Beyond I&D:
Prelude to an Information Theory of the Future
Towards a strong converse for the quantum capacity (of degradable channels)

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- in preparation -
Outline

1. Quantum channel capacity

2. LSD capacity formula and weak converse

3. Strong converse - and why you would care

4. Additivity for degradable channels

5. "Semi-strong" converse and proof ideas

6. Open question(s)
1. Quantum channel capacity

Channel = cptp map $N: \mathcal{L}(A) \rightarrow \mathcal{L}(B)$.

Stinespring: $N(\rho) = \text{Tr}_E V \rho V^\dagger$

with an isometry $V:A \rightarrow B \otimes E$. 
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Complementary channel:

$\hat{N}(\rho) = \text{Tr}_B V \rho V^\dagger$
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with an isometry $V:A \rightarrow B \otimes E$. 

Complementary channel: 

$\hat{N}(\rho) = \text{Tr}_B V \rho V^\dagger$. 

$N$ is degradable if there exists a ctp map $D$ s.t. $\hat{N} = D \circ N$. 
Quantum capacity $Q(N) := \text{maximum gubit rate } \frac{k}{n} \text{ for asymptotically faithful transmission over } N^\otimes n, \text{ with encoding and decoding (error correction).}$
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$$F(\rho, \sigma) = \| \sqrt{\rho} \sqrt{\sigma} \|_1$$

$$= \max \ |\langle \psi | \varphi \rangle| \text{ s.t. }$$

$|\psi\rangle$ purifies $\rho$, $|\varphi\rangle$ purifies $\sigma$. 
Digression on fidelity:

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\( D(\rho, \sigma) := \sqrt{1 - F(\rho, \sigma)^2} \) is a metric on states;
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\[ \mathcal{P}(\rho, \sigma) := \sqrt{1 - \mathcal{F}(\rho, \sigma)^2} \text{ is a metric on states;} \]
\[ \ldots \text{and so is } \mathcal{A}(\rho, \sigma) := \arcsin \mathcal{P}(\rho, \sigma). \]
**Digression on fidelity:**

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\[ \mathcal{P}(\rho, \sigma) := \sqrt{1 - F(\rho, \sigma)^2} \text{ is a metric on states; } \]

...and so is \( A(\rho, \sigma) := \arcsin \mathcal{P}(\rho, \sigma). \)

*Note: Both are equivalent to the trace distance \( \| \rho - \sigma \|_1. \)
With cptp en- and decoding operations $E, D$:
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$k$ EPR pairs $(|C|=|C'|=2^k)$

$max \ k = K(n, \epsilon)$
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$k$ EPR pairs

$\left|C\right| = \left|C'\right| = 2^k$

$\max k = K(n, \epsilon)$

Approximates input:

$\rho(\Phi, \sigma) \leq \epsilon$. 
\[ \varphi(N) := \inf_{\epsilon > 0} \limsup_{n \to \infty} \frac{1}{n} k(n, \epsilon) \]
\[ \Phi_{cc'} : Q(N) := \inf_{\epsilon > 0} \limsup_{n \to \infty} \frac{1}{n} K(n, \epsilon) \]

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2. LSD capacity formula and weak converse

Thm (Lloyd-Shor-Devetak, 1996-2003):
\[ Q(N) = \lim_{n \to \infty} \frac{1}{n} Q^{(1)}(N \otimes^n), \text{ with} \]
2. **LSD capacity formula and weak converse**

**Thm (Lloyd-Shor-Devetak, 1996-2003):**

\[ Q(N) = \lim_{n \to \infty} \frac{1}{n} Q^{(1)}(N^\otimes n), \text{ with} \]

\[ Q^{(1)}(N) = \max_{\phi} I(A>B)_{\rho} \quad \text{s.t.} \]

\[ \rho_{A'A} \]
2. LSD capacity formula and weak converse

Thm (Lloyd-Shor-Devetak, 1996-2003):
\[ Q(N) = \lim_{n \to \infty} \frac{1}{n} Q^{(1)}(N^\otimes n), \]
with
\[ Q^{(1)}(N) = \max_{\phi} I(A\triangleright B) , \]
s.t.

\[ I(A\triangleright B) = -S(A|B) \]
\[ := S(B) - S(AB), \]
coherent info.
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Using a channel $N^\otimes n$ to transmit $k$ qubits with error $\leq \epsilon$, he showed:

$$k (1-\epsilon') \leq 1 + Q^{(1)}(N^\otimes n) \leq 1 + n Q(N)$$
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Using a channel $N^\otimes n$ to transmit $k$ qubits with error $\leq \epsilon$, he showed:

$$k (1-\epsilon') \leq 1 + Q(1)(N^\otimes n) \leq 1 + n Q(N)$$
Schumacher's weak converse:

\[ \frac{k}{n} \leq (1 + \epsilon') \mathcal{O}(N) \]
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\frac{k}{n} \lesssim (1 + \epsilon') \alpha(n)
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...suggests a rate-error trade-off.
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Is it real?
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\[ \frac{k}{n} \lesssim (1 + \epsilon') Q(N) \]

...suggests a rate-error trade-off.

Is it real?

Note: For rates \( R = \frac{k}{n} < Q(N) \), i.e. below the capacity, the LSD (random) coding theorem guarantees error exponentially small in \( n \).
3. Strong converse?

The strong converse - in the sense of Wolfowitz [Ill. J. Math. 1:591 (1957)] - is the statement that there is no rate-error trade-off. Viz., for rates $R$ above the capacity, the error converges to 1.
3. Strong converse?

The strong converse - in the sense of Wolfowitz [Ill. J. Math. 1:591 (1957)] -, is the statement that there is no rate-error trade-off. Viz., for rates $R$ above the capacity, the error converges to 1.

By contrapositive: If error < 1, then asymptotically the rate $\frac{k}{n}$ is bounded by the capacity.
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- Classical channels [Shannon-Wolfowitz]
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- Entanglement-assisted capacity [Bennett et al., 0912.5537]
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Conceptually: capacity concept more meaningful with sharp “phase” transition, rather than trade-off.
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Applications: E.g. bounded/noisy storage quantum cryptography...
For quantum capacity $Q(N)$:

$P(\text{error})$ vs $R: \text{qubit rate}$

Definition/coding theorem (LSD)
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- **Definition/coding theorem (LSD)**
- **Weak converse: error bound**
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- **Definition/coding theorem (LSD)**
- **Weak converse:** error bound
- **Strong converse?**
4. Additivity for degradable channels

[Devetak/Shor, CMP 2004; quant-ph/0311131]
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Apply degrading map (Stinespring form)

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\[ I(A|B) = -S(A|B) = -S(A|E'F) = S(E'F) - S(E') \]

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4. Additivity for degradable channels

\[
I(A > B) = -S(A|B) = -S(A|E'F) = S(E'F) - S(E') = S(F|E')
\]

[Devetak/Shor, CMP 2004; quant-ph/0311131]
\( I(A \gg B)_\psi = S(FIE')_\varphi \) for one channel use.
\[ I(A \rightarrow B)_\psi = S(F(1E'))_\phi \] for one channel use.

For two degradable channels:
\[ I(A_1A_2 \rightarrow B_1B_2) = S(F_1F_2|E_1'E_2') \]
\[ = I(A_1 \rightarrow B_1) + I(A_2 \rightarrow B_2) \]
I(A>B)_ψ = S(FIE")_φ for one channel use.

For two degradable channels:
I(A_1A_2>B_1B_2) = S(F_iF_2E_i'E_2')
≥ S(F_1E_i'E_2') + S(F_2E_i'E_2')
≤ S(F_iE_i') + S(F_2E_2')
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For two degradable channels:

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$\leq S(F_1E_1'E_2') + S(F_2E_1'E_2')$

$\leq S(F_1E_1') + S(F_2E_2')$

By strong subadditivity
$I(A \triangleright B)_\psi = S(F_1 E')_\psi$ for one channel use.

For two degradable channels:

$I(A_1 A_2 \triangleright B_1 B_2) = S(F_1 F_2 E_1' E_2')$

$\leq S(F_1 E_1' E_2') + S(F_2 E_1' E_2')$

$\leq S(F_1 E_1') + S(F_2 E_2')$

$= I(A_1 \triangleright B_1) + I(A_2 \triangleright B_2)$

By strong subadditivity
\[ I(A>B)_\psi = S(F|E')_\phi \text{ for one channel use.} \]

For two degradable channels:
\[ I(A_1A_2>B_1B_2) = S(F_1F_2|E_1'E_2') \]
\[ \leq S(F_1|E_1'E_2') + S(F_2|E_1'E_2') \]
\[ \leq S(F_1|E_1') + S(F_2|E_2') \]
\[ = I(A_1>B_1) + I(A_2>B_2) \]

By strong subadditivity

Proves: \[ Q^{(1)}(N_1 \otimes N_2) = Q^{(1)}(N_1) + Q^{(1)}(N_2). \]
\( I(A \!>\! B)_\psi = S(\mathcal{FIE'})_{\psi} \) for one channel use.

For two degradable channels:
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Proves: \( Q^{(1)}(N_1 \otimes N_2) = Q^{(1)}(N_1) + Q^{(1)}(N_2) \).

...and hence \( Q(N) = Q^{(1)}(N) \), and the latter can be found by convex optimisation.
$I(A>B)_\psi = S(F_{I:E'})_\phi$ for one channel use.

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...and hence $Q(N) = Q^{(1)}(N)$, and the latter can be found by convex optimisation.

G. Smith (0705.3838): Also private classical capacity, $P(N) = Q(N) = Q^{(1)}(N)$. 
5. Results & proof ideas


If $N$ is PPT entanglement-binding, then of course $Q(N)=0$, and strong converse holds (with error converging exponentially to 1).
5. Results & proof ideas

Previous [via E. Rains, IEEE-IT 47(7): 2921-2933 (2001) - thanks to R. Koenig!]: If $N$ is PPT entanglement-binding, then of course $Q(N)=0$, and strong converse holds (with error converging exponentially to 1).

Note: Already for symmetric (degradable & anti-degradable) channels - for which also $Q(N)=0$ - not clear at all.
Thm: For any degradable channel $N$, codes with rate $R > Q(N)$ have error at least 0.707, asymptotically.
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\[
\begin{array}{c|c c c c}
R & 0 & Q(N) & 1 \\
\hline
P(\text{error}) & 0 & 0.707 & 1 \\
\end{array}
\]
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Ideas: (smooth) min-entropies, symmetrisation, de Finetti theorem, AEP
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1) Use code - for simplicity subspace - with maximally entangled state $\Phi$ of $k$ qubits:
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$$k \leq \frac{\epsilon}{\min} (A|E)$$
Maximally entangled state $\Phi$ of $k$ qubits:

$$
\Phi_{A'A'} \rightarrow V \rightarrow E, E', F, A
\Psi_{ABE} \rightarrow W \rightarrow \Phi_{AEE'F}
$$

$$
k \leq H^\epsilon_{\min}(AIE) = -H^\epsilon_{\max}(AIE'F)
$$
Maximally entangled state $\Phi$ of $k$ qubits:

$$k \leq H_{\min}^{\epsilon}(A|E) = -H_{\max}^{\epsilon}(A|E'|F)$$

\[ k \leq \min_{\text{min}} H^\varepsilon (AIE) \]
\[ = -\max_{\text{max}} H^\varepsilon (AIE'F) \]
\[
k \leq H^\varepsilon_{\min}(AIE) = -H^\varepsilon_{\max}(AIE^c F)
\]

[Cf. also Buscemi/Datta, IEEE-IT 56(3), 2010; Datta/Hsieh, 1103.1135]
\[ k \leq H_{\min}^{\epsilon}(AIE) = -H_{\max}^{\epsilon}(AIE'F) \]

Note: If we knew that for \( n \) channel uses, the maximum min-entropy is attained on a tensor product input, we'd be done by AEP (= asymptotic equipartition property) ...
\[ k \leq H_{\min}^{\epsilon}(AIE) \]
\[ = -H_{\max}^{\epsilon}(AIE'F) \]
\[ \leq H_{\max}^{\lambda}(FIE') - H_{\max}^{\delta}(AFIE') + O(1) \]
\( k \leq H^\epsilon_{\min}(AIE) \)
\( = -H^\epsilon_{\max}(AIE'F) \)
\( \leq H^\lambda_{\max}(FIE') - H^\delta_{\max}(AFIE') + O(1) \)

Chain rule, \( \delta = \epsilon + 3 \lambda \).
\[ k \leq H_{\min}^\epsilon(AIE) \]
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\[ \leq H_{\max}^\lambda(FIE') - H_{\max}^\delta(AFIE') + O(1) \]

*Chain rule, \( \delta = \epsilon + 3\lambda \).*

\[ \leq H_{\max}^\lambda(FIE') + O(1) \]
\[ k \leq H_{\min}^\epsilon(AIE) \]
\[ = -H_{\max}^\epsilon(AIEF) \]
\[ \leq H_\lambda^\delta(FIE) - H_\delta^\delta(AFIE) + O(1) \]

Chain rule, \( \delta = \epsilon + 3\lambda \).

\[ \leq H_\lambda^\lambda(FIE) + O(1) \]

...if \( \delta < 0.707 \), by inequality \( H_{\min} \) vs. \( H_{\max} \), and using symmetry between \( E \) and \( E' \)...
2) For $n$ channel uses, have restricted concavity of $H^{\lambda}_{\max}$:

$$k \leq H^{\lambda}_{\max}(E^n I E^n) + O(1)$$
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$$\leq H_{\lambda}^* (F^n | E^n)_{\rho_{(n)}} + O(1)$$
2) For $n$ channel uses, have restricted concavity of $H_{\text{max}}^\lambda$:

$$k \leq H_{\text{max}}^\lambda(F^n|E^n) + O(1)$$

$$\leq H_{\text{max}}^{\lambda'}(F^n|E^n)_{\rho_A^{(n)}} + O(1)$$

w.r.t. a permutation symmetric input state and $\lambda' = \lambda / \sqrt{2}$
2) For \( n \) channel uses, have restricted concavity of \( H_{\max}^\lambda \):

\[
k \leq H_{\max}^\lambda(F^n | E^n) + O(1)
\]

\[
\leq H_{\max}^{\lambda'}(F^n | E^n)^{(n)}_{\rho_{A^n}} + O(1)
\]

3) By exponential de Finetti theorem


\[
k \leq \max_{\rho_A} H_{\max}^{\lambda''}(F^n | E^n)^{\otimes n} + o(n)
\]
4) By AEP (asymptotic equipartition property) [M. Tomamichel, arXiv:1203.2142]:

$$k \leq \max_{\rho_A} \max_{\rho_{\lambda''}} H_{\rho_{\lambda''}}^\rho(F^n I E^n)_{\rho^\otimes n} + o(n)$$
4) By AEP (asymptotic equipartition property) [M. Tomamichel, arXiv:1203.2142]:

\[ k \leq \max_{\rho^A} \max_{\lambda''} H^{\lambda''}(F^n I E^n)_{\rho^A \otimes n} + o(n) \]

\[ = \max_{\rho^A} n S(F IE^{\rho^A})_{\rho^A} + o(n) \]
4) By AEP (asymptotic equipartition property) [M. Tomamichel, arXiv:1203.2142]:

\[ k \leq \max_{\rho_A}^{\chi''} \max_{\rho} \mathcal{H} (F^n | E^n)_{\rho} \otimes n + o(n) \]

\[ = \max_{\rho_A} n S(FIE')_{\rho} + o(n) \]

\[ = n Q^{(1)}(N) + o(n) \]
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(by the degradability argument)
4) By AEP (asymptotic equipartition property) [M. Tomamichel, arXiv:1203.2142]:

$$k \leq \max_{\rho_A} \max_n H^{\lambda_n'}(F^{n_1}E^{n_2})_{\rho_A^n} + o(n)$$

$$= \max_n n S(F|E)_{\rho_A^n} + o(n)$$

$$= n Q^{(1)}(N) + o(n)$$

(by the degradability argument)
6. Conclusion (sort of...)

- Proved “semi-strong” converse for the quantum capacity of degradable channels;

- [Additional points or details可以直接翻译为中文]

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- Can we get an honest strong converse? (Bottleneck are symmetric channels: the current error threshold is that of a 50% erasure channel.)

- Indeed: Strong converse for degradable channels if it holds for symmetric ch.
Sharma/Warsi (arXiv:1205.1712) propose a different methodology, based on certain generalised divergences. They can’t prove a strong converse, but if an additivity problem could be solved for some channels (degradable?), it would follow.
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How to prove strong converses without additivity? Note that neither $Q$ nor $Q^{(1)}$ are additive! Similar for other capacities...