of the underlying stock is not constant (see e.g. (BLATTBERG and GONEDES 1974)), and implied volatility varies across strikes. In a famous paper (RUBINSTEIN 1985) proved that short maturity out of the money calls are priced higher relative to other calls than Black and Scholes would predict, and that strike price biases are statistically significant and can reverse over time.

The specification of volatility as a stochastic process is extremely natural, since it can explain a number of different empirical findings, such as distributional properties of the underlying or the presence of transaction costs. For example the observed correlation between volatility and asset prices can be explained by 'level dependent' volatility models like the Constant Elasticity of Variance (CEV)[1] model of (COX and ROSS 1976). Stochastic volatility can also arise endogenously: (PLATEN and SCHWEIZER 1998) obtain it through an equilibrium argument, modelling the behaviour of market participants.

Fully stochastic volatility models (introduced in 1987 by (HULL and WHITE 1987), (SCOTT 1987), and (WIGGINS 1987)) are again motivated by the idea that a sensible specification of the volatility process can offer generality and analytical tractability. Sadly it is often the case that using 'plausible' parameters, like the negative correlation between volatility and prices observed in practice, it is difficult to match actual option prices. In their empirical study (BAKSHI, CAO, and CHEN

[1] Although widely used as an alternative to the standard log-Brownian model, it is not hard to see that if the exponent is less than 1 then the CEV stock price will hit zero in finite time almost surely; in some applications it may be hard to justify this property.

1997) find indeed that structural parameters obtained via calibration are significantly different from their historical time-series estimated counterparts.

In this sense the fact that the literature is increasingly concerned with the consistency of models both cross-sectionally and intertemporally is not surprising. The study in Section 2 is intended to support the volatility structure of our model before the investigation of its option pricing implications.

Generalized Autoregressive Conditional Heteroskedastic (GARCH) processes are in a limit sense the discrete time counterpart of stochastic volatility models (see (CORRADI 2000)). Under assumptions on the utility of the investor, (DUAN 1995) derives unique option prices. Nonetheless without the familiar (complete) continuous time framework it is impossible to define an exact replicating strategy.

Usually, a pricing model has to balance theoretical generality and consistency of the volatility structure against the ability to estimate its parameters efficiently and precisely. In the first sense the literature achieved some encouraging results, raising new important issues and developing different approaches (see (BATES 2003) and (GHYSELS, HARVEY, and RENAULT 1996) for an insightful review). Recently (FOUQUE, PAPANICOLAU, and SIRCAR 2000) proposed an efficient and robust (almost specification free) method for the modeling, analysis and stable estimation of important groupings of market parameters, exploiting the fast mean reversion of volatility. Their key idea is to compute a simple correction of Black-Scholes model which reflects the effect of stochastic volatility on derivative prices.

The paper is organized as follows: in Section 1 we present the model, underlying the particular case used in the numerical procedures. In Section 2 we propose a simple empirical analysis to support our main modelling assumption. In Section 3 we explain both the finite difference and the Monte Carlo methods used to calibrate the model. In Section 4 we perform a limited but instructive comparison between some well known pricing models. In Section 5 we discuss the use of asymptotic expansion for our model. Finally in Section 6 we conclude the paper and we suggest some remaining research issues.

1 The Hobson-Rogers model

(HOBSON and ROGERS 1998) (henceforth HR) specify local volatility in terms of weighted moments of past returns. Let us denote with $Z_t = \log(e^{-rt}P_t)$ the log-discounted price process, and define the offset function of order m as

$$S_t^{(m)} = \lambda \int_0^{\infty} e^{-\lambda u}(Z_t - Z_{t-u})^m \mathrm{d}u, \tag{1}$$

where the parameter λ describes the weight of historic observations. Stock prices are driven by the stochastic differential equation

$$\mathrm{d}Z_t = \sigma\left(t, Z_t, S_t^{(1)}, \ldots, S_t^{(n)}\right)\mathrm{d}B_t + \mu\left(t, Z_t, S_t^{(1)}, \ldots, S_t^{(n)}\right)\mathrm{d}t$$

for some smooth functions $\sigma(\cdot) > 0$ and $\mu(\cdot)$.

Since $\sigma(\cdot)$ can eventually depend on P_t, the model includes as a subclass the case when the volatility rate is a deterministic function of the underlying. Furthermore the hypotheses preserve completeness, allowing for preference independent option pricing. This last feature constitutes an advantage over fully stochastic volatility processes, where arbitrage considerations are not sufficient to identify 'risk premia' uniquely.

In the following, we will assume the instantaneous volatility is a function of the first order offset $S_t = S_T^{(1)}$ only, since we want to obtain a tractable PDE and to solve it with reliable precision. HR showed that even in this case the model has the potential to explain volatility smiles and skews, and our simulation studies seem to suggest that including higher order offset functions does not improve the results significantly.

Using equation (1) we readily decompose S_t as the deviation of the current price from an exponentially weighted average of past records

$$S_t = Z_t - \lambda \int_0^{\infty} e^{-\lambda u} Z_{t-u} \mathrm{d}u. \tag{2}$$

The latter says that λ determines the horizon of the 'moving time window' of the integral on the right. For bigger values of this parameter, S_t is more dependent on the recent past, while small values almost identify the offset increments with price changes. Obviously in this case a level dependent volatility assumption would be numerically more convenient.

To build the basic model used in the numerical procedures, consider the risk neutral measure $\tilde{\mathbb{P}}$ and the $\tilde{\mathbb{P}}$-Brownian motion $\tilde{B}_t$. Let $e^{-rt}P_t$ be a $\tilde{\mathbb{P}}$-martingale solving $\mathrm{d}\tilde{P}_t = \sigma \tilde{P}_t \mathrm{d}\tilde{B}_t$, so that by Itô's formula

$$\mathrm{d}Z_t = \sigma \mathrm{d}\tilde{B}_t - \frac{1}{2}\sigma^2 \mathrm{d}t. \tag{3}$$

With the substitution $u = t - s$ equation (2) gives

$$S_t = Z_t - \lambda e^{-\lambda t} \int_{-\infty}^{t} e^{\lambda s} Z_s \mathrm{d}s,$$

and we can easily compute the differential of S_t

$$\mathrm{d}S_t = \mathrm{d}Z_t - \lambda S_t \mathrm{d}t. \tag{4}$$

HR find a general formula for the differential of higher order offsets and prove that $\left(Z_t, S_t^{(1)}, \ldots, S_t^{(n)}\right)$ forms a Markov process.

Finally take $U = Z - S$ and denote with $f(t, U_t, Z_t)$ the price at time $t < T$ of a contingent claim worth

$$f(T, U_T, Z_T) = q(Z_T) \tag{5}$$

at maturity T. Using standard arguments we derive HR PDE

$$0 = f_t + \lambda(Z - U) f_U - \frac{1}{2}\sigma(Z - U)^2 (f_{ZZ} - f_Z), \tag{6}$$

with boundary condition (5).

2 Does volatility depend on the offset?

Volatility changes over time: the explanation of its movements represent a key issue in finance theory, and various modelling attempts have been proposed in literature (see for instance (SCHWERT 1989)).

The dependence between volatility and past returns is intuitively appealing. In their conclusions (DUMAS, FLEMING, and WHALEY 1998) suggest to relate the volatility surface to past changes in the index level, while pratictioners commonly use exponentially weighted moving averages to forecast volatility. Moreover, this hypothesis introduces an effect of volatility clustering. It is in fact clear from Equation (1) that, depending on the values of λ, large changes in the price will cause the offset to substantially modify for a certain period. Many other characteristics of the volatility dynamics are naturally explained, including the local persistence and state dependent volatility of volatility recently observed by (CHERNOV, GALLANT, GHYSELS, and TAUCHEN 2003).

We carry out a simple analysis using SP500 index settlement prices collected from the CME, with sample period between Jan 1993 and Dec 2002. Consider the approximation for the offset process S given by

$$\hat{S}_t = \sum_{i=0}^{M} \frac{w_i}{W}(Z_t - Z_{t-i}), \tag{7}$$

where the weights are $w_i = e^{-\lambda i \Delta t}$ and W is their sum. As a corresponding estimate of volatility take

$$\hat{\sigma}_t = \sqrt{k \sum_{i=1}^{M} \frac{w_i}{W}(Z_{t-i+1} - Z_{t-i} - \hat{\mu}_t)^2}, \tag{8}$$

where $\hat{\mu}_t$ is the weighted mean of log returns between time $t - M$ and time t, and $k = W^2 / \sum \left(\frac{W^2}{n} - w_i^2\right)$ is a correction to make $\hat{\sigma}_t^2$ unbiased. All estimates are computed on a daily basis, and we consider a calendar of 242 trading days implying a unit of time $\Delta = 1/242$.

We fixed the lookback as $M = 2000$ using overlapping data[2], that is we dynamically compute an estimate based on 2000 historical observations for each trading day. Note that thanks to their particular construction (7) and (8) are based on the same amount of past information.

[Figure 1 about here]

The six plots in Figure 1 show the relationship between our estimate (7) of the offset and (8) of the volatility for various values of λ. Each subplot includes the curve obtained regressing the volatility $\hat{\sigma}_t$ over a second order polynomial in the offset $a + b\hat{S} + c\hat{S}^2$. The resulting minimum squared error parameters are reported

[2]This huge M is to ensure that both $\hat{S}_t$ and $\hat{\sigma}_t$ are computed with a sufficient number of data even in the case where λ is small.

in Table 1, together with their 95% confidence intervals and the corresponding λ values.

[Table 1 about here]

Nonetheless a simple parabola is unlikely to capture the relation between offset and volatility. Extensive numerical studies prove that a very effective specification for the volatility function is given by

$$\sigma(s) = \frac{1 + as + bs^2}{c + ds + es^2}\,, \tag{9}$$

combining the flexibility of rational polynomials with a reasonable number of parameters. The alternative

$$\sigma(s) = \sqrt{a + bs^2} \wedge N\,, \tag{10}$$

originally proposed in HR and discussed in Section 4, turned out to be particularly effective because it largely reduced efforts involved in the optimization problem.

This qualitative study provides empirical evidence in favour of our conjectures and help us in the selection of a proper volatility function.

3 Empirical Methodology

The parameters of HR models can be estimated using different strategies. The likelihood function cannot be computed, but calibration could be based on statistically respectable procedures thanks to the developements of econometric literature during the last decade. (ANDERSEN, CHUNG, and RENSEN 1999) make an extensive investigation of the EMM technique and provide a source of related literature. (TOMPKINS 2001) applies a similar method to estimate the parameters of a class of Heston related models, and finds that the inclusion of a stochastic volatility process consistent with the objective process alone does not explain volatility smiles. (CHERNOV, GALLANT, GHYSELS, and TAUCHEN 2003) compare many diverse specifications on a long daily Dow Jones industrial average index return series and suggest that the choice between models should be based upon practical criteria.

These approaches benefit from the existence of a proper statistical theory, but some drawbacks remain. The choice of the score generator or of the key attributes is often critical and influences the precision in terms of standard errors. The presence of local optima, common to every optimization problem, has to be tackled when maximizing the quasi-likelihood. Moreover, (ROGERS and SATCHELL 2000) show why linking distributional properties of real world and risk neutral measures should be considered carefully.

Calibration has been widely considered in the literature. (SCOTT 1987) uses Monte Carlo simulation to estimate his stochastic volatility model, optimizing over the sum of squared errors between the model and actual prices. He finds that his model outperforms Black and Scholes, but tends to overprice out of the money options, and he suggests to use a larger sample. The so called 'indirect inference' techniques obtained varying success. (JACKWERTH and RUBINSTEIN 1996) derive underlying asset risk neutral probability distributions implied by index option and underlying asset prices. Their results show robustness over alternative optimization specifications and stability of implied levels of skewness and kurtosis over time. Furthermore they outperform Black and Scholes lognormality assumption in the explanation of rare events. More recently (DUMAS, FLEMING, and WHALEY 1998) calibrate a model with deterministic volatility function and find that its predictive performance is no better than ad-hoc smoothings of Black and Scholes.

We used two different approaches to calibrate our model. The solution of PDE (6) is supposed to be fast and accurate when it converges, but the idea of simulating SDEs (3) and (4) is also attractive since the model is driven by one single source of uncertainty.

3.1 Finite difference methods

The application of finite difference methods to option pricing goes back at least to (SCHWARTZ 1977) and (BRENNAN and SCHWARTZ 1978), and later (COURTADON 1982) and (HULL and WHITE 1990). A recent textbook treatment of the subject is given by (TAVELLA and RANDALL 2000).

The main idea of finite difference methods is to convert a PDE into a set of difference equations and to solve them iteratively.

In order to obtain a numerical solution of the PDE (6) by finite differencing we need first of all to build a grid containing a discrete set of points for each variable. We determined the range for the logprice and the offset according to the laws of the solutions of the corresponding SDEs, thus linking the grid size to the parameters used for each solution. Then we fixed a reasonable number of steps in each direction testing our results in the Black and Scholes and other known settings.

More advanced techniques use combinations of explicit and implicit finite difference methods. While the hopscotch method alternates between them, the θ-method uses a weighted combination of explicit and implicit approximations. The typical final system has the form

$$A(\theta)f_i = B(\theta)f_{i+1}, \qquad i = 0, \ldots, T - \Delta t, \tag{11}$$

where A and B are typically huge sparse matrices depending on both the weight θ and the problem's parameters. We choose the well known Crank-Nicholson method, that uses the uniform weight $\theta = \frac{1}{2}$. This scheme is stable and high-order accurate. As with any implicit method, the linear system (11) has to be built and solved. In this task it is essential to take full advantage of the structure of the matrices A and B, using routines for sparse linear systems such as the freely distributed UMFPACK 4.0 in order to avoid the direct computation of A's inverse.

Another important problem is how to compute the value of the derivatives on those points where we are forced to cut the range of our variables for computational needs; in other words we must provide some numerical boundaries for the edges of our finite grid.
Various easy approaches are possible, such as forcing the exercise of the derivative, or killing or reflecting the diffusion outside the edges. Unfortunately, this is not sufficiently precise especially in a two dimensional scenario.
The undetermined coefficient method approximates the value of a partial derivatives at a point using a weighted sum of the solution values on the nearest grid points, for example

$$f_z(t, S_t, Z_t) = af(t, S_t, Z_t + \Delta Z) + bf(t, S_t, Z_t + 2\Delta Z) + cf(t, S_t, Z_t + 3\Delta Z),$$

where $f_z(\cdot)$ is the partial derivative of f with respect to Z and ΔZ is the Z increment for our grid. We can now expand $f(\cdot)$ in Taylor series and match the two sides of the equation, determining the unknown coefficients through a linear system. This is more difficult for the mixed derivative $f_{sz}(\cdot)$, where we used a multivariate Taylor expansion with fifteen terms (and coefficients to determine): in this case we used some software to manipulate symbolic algebra.

3.2 Monte Carlo simulations

Since their first appearance in (BOYLE 1977), Monte Carlo techniques proved to be a very flexible tool in financial applications. (BOYLE, BROADIE, and GLASSERMAN 1997) discuss some of them and describe a number of useful variance reduction techniques studying their efficiency. For complete and detailed references, see (KLOEDEN and PLATEN 1999) and (GLASSERMAN 2003).

The price of a derivative security is the expected value under the risk neutral $\tilde{\mathbb{P}}$ of its discounted payoff. The Monte Carlo method lends itself naturally to this evaluation. The main principle is to generate sample paths of the underlying assets over the time interval of interest, compute the discounted payoff of the derivative according to its definition and take the average over the sample paths. By the strong law of large numbers this estimate converges to the true price, and we easily obtain an estimate of its error from the central limit theorem.

In order to obtain a remarkable precision we used Richardson extrapolation, Milstein's method to improve the strong order of convergence, the antithetic variable technique and the control variate technique, using Black and Scholes price as a control variate.

Monte Carlo techniques are not very precise for calibration, for example changing the initial random seed prices can vary significantly. Nonetheless thanks to this approach we were able to answer a number of questions. First of all, we realized that in the data sets we considered higher order offset inclusion is not needed. Furthermore Monte Carlo prices provide a rough comparison to PDE prices, and the simulated paths help to fix the size of the finite difference schemes grids. In the following, all reported prices are obtained via finite differencing.

3.3 Cost function specification

The objective to be minimized was the sum over the different strikes of the relative errors. We do this because it represents a measure of the success of an investment.

Formally, let θ be the vector of parameters of the volatility function (9) together with λ, and denote by Θ its parametric space. Let $f_{HR}(t, K_i, Z_t, \hat{S}_t, \theta)$ be the HR price when the strike is K_i for $i = 1, 2, \ldots, N$ and the estimate $\hat{S}_t$ is computed via (7), and let $v(t, K_i, Z_t)$ be the actual price. We want to find

$$\min_{\theta \in \Theta} \sum_{i=1}^{N} \frac{|v(t, K_i, Z_t) - f_{HR}(t, K_i, Z_t, \hat{S}_t, \theta)|}{v(t, K_i, Z_t)},$$

or the minimum of the logarithm of the sum when we expect some percentage errors to be particularly big. We used many different routines to solve this global optimization problem, but we obtained the best results with CFSQP, a set of C functions based on Sequential Quadratic Programming. Actually, each evaluation of the cost function takes about 0.1 seconds on a PENTIUM 3.6 Ghz Linux machine both for the finite difference and the Monte Carlo approaches, while the whole optimization needs between one and five minutes.

4 Comparisons with different models

Testing alternative models is a challenging task, requiring careful procedural choices to build a reliable empirical methodology. The purpose of the present section is to gain a rough idea of the empirical performances of some well known pricing models in the limited scenario of a cross-section of option prices. Let us briefly introduce the proposed alternatives.

Constant Elasticity of Variance The CEV model defines volatility as a deterministic function of the stock price, trying to capture the leverage effect observed by (BLACK 1976). The stock price follows the diffusion

$$\mathrm{d}P_t = \mu P_t \mathrm{d}t + \delta P_t^{\beta/2} \mathrm{d}B_t,$$

so that for $\beta < 2$ the volatility $\sigma(S_t, t) = \delta P_t^{(\beta-2)/2}$ is inversely related with prices, while if $\beta = 2$ we recover the Black and Scholes case.

(SCHRODER 1989) expresses the CEV pricing formula in simple terms of the non-central chi-square distribution[3], and the optimization over the two parameters requires little effort too.

Heston model In (HESTON 1993) model the spot asset is assumed to be governed by the diffusion

$$\mathrm{d}P_t = \mu P_t \mathrm{d}t + \sigma_t P_t \mathrm{d}B_t^{(1)}$$

and the volatility is an Ornstein-Uhlenbeck process. This can be written as the square-root process

$$\mathrm{d}\nu_t = \kappa\,(\phi - \nu_t)\,\mathrm{d}t + \varsigma \sigma_t \mathrm{d}B_t^{(2)},$$

where $\nu_t = \sigma_t^2$. The model allows correlation ρ between $B_t^{(1)}$ and $B_t^{(2)}$, and it is possible to obtain a close form solution for vanilla options via Fourier inversion (see (DUFFIE 2001) for a detailed exposition of the transform analysis approach). This involves just the computation of a non trivial integral, but a further assumption that gives the price of volatility risk has to be made to price contingent claims.
We performed the optimization over six parameters: λ specifies the risk premium, $\kappa^* = \kappa + \lambda$ the mean reversion, $\phi^* = \kappa\phi/(\kappa+\lambda)$ the long run variance, ν_0 the starting (current) variance, ρ the correlation and ς the volatility of the volatility.

[Table 2 about here]

Table 2 shows the optimization results on a cross section of option prices for Black and Scholes, CEV, Heston, and HR models for both volatility functions (9) (label HR1) and (10) (label HR2). The second column shows the market prices of a European put option on SP500 index on the 20th of March 2003, with strikes specified in the first column and maturity September 2003. As an annual interest rate we considered the corresponding T-bill $r = 0.0135$ available from Federal Reserve Statistical Releases. We choose a quite long time to maturity[4] remembering that smile effect decreases with it (see e.g. (DERMAN and KANI 1994)), thus simplyfing the task of pricing on such a broad range of moneyness values. On the right of each estimated price we report the percentage relative error between parentheses, while

[3] in order to avoid difficulties we used a robust interpolation technique, see the reference for details.
[4] $\tau = 0.5289256$ considering one year of 242 trading days.

the last row shows the mean of the absolute percentage relative error across the strikes for each model.

We see that the Black and Scholes model can be reasonably good on a small number of strikes around the current stock value, but completely fails to fit a large cross section of option prices. On the other hand, considering the small number of parameters used, the CEV model performs far better and it is easy to calibrate. Sadly, theoretically it is not able to explain the change over time in the direction of the implied volatility skews observed by (RUBINSTEIN 1985). Both the Heston model and the HR model provide an excellent fit at the cost of an increased computational burden. However, for this dataset the simple specification HR2 provides the best fit using the simple volatility specification (10) and just three parameters.

The model performance was tested against a variety of other datasets, considering different underlying assets, multiple maturities and the behaviour of parameters through time. Without entering into the econometric details, in general HR performances are comparable with if not superior to other commonly used stochastic volatility models.

5 Why no asymptotics?

We have seen how the price $f(t, U, Z)$ at time t of a European derivative solves a PDE, which we can compute numerically, but this is a time-consuming business, and the model would be much easier to use if there were some simple asymptotics. The natural form to look for would be to express the derivative price as the Black-Scholes price plus some (smaller order) correction, since the Black-Scholes price is what we could obtain if the volatility were constant. But to obtain an asymptotic, we need to identify some small parameter and expand in that.

One possibility would be to take λ^{-1} to be the small parameter, but this has certain drawbacks. It is true that if λ^{-1} were small then the offset would typically be small, and so the volatility would be close to $\sigma(0)$; but then in times that are $O(1)$ the diffusion S will quickly settle down to its invariant distribution, and the price will lose dependence on the initial value of the offset, becoming in effect a Black-Scholes price with an averaged variance. Moreover, as Figure 1 shows, the volatility does not appear to be close to a constant (at least in the real-world measure).

Another possibility would be to keep λ fixed, but instead to take the volatility of the form $\sigma(\varepsilon S)$ for small ε; this way, the initial value of the offset could continue to influence prices for some time, but the correction will itself solve a PDE, which is no easier than the PDE for calculating the price.

But however one tries to develop an asymptotic, Figure 1 ruins the attempt; offsets are not generally small, and volatility is not approximately constant, so no asymptotic based on assuming either of these properties can succeed.

The paper of (HUBALEK, TEICHMANN, and TOMPKINS 2004) obtains an asymptotic for a model where $\sigma(s) = \sigma_0(\sqrt{\varepsilon}s)$ for some small ε; again in view of Figure 1 it seems unlikely that any such asymptotic will explain the data well, and few results of fitting this model are reported in their work.

6 Conclusions

This study has tested the model of HR at two levels. Firstly, we have investigated whether there is any apparent dependence of volatility on offset; fitting quadratic functions of offset to volatility data shows a conclusive dependence. Secondly, we have attempted to fit some simple functional forms to the dependence of volatility on offset, using options data on the SP500. The quality of fit is excellent, generally twice as good as the popular Heston model, and within the bid-ask spread. The numerics involved are somewhat more complicated than (for example) pricing in the Heston model using transform methods; nevertheless, given that one has available good finite-difference code for sparse linear systems, the development time is not great, and the run times for solving the PDE are of the order of 0.1s. In addition to the good fit, the model has two advantages at a theoretical level: it is complete (so no arbitrary choices of market price of risk are needed); and option prices do not depend solely on spot, strike and expiry, so that different skew/smile profiles may be generated for the same spot, depending on the value of the offset. This means in particular that it may be possible to avoid time-dependent implied-volatility surfaces as a way to explain observed prices. Further work is needed to evaluate this model in this respect, but we conclude from the present study that its performance is good enough already to justify it.

References

ANDERSEN, T. G., H. CHUNG, and B. E. SØRENSEN, 1999, Efficient method of moments estimation of a stochastic volatility model: a monte carlo study, *Journal of Econometrics* 91, 61–87.

BAKSHI, G., C. CAO, and Z. CHEN, 1997, Empirical performance of alternative option pricing models, *The Journal of Finance* 52, 2003–2049.

BATES, D. S., 2003, Empirical option pricing: a retrospection, *Journal of Econometrics* 116, 387–404.

BLACK, F., 1976, Studies of stock price volatility changes, Proceedings of the 1976 Meetings of the American Statistical Association, 177-181.

——— , and M. SCHOLES, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–659.

BLATTBERG, R. C., and N. J. GONEDES, 1974, A comparison of the stable and student distributions as statistical models for stock prices, *Journal of Business* 47, 244–280.

BOYLE, P., 1977, Options: A monte carlo approach, *Journal of Financial Economics* 4, 323–38.

——— , M. BROADIE, and P. GLASSERMAN, 1997, Monte Carlo methods for security pricing, *Journal of Economic Dynamics and Control* 21, 1267–1321.

BRENNAN, M. J., and E. S. SCHWARTZ, 1978, Finite difference method and jump processes arising in the pricing of contingent claims, *Journal of Financial and Quantitative Analysis* 13, 461–474.

CHERNOV, M., A. R. GALLANT, E. GHYSELS, and G. TAUCHEN, 2003, Alternative models for stock price dynamics, .

CORRADI, V., 2000, Reconsidering the continuous time limit of the GARCH(1,1) process, *Journal of Econometrics* 96, 145–153.

COURTADON, G., 1982, A more accurate finite difference approximation for the valuation of options, *Journal of Financial and Quantitative Analysis* 17, 697–703.

COX, J. C., and S. A. ROSS, 1976, The valuation of options for alternative stochastic processes, *Journal of Financial Economics* 3, 145–166.

DERMAN, E., and I. KANI, 1994, Riding on the smile, *RISK* 7, 32–39.

DUAN, J. C., 1995, The GARCH option pricing model, *Mathematical Finance* 5, 13–32.

DUFFIE, D., 2001, *Dynamic Asset Pricing Theory* (Princeton University Press: Princeton) third edn.

DUMAS, B., J. FLEMING, and R. E. WHALEY, 1998, Implied volatility functions: Empirical tests, *The Journal of Finance* 53, 2059–2106.

FOUQUE, J. P., G. PAPANICOLAU, and K. R. SIRCAR, 2000, *Derivatives in Financial Markets with Stochastic Volatility* (Cambridge University Press: Cambridge).

GHYSELS, E., A.C. HARVEY, and E. RENAULT, 1996, *Stochastic Volatility*, vol. 14 of *Statistical Methods in Finance* . pp. 119–191 (Handbook of Statistics, North-Holland).

GLASSERMAN, P., 2003, *Monte Carlo Methods in Financial Engineering* (Springer-Verlag: New York).

HESTON, S. L., 1993, A closed-form solution for options with stochastic volatility with applications to bond and currency options, *Review of Financial Studies* 6, 327–343.

HOBSON, D. G., and L. C. G. ROGERS, 1998, Complete models with stochastic volatility, *Mathematical Finance* 8, 27–48.

HUBALEK, F., J. TEICHMANN, and R. TOMPKINS, 2004, Flexible complete models with stochastic volatility generalizing Hobson-Rogers, submitted.

HULL, J., and A. WHITE, 1987, The pricing of options on assets with stochastic volatilities, *Journal of Finance* 42, 281–300.

———, 1990, Valuing derivative securities using the explicit finite difference method, *Journal of Financial and Quantitative Analysis* 25, 87–100.

JACKWERTH, J. C., and M. RUBINSTEIN, 1996, Recovering probability distributions from option prices, *The Journal of Finance* 51, 1611–1631.

KLOEDEN, P. E., and E. PLATEN, 1999, *Numerical Solution of Stochastic Differential Equations* (Springer: Berlin).

PLATEN, E., and M. SCHWEIZER, 1998, On feedback effects from hedging derivatives, *Mathematical Finance* 8, 67–84.

ROGERS, L. C. G., and S. E. SATCHELL, 2000, Does the behaviour of the asset tell us anything about the option price formula?, *Applied Financial Economics* 10, 37–39.

RUBINSTEIN, M., 1985, Nonparametric tests of alternative option pricing models using all reported trades and quotes on the 30 most active CBOE option classes from august 23, 1976 through august 31, 1978, *The Journal of Finance* 11, 455–480.

SCHRODER, M., 1989, Computing the constant elasticity of variance option pricing formula, *The Journal of Finance* 44, 211–219.

SCHWARTZ, E. S., 1977, The valuation of warrants: Implementing a new approach, *Journal of Financial Economics* 4, 79–93.

SCHWERT, G. W., 1989, Why does volatility change over time?, *The Journal of Finance* 44, 1115–1153.

SCOTT, L. O., 1987, Option pricing when the variance changes randomly: Theory, estimation, and an application, *Journal of Financial and Quantitative Analysis* 22, 419–438.

TAVELLA, D., and C. RANDALL, 2000, *Pricing Financial Instruments: the Finite Difference Method* (Wiley: New York).

TOMPKINS, R. G., 2001, Stock index futures markets: Stochastic volatility models and smiles, *Journal of Future Markets* 21, 43–78.

WIGGINS, J. B., 1987, Option values under stochastic volatility. theory and empirical estimates, *Journal of Financial Economics* 19, 351–372.

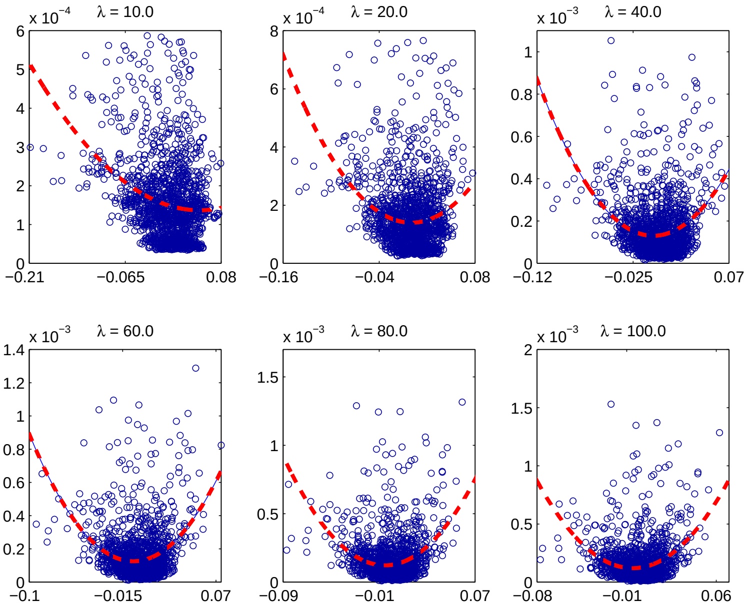

Figure 1: Offset vs volatility, SP500 index 1993-2002.

λ	Term	Bounds		PE	λ	Term	Bounds		PE
10.0	a	1.4062	1.5078	1.4570	60.0	a	1.2636	1.4329	1.3483
	b	-10.4201	-8.4427	-9.4314		b	-5.6929	1.0993	-2.2968
	c	33.0946	53.5770	43.3358		c	326.2475	463.6645	394.9560
20.0	a	1.3570	1.4824	1.4197	80.0	a	1.2360	1.4177	1.3269
	b	-11.4611	-7.9622	-9.7116		b	-4.2323	3.7437	-0.2443
	c	83.8240	129.0635	106.4437		c	449.9151	630.4401	540.1776
40.0	a	1.2643	1.4128	1.3386	100.0	a	1.2167	1.4097	1.3132
	b	-9.5973	-4.3470	-6.9722		b	-2.9359	6.1401	1.6021
	c	249.9483	339.9986	294.9734		c	559.4291	782.6407	671.0349

Table 1: SP500 index 1993-2002 regressions. Bounds are the 95% confidence intervals for the point estimates PE, all numbers are multiplied by 1.0E4.

Strike	MK	BS		CEV		Heston		HR1		HR2	
700	13.20	10.59	(-19.8)	13.83	(4.8)	13.56	(2.7)	13.36	(1.2)	13.82	(4.7)
725	17.45	14.96	(-14.3)	16.61	(-4.8)	17.17	(-1.6)	16.97	(-2.8)	17.36	(-0.5)
750	21.70	20.49	(-5.6)	20.47	(-5.7)	21.54	(-0.7)	21.44	(-1.2)	21.70	(0.0)
775	27.00	27.29	(1.1)	25.64	(-5.0)	26.81	(-0.7)	26.88	(-0.4)	26.98	(-0.1)
800	33.40	35.43	(6.1)	32.32	(-3.2)	33.16	(-0.7)	33.42	(0.1)	33.37	(-0.1)
825	41.00	44.97	(9.7)	40.69	(-0.8)	40.74	(-0.6)	41.22	(0.5)	41.03	(0.1)
850	50.10	55.92	(11.6)	50.83	(1.5)	49.79	(-0.6)	50.46	(0.7)	50.13	(0.1)
875	60.80	68.28	(12.3)	62.79	(3.3)	60.52	(-0.4)	61.01	(0.3)	60.82	(0.0)
900	73.40	82.01	(11.7)	76.53	(4.3)	73.11	(-0.3)	73.58	(0.2)	73.22	(-0.2)
925	87.70	97.04	(10.7)	91.97	(4.9)	87.61	(-0.1)	87.59	(-0.1)	87.41	(-0.3)
950	103.4	113.30	(9.6)	108.98	(5.4)	103.95	(0.5)	103.40	(-0.0)	103.36	(-0.0)
975	121.0	130.69	(8.0)	127.39	(5.3)	121.91	(0.7)	120.85	(-0.1)	121.01	(0.0)
995	136.2	145.36	(6.7)	143.03	(5.0)	137.27	(0.7)	135.96	(-0.1)	136.22	(0.0)
1025	160.5	168.48	(5.0)	167.77	(4.5)	161.65	(0.7)	160.33	(-0.1)	160.67	(0.1)
1050	182.0	188.66	(3.7)	183.37	(4.0)	182.93	(0.5)	182.00	(-0.0)	182.27	(0.1)
1075	204.5	209.58	(2.5)	211.70	(3.5)	204.91	(0.2)	204.65	(-0.1)	204.74	(0.1)
1100	227.8	231.12	(1.5)	234.63	(3.0)	227.43	(-0.2)	228.08	(-0.1)	227.87	(0.0)
1125	251.6	253.22	(0.6)	258.02	(2.5)	250.38	(-0.5)	252.04	(0.2)	251.50	(-0.0)
1150	275.7	275.79	(0.0)	281.77	(2.2)	273.68	(-0.7)	276.34	(0.2)	275.50	(-0.1)
1175	300.2	298.75	(-0.4)	305.80	(1.9)	297.25	(-1.0)	300.84	(0.2)	299.77	(-0.1)
			(7.05)		(3.78)		(0.66)		(0.43)		(0.33)

Table 2: SP500 $P_0 = 875.67, r = 0.0135, \tau = 0.5$