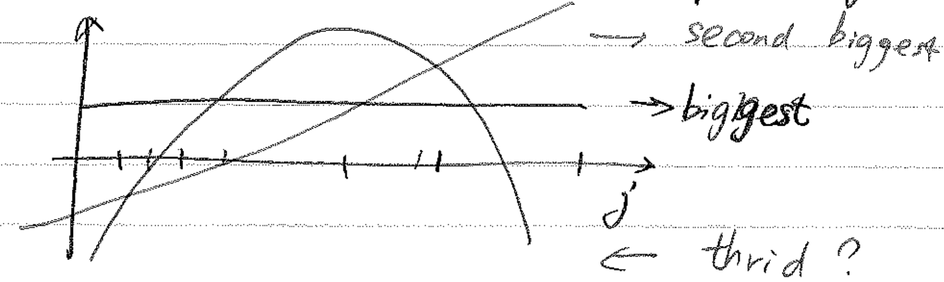
$$y_t^j \equiv -\frac{1}{T_j} \log B(t, t+T_j)$$

$j=1, 2, \cdots, J$. Now we consider y_t as a J-vector, and we can do some basic statistics, like ~~ea~~ calculate the sample means $\overline{y_t} = \frac{1}{t}\sum_{i=1}^{t} y_i$ and sample covariance

$$V_t = \frac{1}{t}\sum_{i=1}^{t} (y_i - \overline{y_t})(y_i - \overline{y_t})^T$$

and then look at the eigen~~value~~ structure of V_t. Typically, we find that there are 2-3 eigenvalues that "matter" with corresponding e-vectors



A 1-factor model could not generate such dynamics...

Nov 24 Thursday AFM

HJM

$$\mathbb{E}_t \exp\left(-\int_0^T r_s\, ds\right) = \exp\left[-\int_0^t r_s\, ds - \int_t^T f_{ts}\, ds\right]$$
$$= \exp\left(-\int_0^t f_{0s}\, ds\right)$$
$$\cdot \exp\left(-\int_0^t \Sigma_{sT}\, dB_s - \frac{1}{2}\int_0^t \Sigma_{sT}^2\, ds\right.$$

$$d f_{ts} = \sigma_{ts}\, dB_t + \alpha_{ts}\, dt$$

Taking logs, we get

$$\int_0^T f_{0s}\, ds + \int_0^t \Sigma_{sT}\, dB_s + \int_0^t \frac{1}{2}\Sigma_{sT}^2\, ds$$
$$= \int_0^t r_s\, ds + \int_t^T \left\{ f_{0s} + \int_0^t \sigma_{us}\, dB_u + \int_0^t \alpha_{us}\, du \right\} ds$$
$$= \int_0^t r_s\, ds + \int_t^T f_{0s}\, ds + \int_0^t \left(\int_t^T \sigma_{us}\, ds\right) dB_u$$
$$+ \int_0^t \left(\int_t^T \alpha_{us}\, ds\right) du\, .$$

Hence

$$\int_0^t f_{0s}\, ds - \int_0^t r_s\, ds + \boxed{\int_0^t \Sigma_{sT}\, dB_s} + \int_0^t \frac{1}{2}\Sigma_{sT}^2\, ds$$
$$= \int_0^t \left(\int_t^T \sigma_{us}\, ds\right) dB_u + \int_0^t \left(\int_t^T \alpha_{us}\, ds\right) du,$$

$$= \int_0^t \left(\int_t^T \sigma_{us}\, ds - \underline{\int_0^t \sigma_{us}\, ds}\right) dB_u + \int_0^t \left(\int_0^T \alpha_{us}\, ds - \int_0^t \alpha_{us}\, ds\right) du$$

Now

$$\int_0^t (f_{0s} - r_s)\, ds = \int_0^t (f_{0s} - f_{ss})\, ds = -\int_0^t \left\{\int_0^s \sigma_{us}\, dB_u + \int_0^s \alpha_{us}\, du\right\} ds$$
$$= -\int_0^t \left(\underline{\int_u^t \sigma_{us}\, ds}\right) dB_u - \int_0^t \left(\int_u^t \alpha_{us}\, ds\right) du.$$

Looking at the integrals dB, we conclude that

$$\boxed{\Sigma_{uT} = \int_0^T \sigma_{us}\, ds}$$

The remaining terms give us