L23 Poor modelling of the codependence of default times of many companies is partially to blame for the crunch.

L24 Credit risk modelling: the main challenge is to find a way to model the default time $\tau_i$ of the $i^{th}$ entity/firm:

a) does not assume independence of the $\tau_i$
b) gives a reasonable story for the dependence.

[Another issue is to model the fraction lost on default]

A possible route: ← hazard rate, intensity of default.

$$\mathbb{P}(\tau > t) = \exp\left(-\int_0^t h_s \, ds\right)$$

If we do that, then we could extend the "potential approach" by introducing default intensity:

$$h^i(X_t)$$

For example, a zero-coupon corporate bond with recovery rate $R$ would be valued at time 0 as

$$\mathbb{E}\Big[\exp\left(-\int_0^T r(X_s)\,ds\right)\mathbb{1}(\tau > T) \quad \leftarrow \text{no default}$$

$$+ \exp\left(-\int_0^\tau r(X_s)\,ds\right) R\,\mathbb{1}(\tau \le T)\Big] \quad \leftarrow \text{default}$$

$$= \mathbb{E}\left[\exp\left(-\int_0^T (r(X_t) + h(X_s))\,ds\right)\right]$$

$$+ \mathbb{E}\left[\int_0^T \underbrace{h(X_t)\, e^{-\int_0^t h(X_s)ds - \int_0^t r(X_s)ds}}_{\text{density of } \tau} R \, dt\right]$$

So this would be very easy to calculate in the context of a finite state Markov chain $X_t$.

## Futures and forwards

There is some underlying process $(Y_t)_{t\geq 0}$ which is the spot price of the commodity / other asset to be delivered in the future / forward contract.

The forward price $F^*_{tT}$ at time $t$ for delivery at $T > t$ is defined by

$$\mathbb{E}\left( (Y_T - F^*_{tT}) e^{-\int_t^T r_s\, ds} \right) = 0$$

so we find that

$$F^*_{tT} = \frac{\mathbb{E}\left[ Y_t e^{-\int_t^T r_s\, ds} \right]}{B(t,T)}$$

Compare with a futures contract: Futures contracts are traded on an exchange.

In effect, what happens is that you set up an account with the exchange, and you pay in at time 0 a sum of money $A_0$ which is going to (eventually) be used to buy $Y$ at time $T$.

The futures price $F_{tT}$ varies, and your account moves with it according to

$$dA_t = r_t A_t\, dt + dF_{tT}$$

The account $A$ is a traded asset, so

$\exp\left(-\int_0^t r_s\, ds\right) A_t$ is a martingale

$$d\left( e^{-\int_0^t r_s\, ds} A_t \right) = e^{-\int_0^t r_s\, ds} \left( -r_t A_t\, dt + dA_t \right)$$
$$= e^{-\int_0^t r_s\, ds}\, dF_{tT}$$

Hence $F_{tT}$ is a martingale.

At expiry $T$, the futures price equals the spot price:

$$F_{TT} = Y_T.$$

So we learn that $F_{tT} = \mathbb{E}(Y_T \mid \mathcal{F}_t)$

compare with forward: $F^*_{tT} = \dfrac{\mathbb{E}\left( Y_T e^{-\int_t^T r_s\, ds} \mid \mathcal{F}_t \right)}{\mathbb{E}\left( e^{-\int_t^T r_s\, ds} \mid \mathcal{F}_t \right)}$

AFM

L24 The two are equal if interest rates are deterministic but not in general otherwise.

Remarks (i) Futures contracts are hugely liquid, because
a) at any time $t$, there is only one December futures contract.
- gains and losses gradually accumulate, whereas in a forward contract all gains/losses happen at $T$.

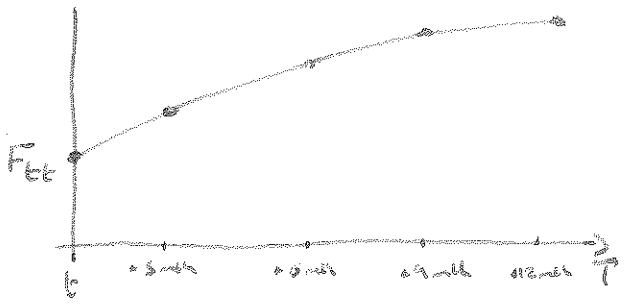
b) no counterparty risk - contract is with the exchange, don't need to spend time going through the books of

c) costs are very small ~2bp

d) you can always effectively cancel a futures contract by entering into an opposite contract.

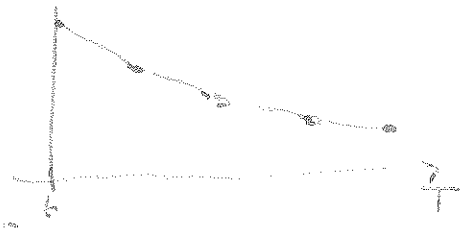
e) To operate a futures contract, you post a margin (~10% of current futures price). If the price moves against you, the exchange makes a margin call (asks for more money or closes position)
- you get leverage (don't need to have 100% of funds to expand)
- you can jump out when things go back, so limit your losses.

Normally, if you plot out $T \to F_{tT}$




Typically rising ("in contango"), as investors are willing to pay a premium for the future commodity because it reduces risk.

Sometimes get backwardation, where perhaps current supply was restricted.



How would we model these things? A simple approach would be to mimic what we did for multifactor yield curve models:

$$dX_t = \sigma\, dB_t - \lambda X_t\, dt,$$

and then propose that

$$Y_t = \exp(-a - b \cdot X_t)$$

So we could calculate prices as before.
Note that there is no reason why

$$\exp\left(-\int_0^t r_s\, ds\right) Y_t$$

should be a martingale.

e.g. Electricity market: 9 - 9:30 while running } very correlated products.
night time in summer