



**Beyond I&D:
Prelude to an Information Theory of the Future**

Towards a strong converse for the quantum capacity (of degradable channels)

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de Barcelona)

- in preparation -

Outline

1. Quantum channel capacity
2. LSD capacity formula and *weak* converse
3. *Strong* converse - and why you would care
4. Additivity for degradable channels
5. "Semi-strong" converse and proof ideas
6. Open question(s)...

1. Quantum channel capacity

Channel = cptp map $\mathcal{N}: L(A) \rightarrow L(B)$.

Stinespring: $\mathcal{N}(\rho) = \text{Tr}_E V \rho V^\dagger$
with an isometry $V: A \hookrightarrow B \otimes E$.

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\mathcal{N} is degradable if there exists a cptp map \mathcal{D} s.t. $\hat{\mathcal{N}} = \mathcal{D} \circ \mathcal{N}$.

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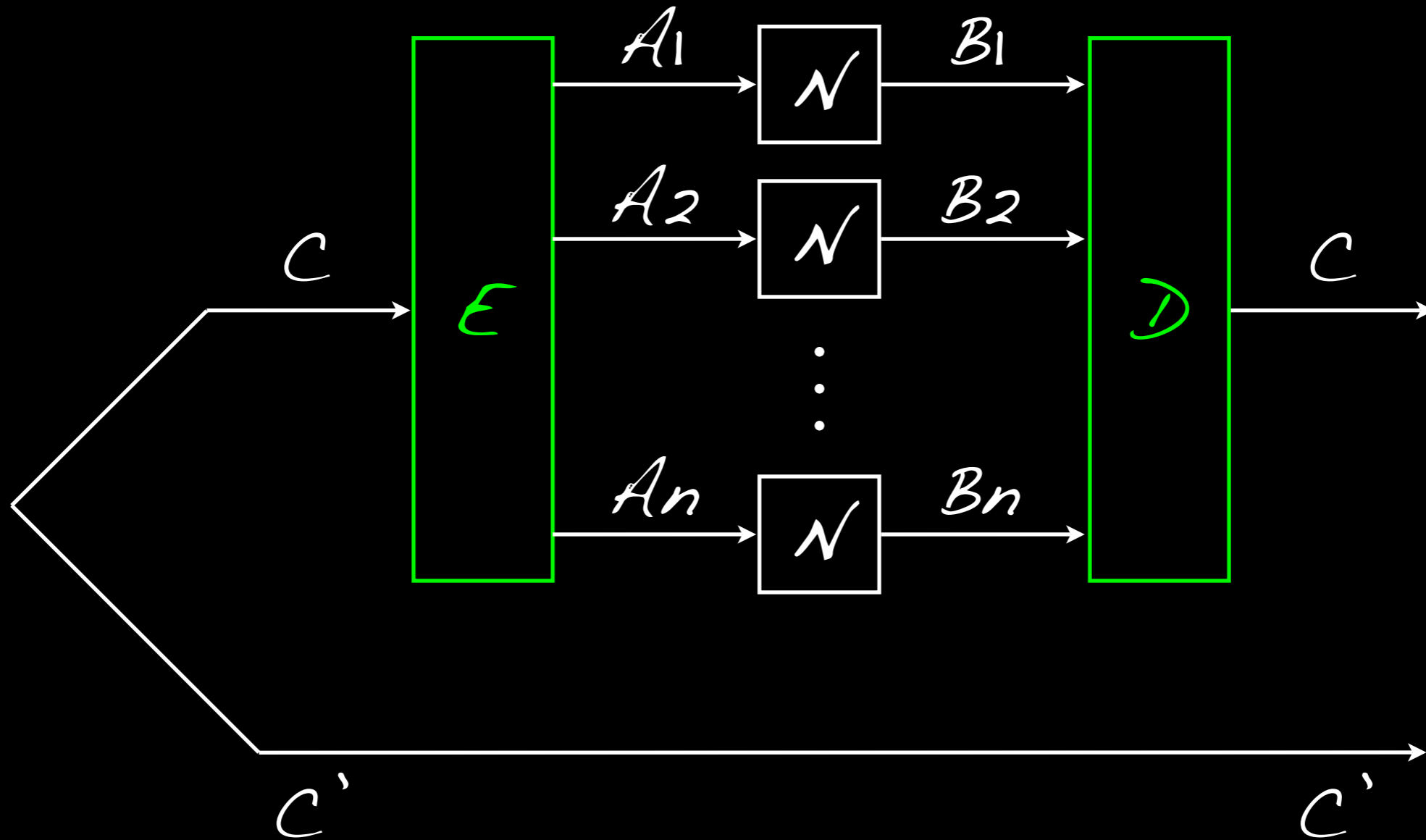
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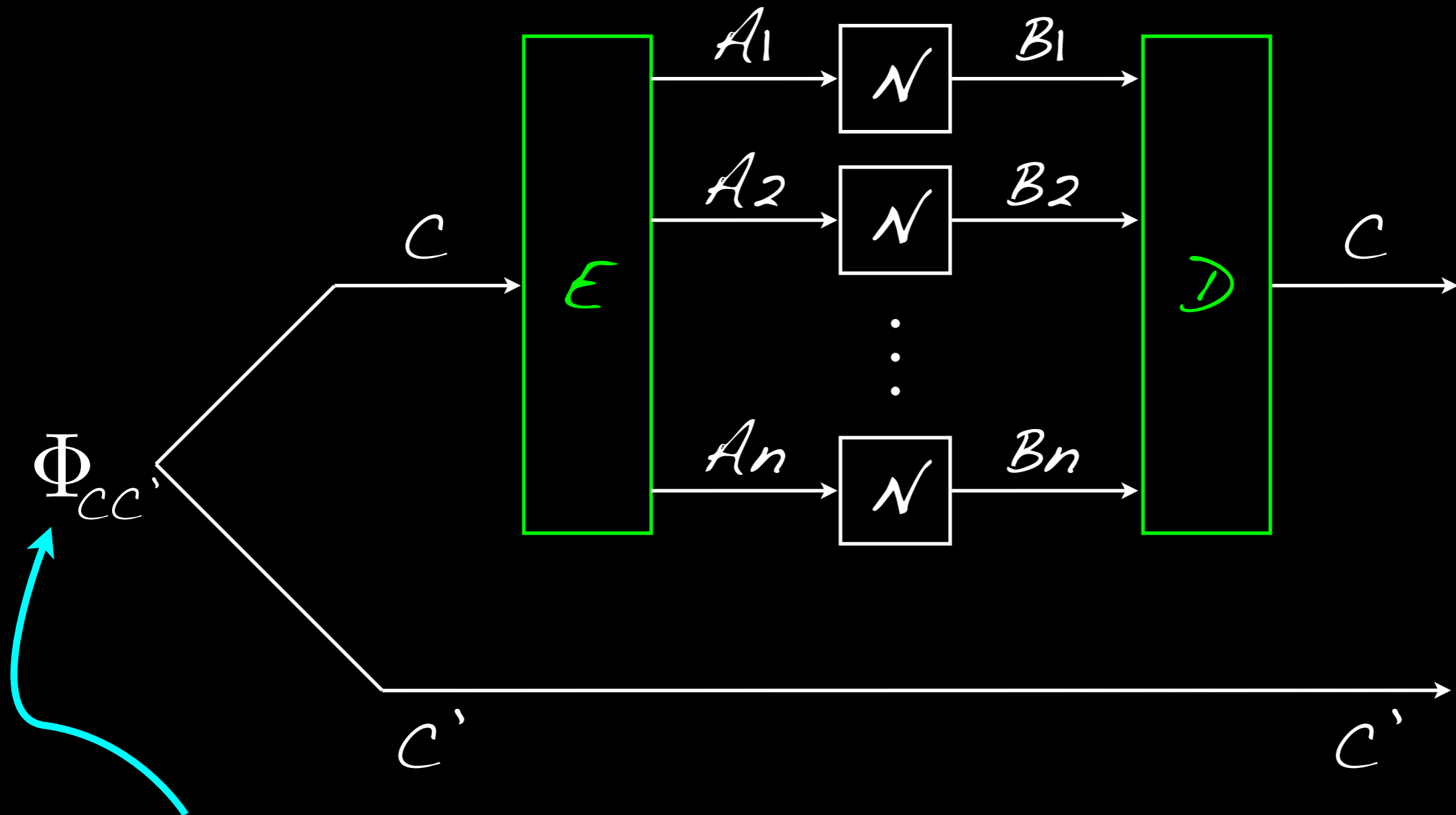
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Note: Both are equivalent to the trace distance $\|\rho - \sigma\|_1$.

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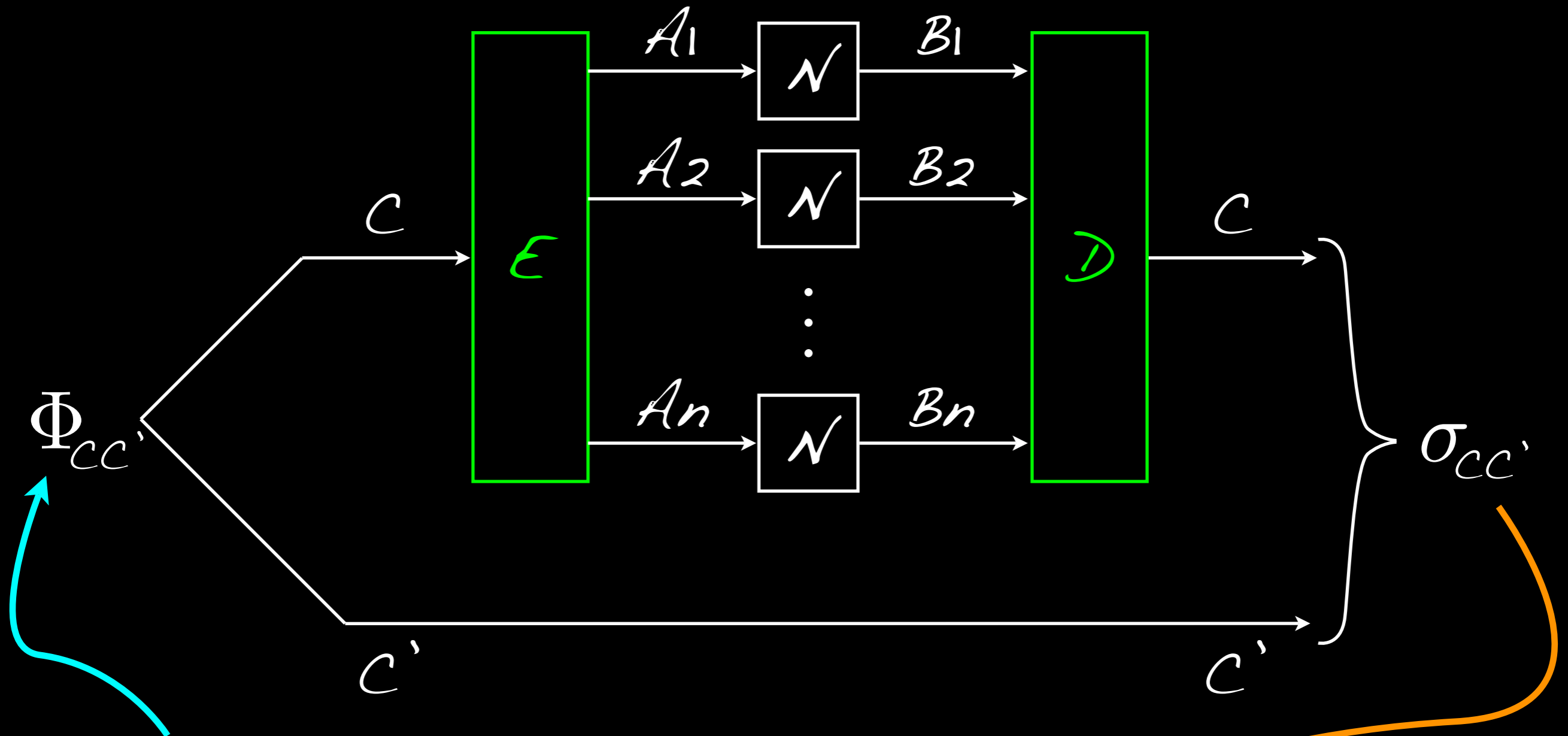


k EPR pairs

$(|C| = |C'| = 2^k)$

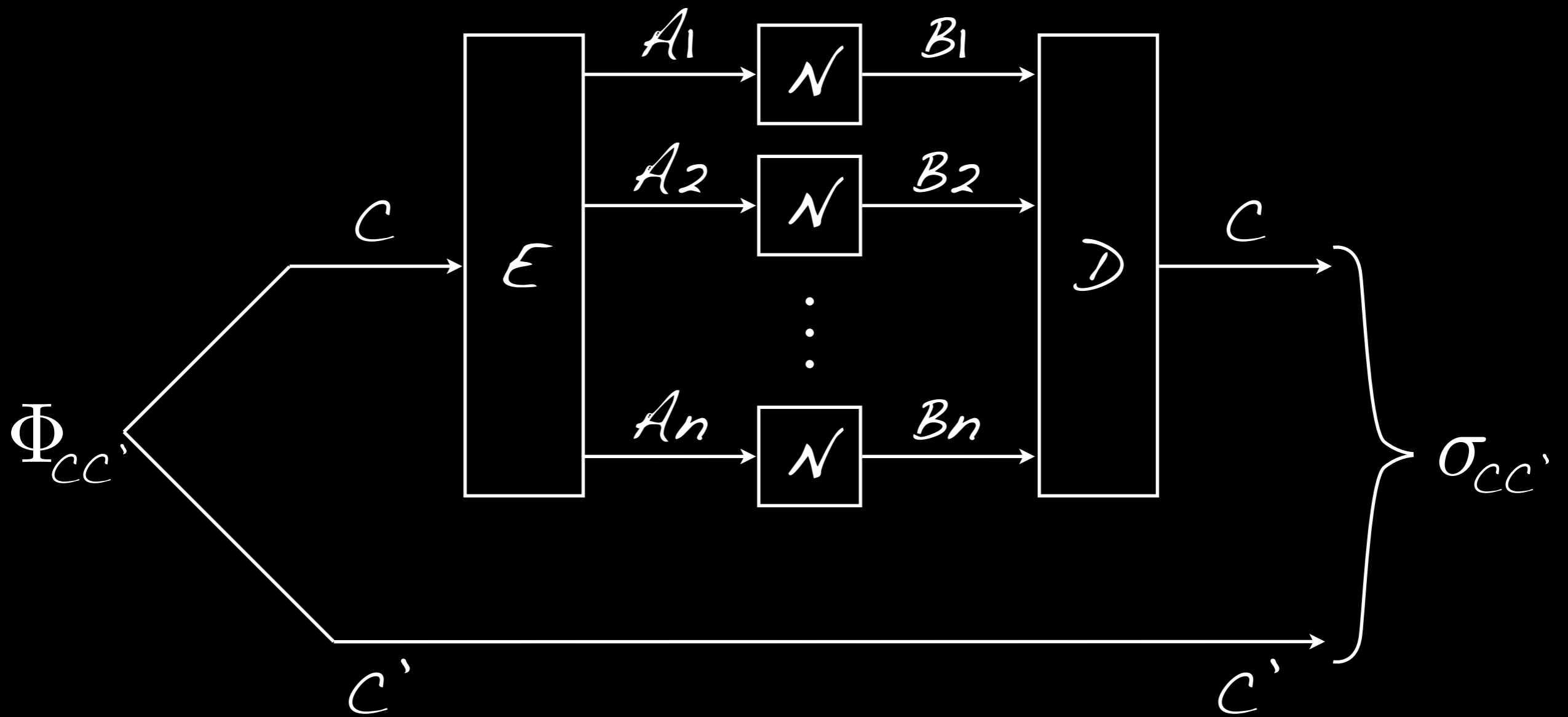
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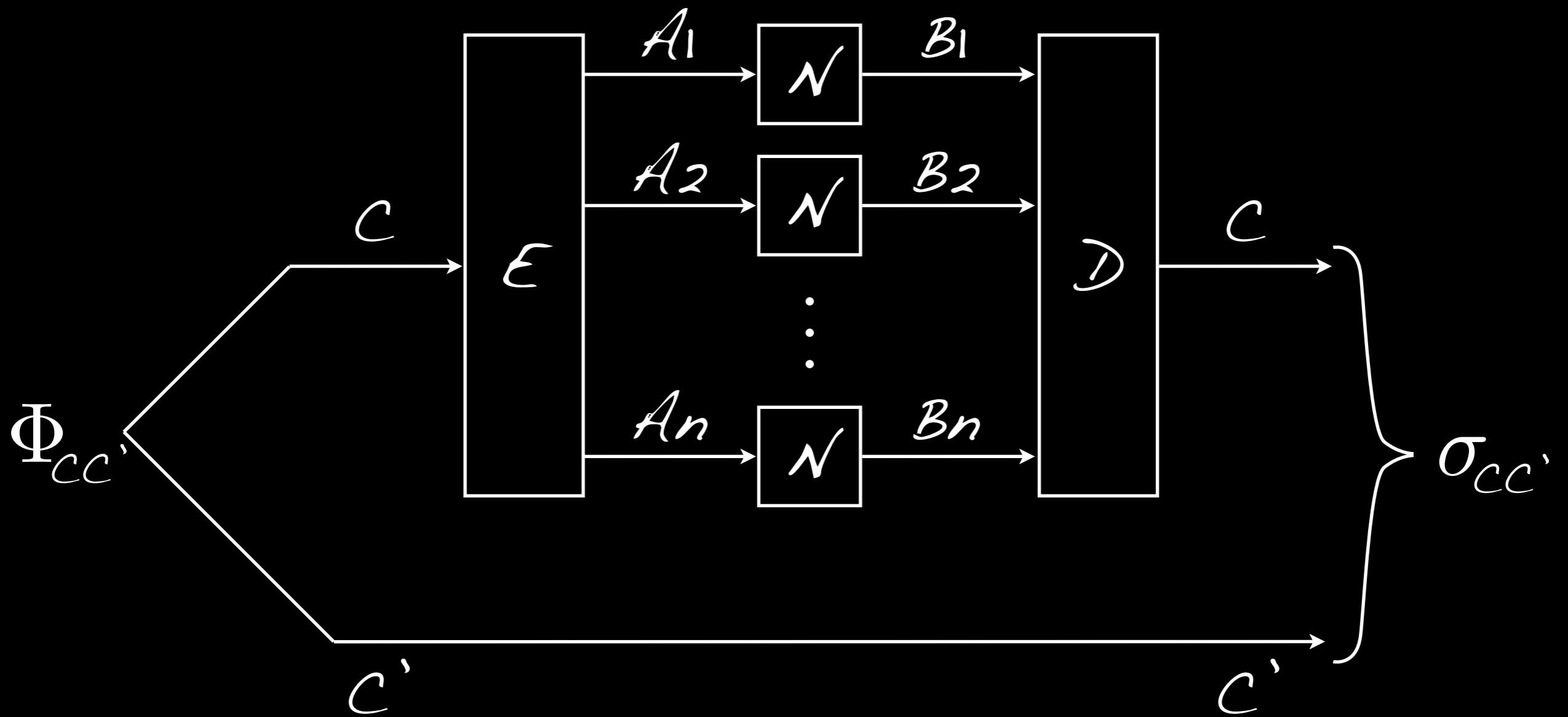


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Approximates input:
 $P(\Phi, \sigma) \leq \epsilon.$



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[Cf. D. Kretschmann
& R.F. Werner,
NJP 6:26 (2004).]

2. LSD capacity formula and weak converse

Thm (Lloyd-Shor-Devetak, 1996-2003):

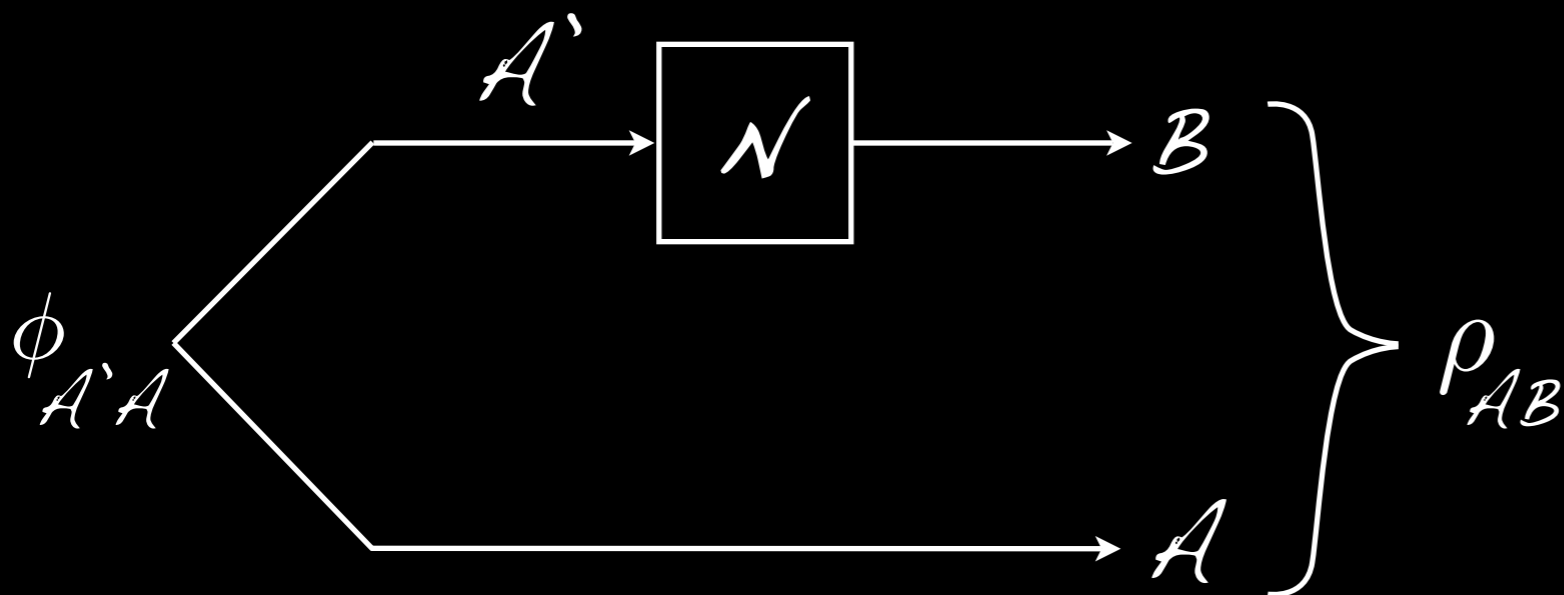
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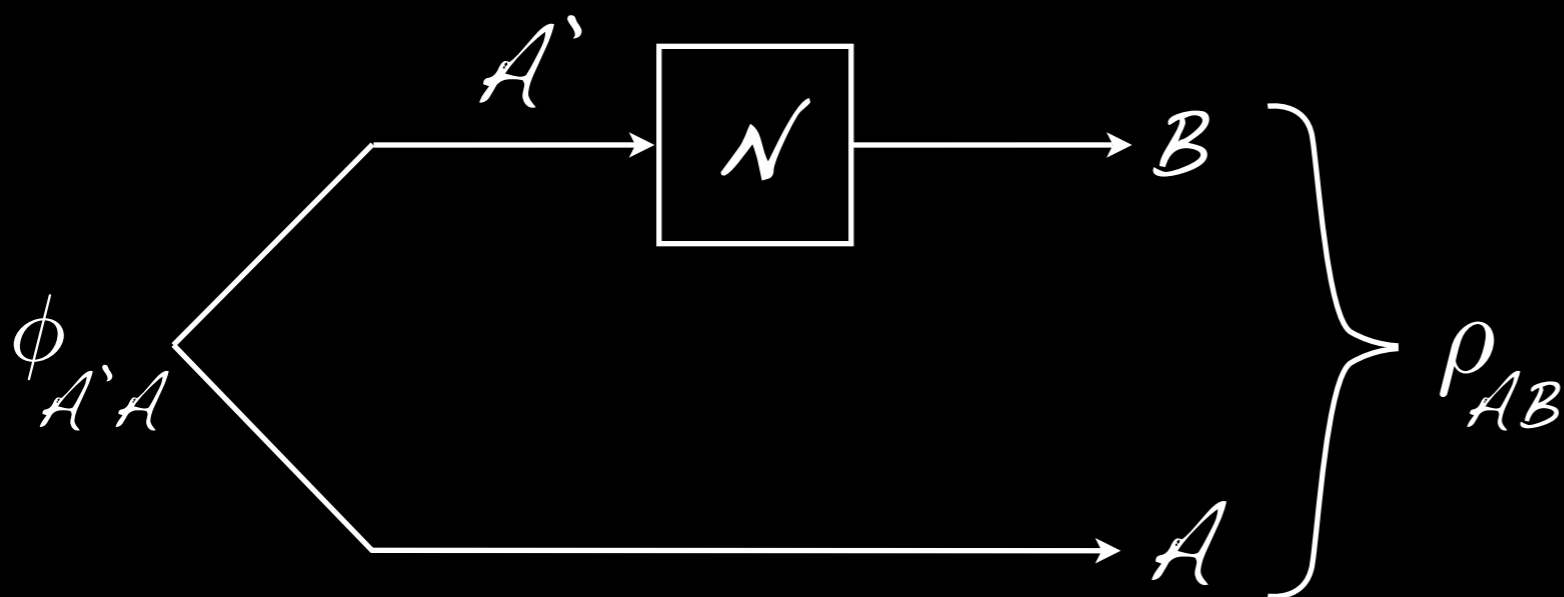


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$$I(A \rangle B) = -S(A|B) \\ := S(B) - S(AB), \\ \text{coherent info.}$$

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Note: For rates $R = \frac{k}{n} < Q(N)$, i.e. below the capacity, the LSD (random) coding theorem guarantees error exponentially small in n !

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The strong converse - in the sense of Wolfowitz [Ill. J. Math. 1:591 (1957)] -, is the statement that there is no rate-error trade-off. Viz., for rates R above the capacity, the error converges to 1.

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By contrapositive: If error < 1 , then asymptotically the rate $\frac{k}{n}$ is bounded by the capacity.

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Strong converse: If error $\leq \epsilon$, then asymptotically the rate $\frac{k}{n}$ is bounded by the capacity.

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- Classical channels [Shannon-Wolfowitz]

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- Entanglement-assisted capacity [Bennett et al., 0912.5537]

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Conceptually: capacity concept more meaningful with sharp "phase" transition, rather than trade-off.

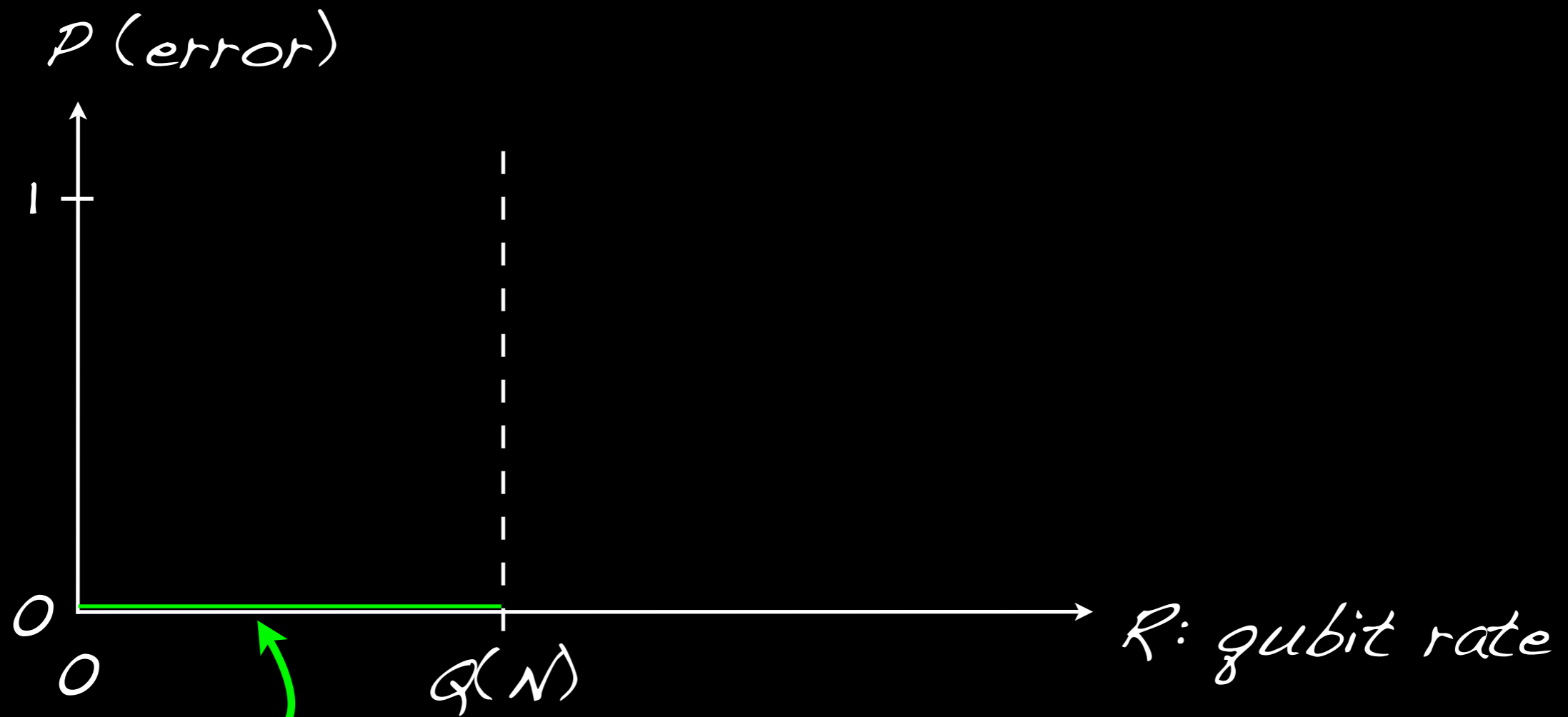
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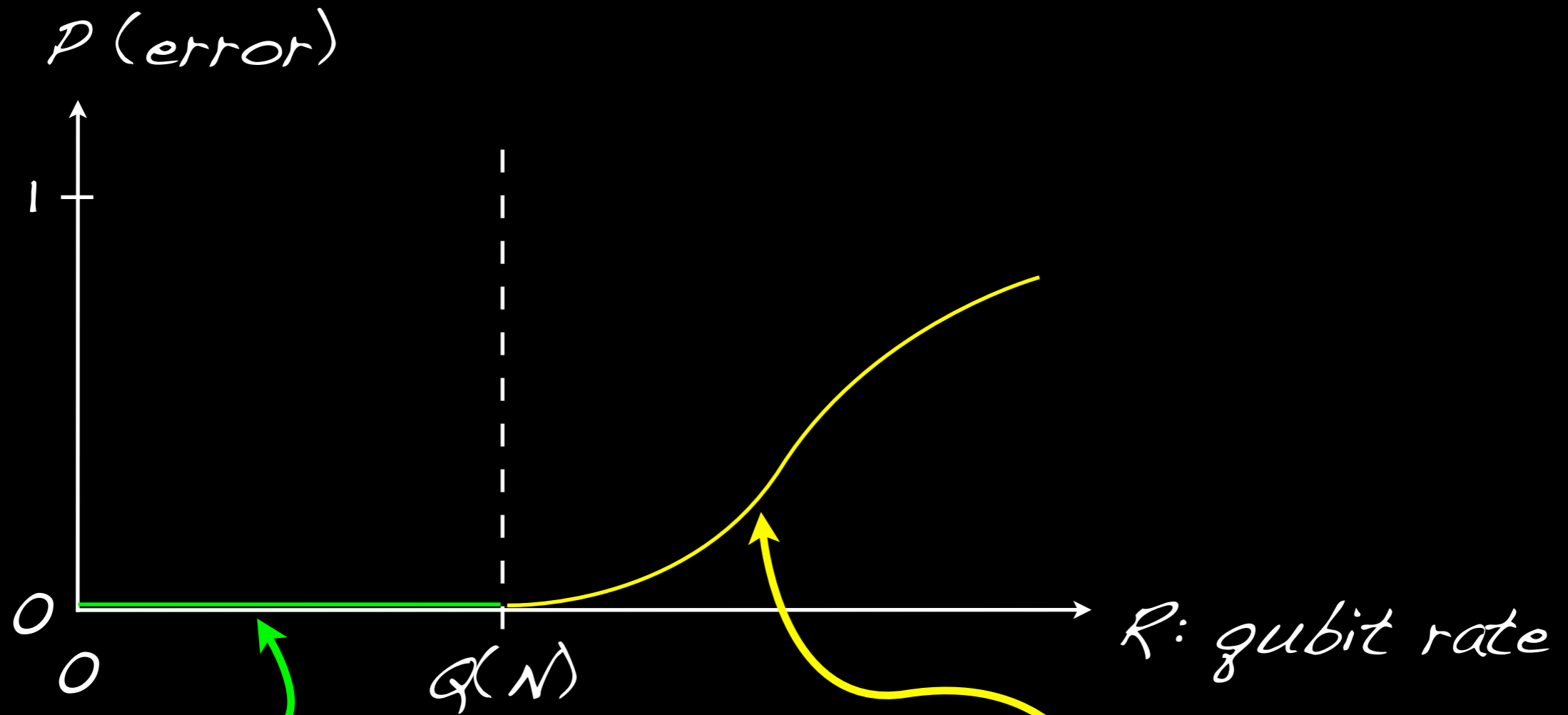
Applications: E.g. bounded/noisy storage quantum cryptography...

For quantum capacity $Q(N)$:



Definition/coding
theorem (LSD)

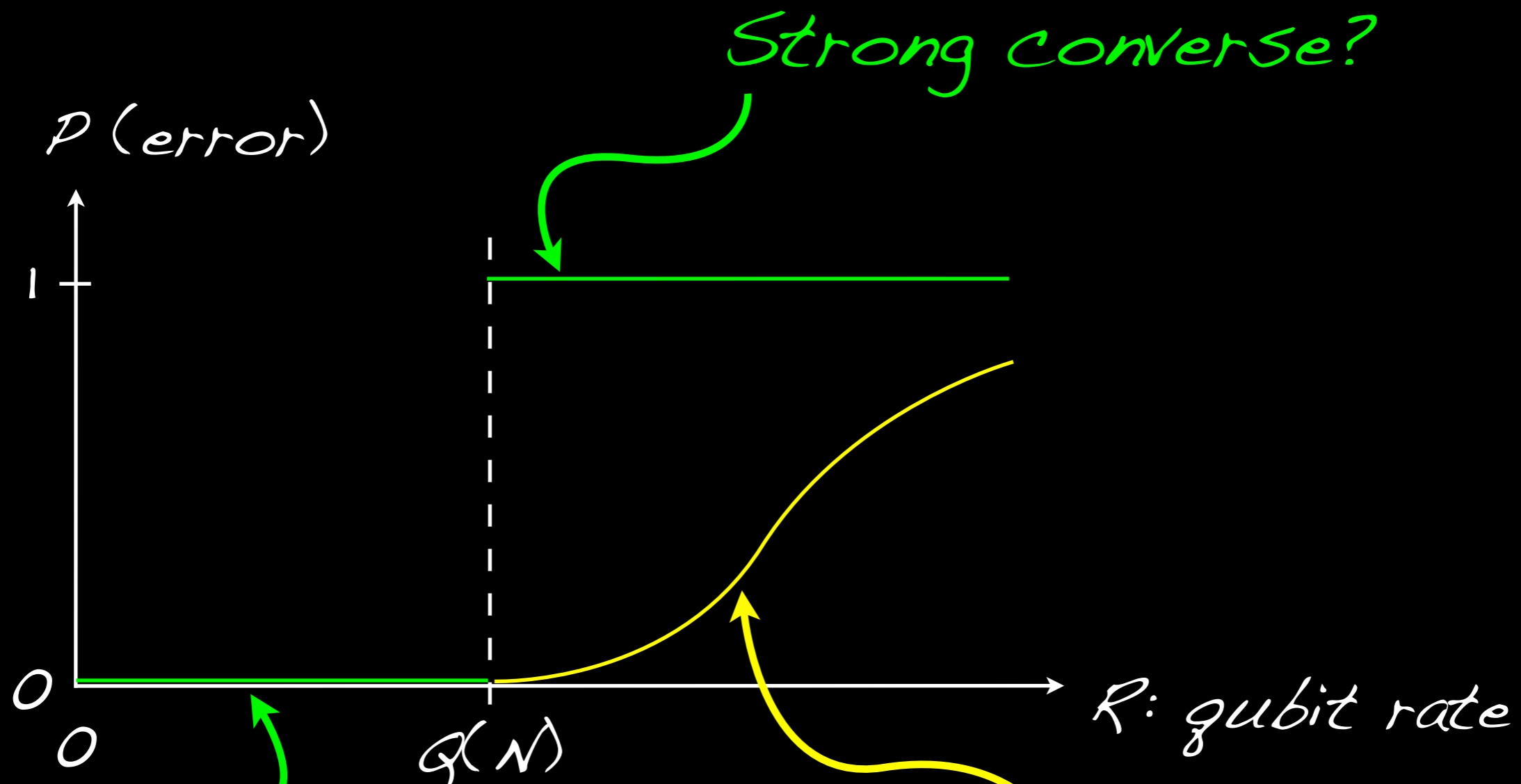
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Weak converse:
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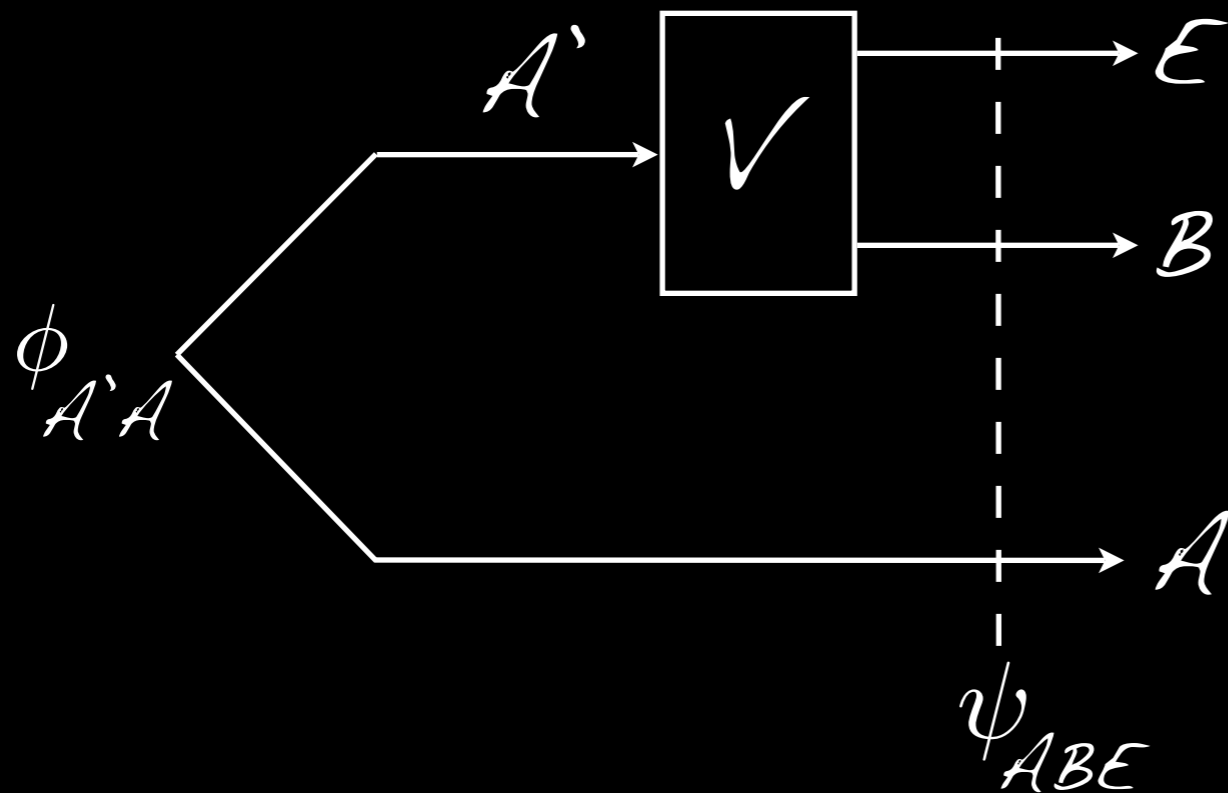
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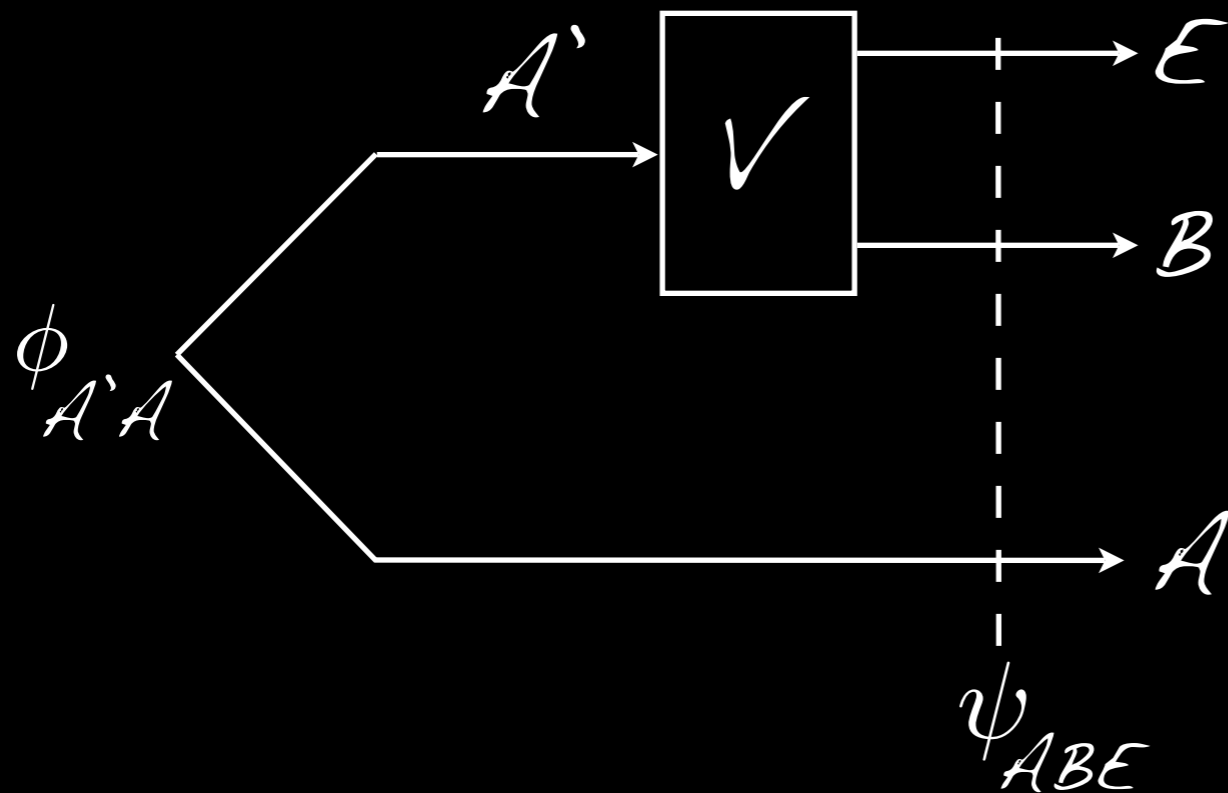
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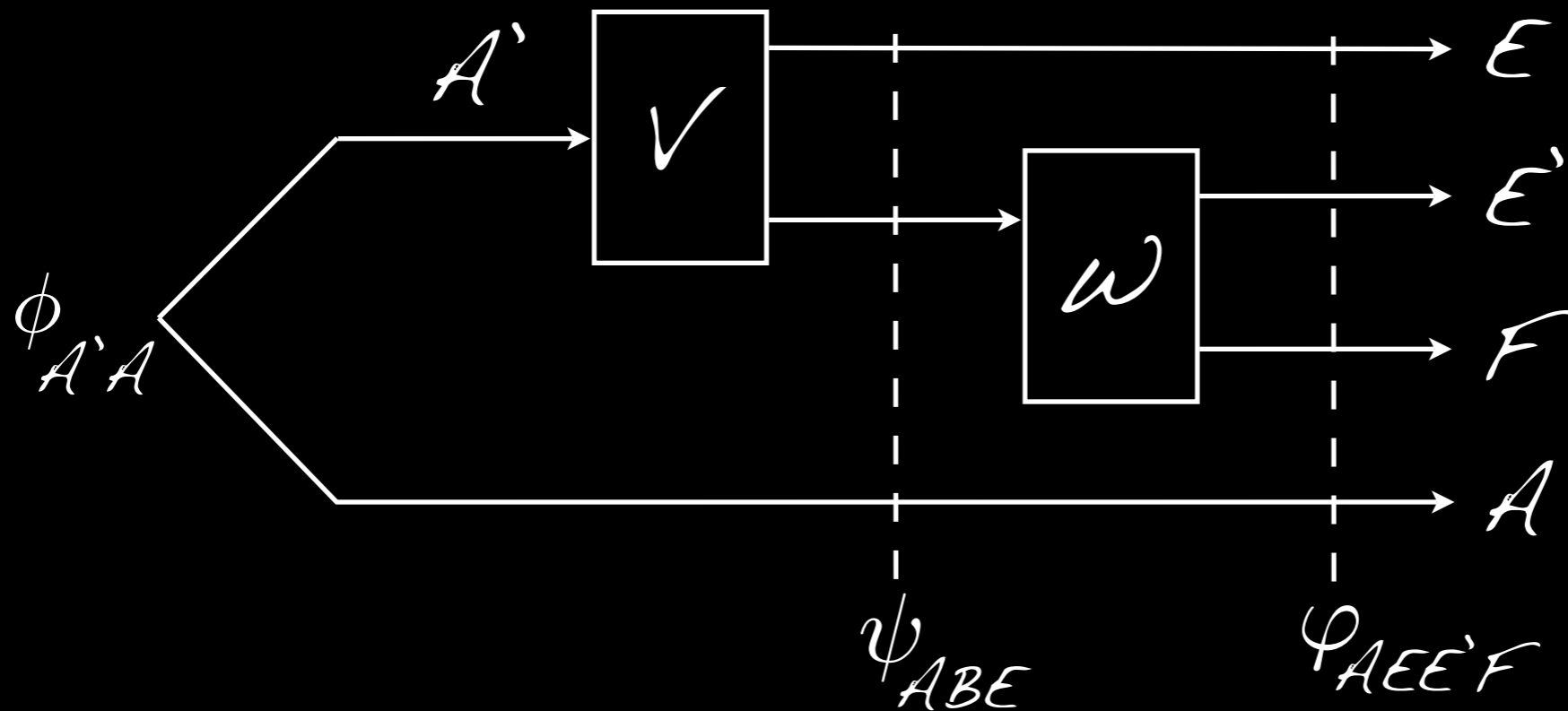
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Apply degrading map
(Stinespring form)

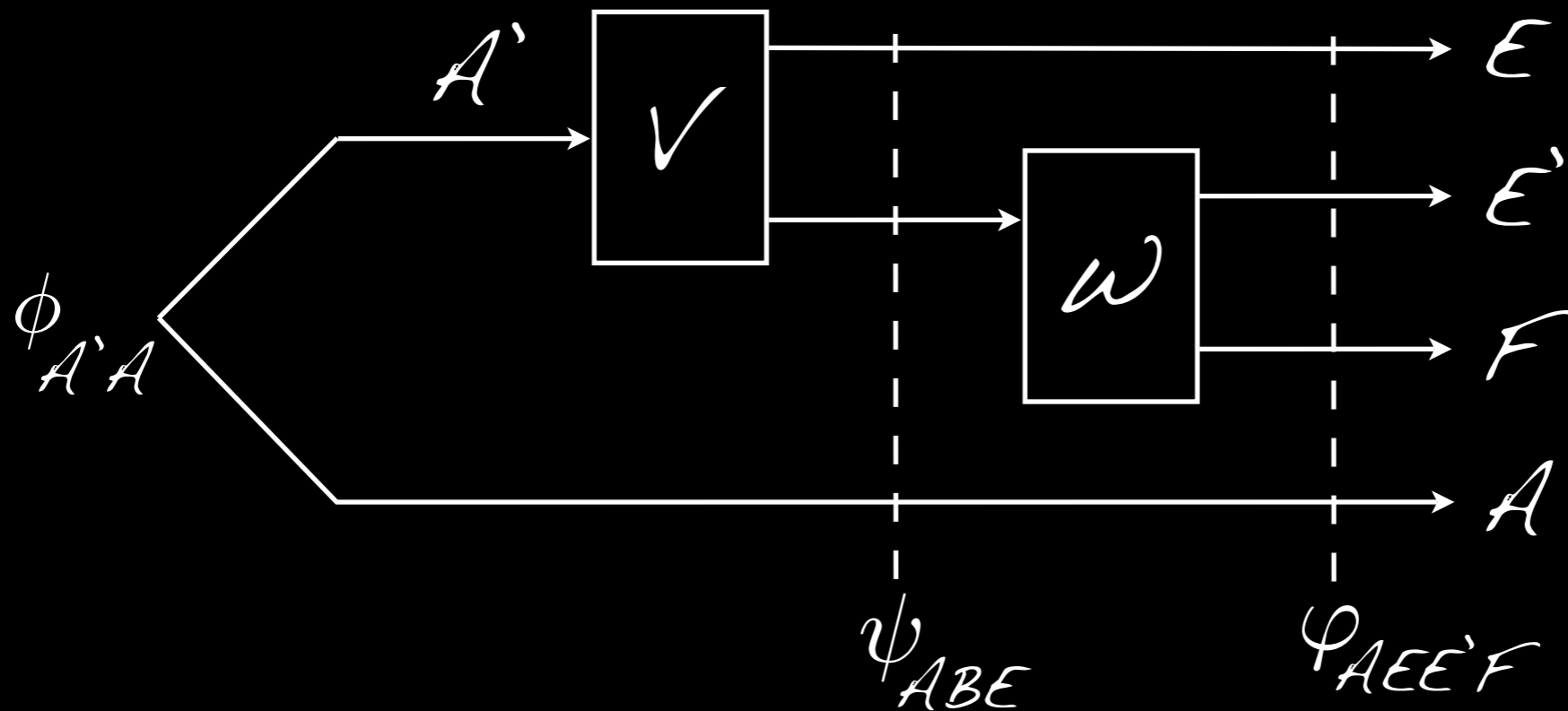
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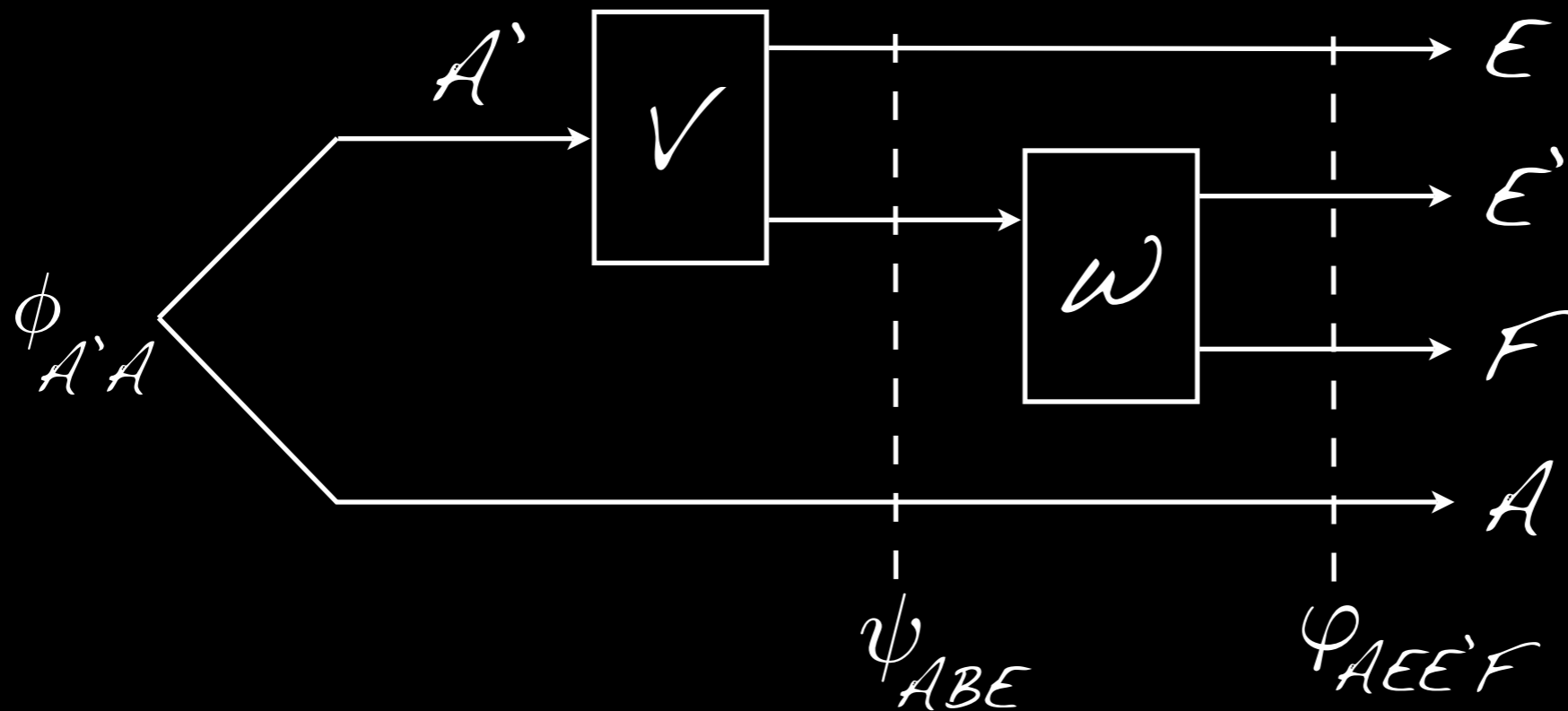
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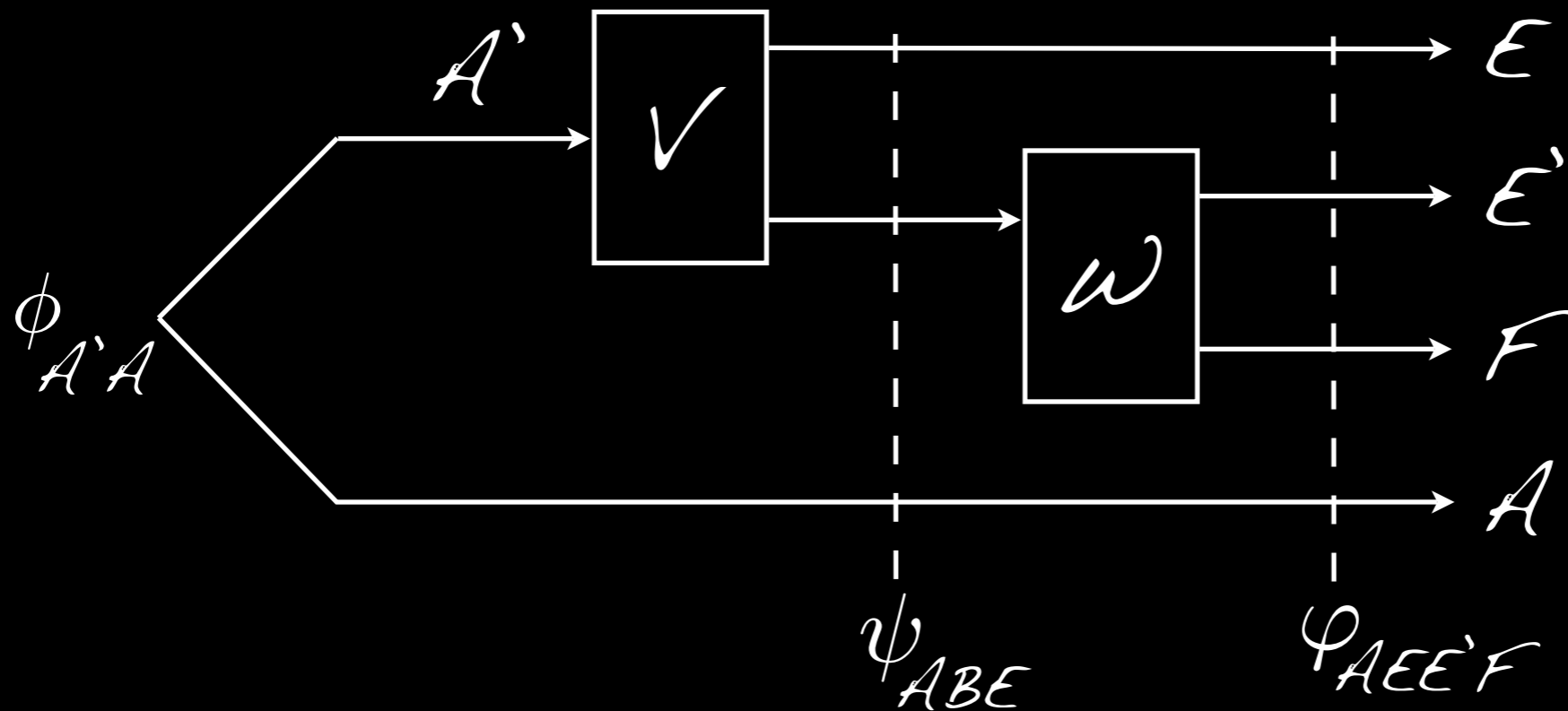
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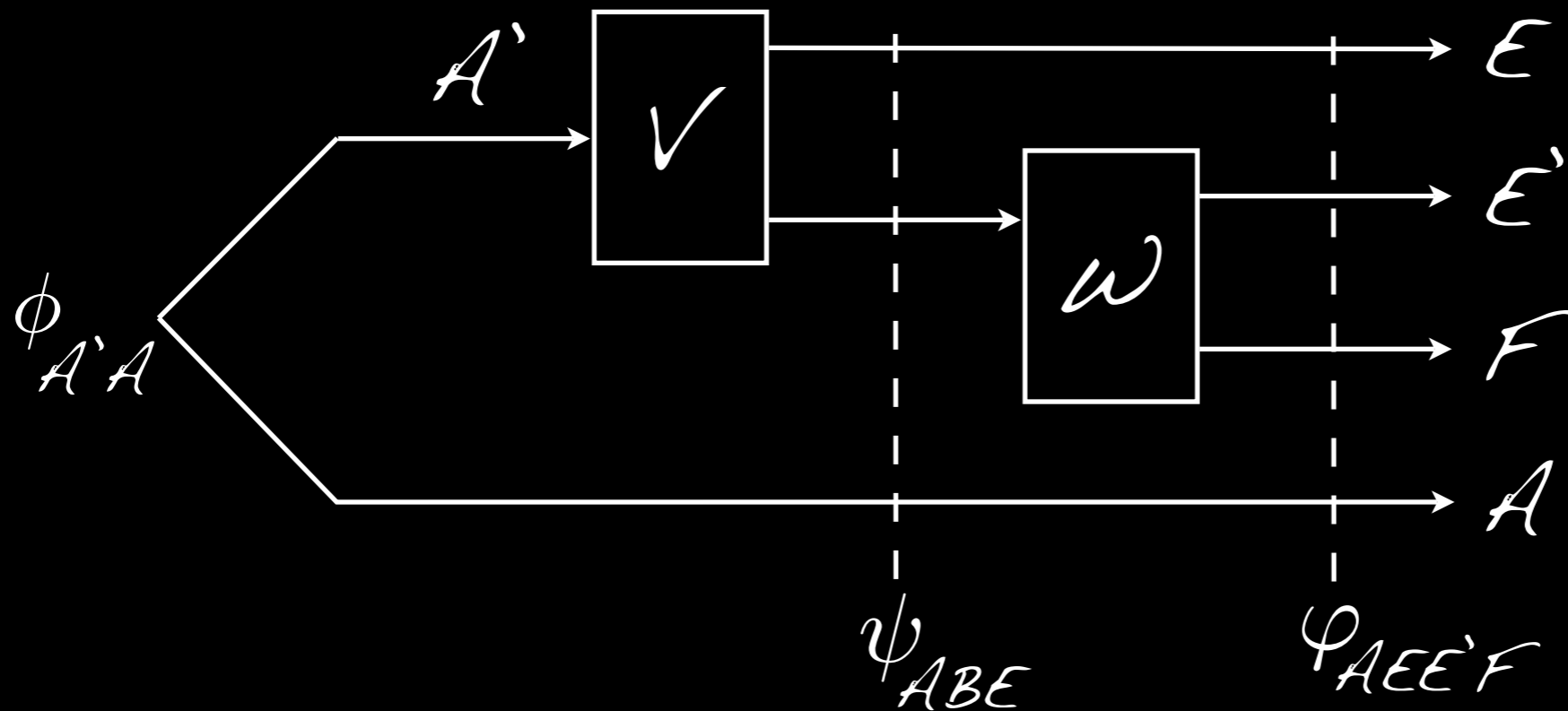
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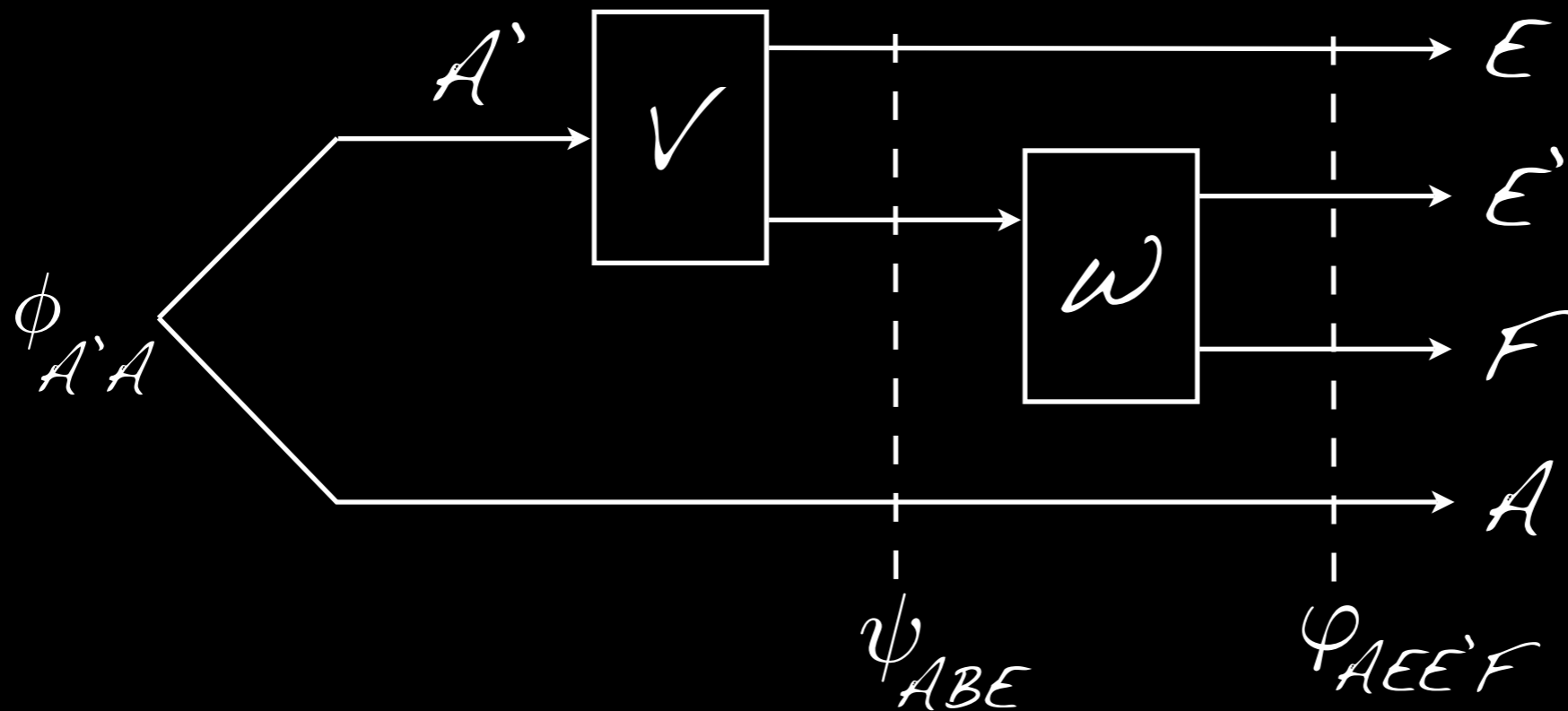
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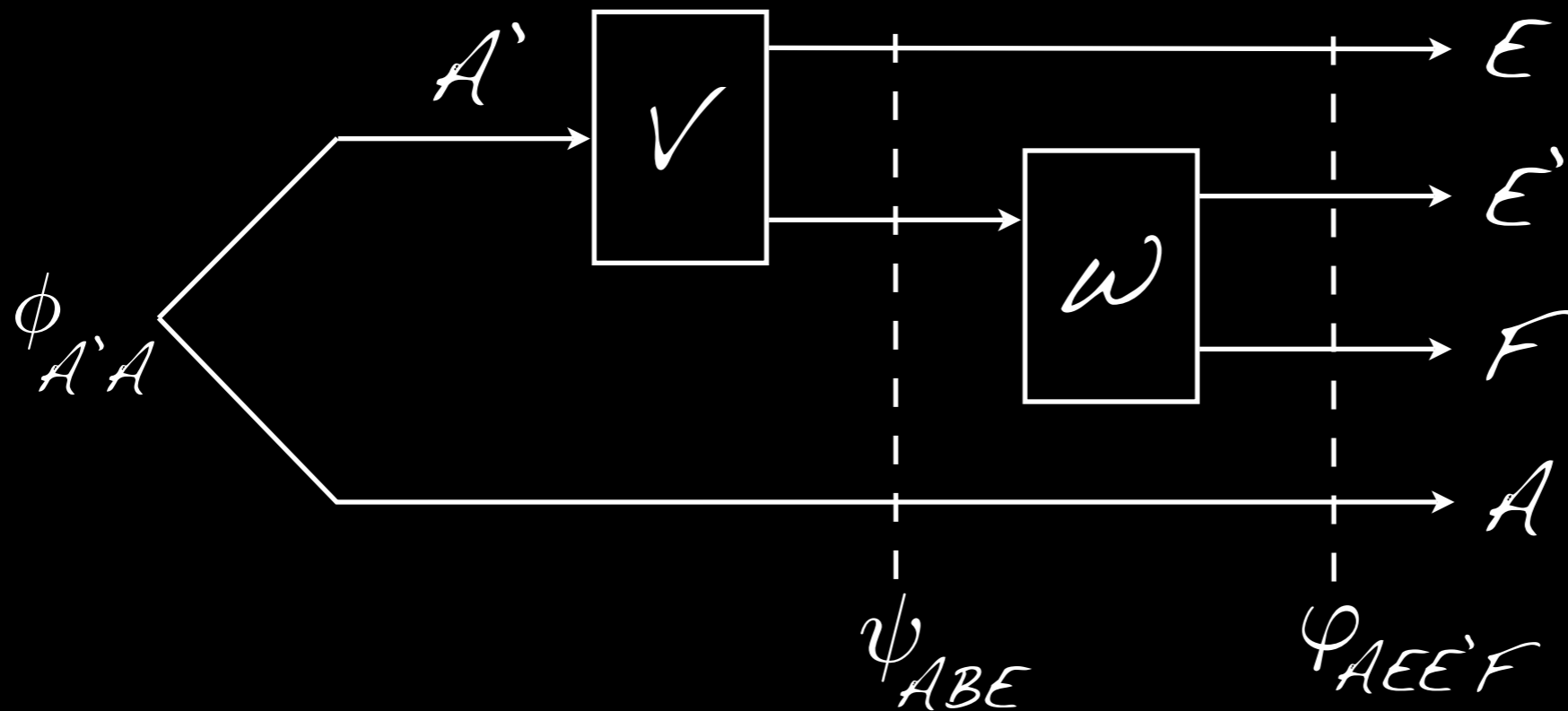
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G. Smith (0705.3838): Also private classical capacity, $P(N) = Q(N) = Q^{(1)}(N)$.

5. Results & proof ideas

Previous [via E. Rains, IEEE-IT 47(7):

2921-2933 (2001) - thanks to R. Koenig!]:

If N is PPT entanglement-binding, then

of course $Q(N)=0$, and strong converse

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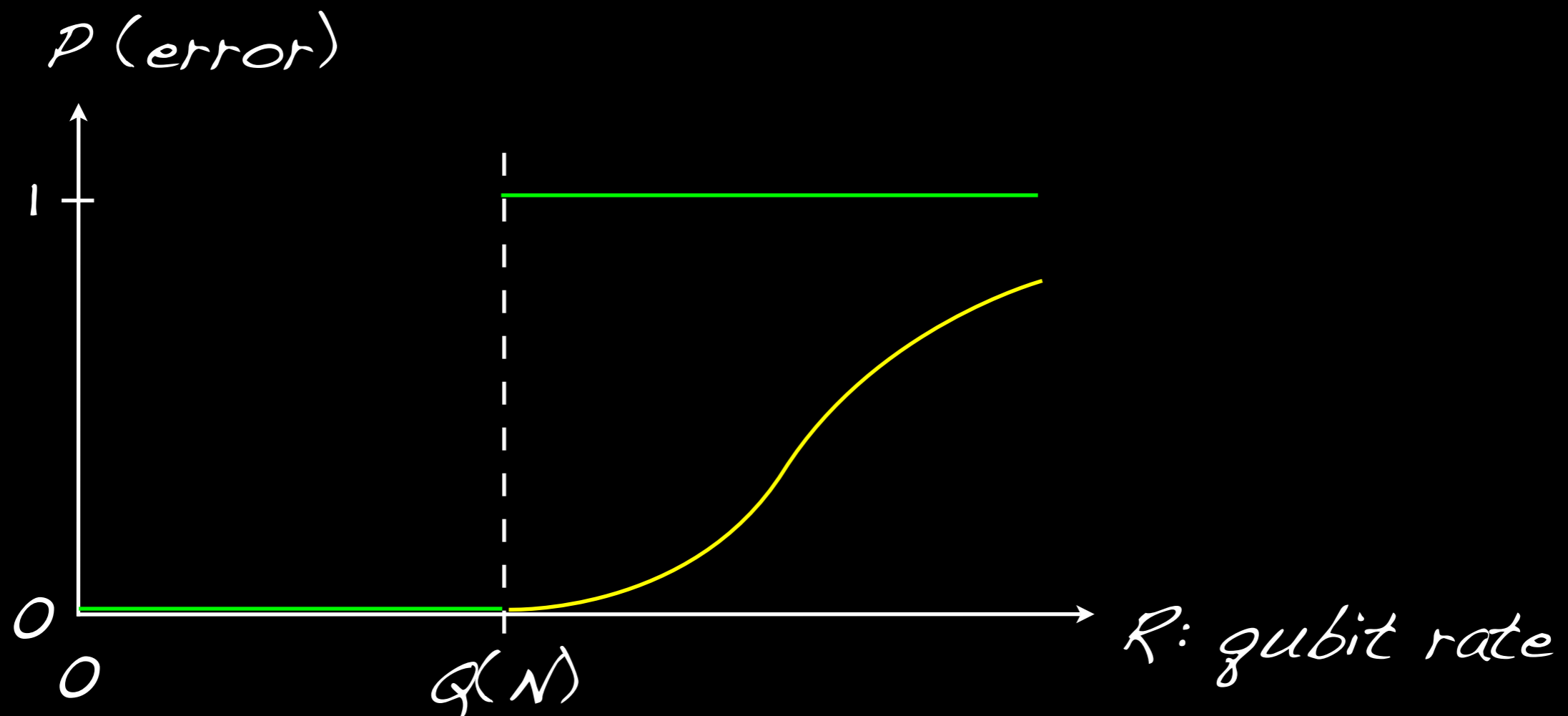
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Note: Already for symmetric (degradable
& anti-degradable) channels - for which
also $Q(N)=0$ - not clear at all.

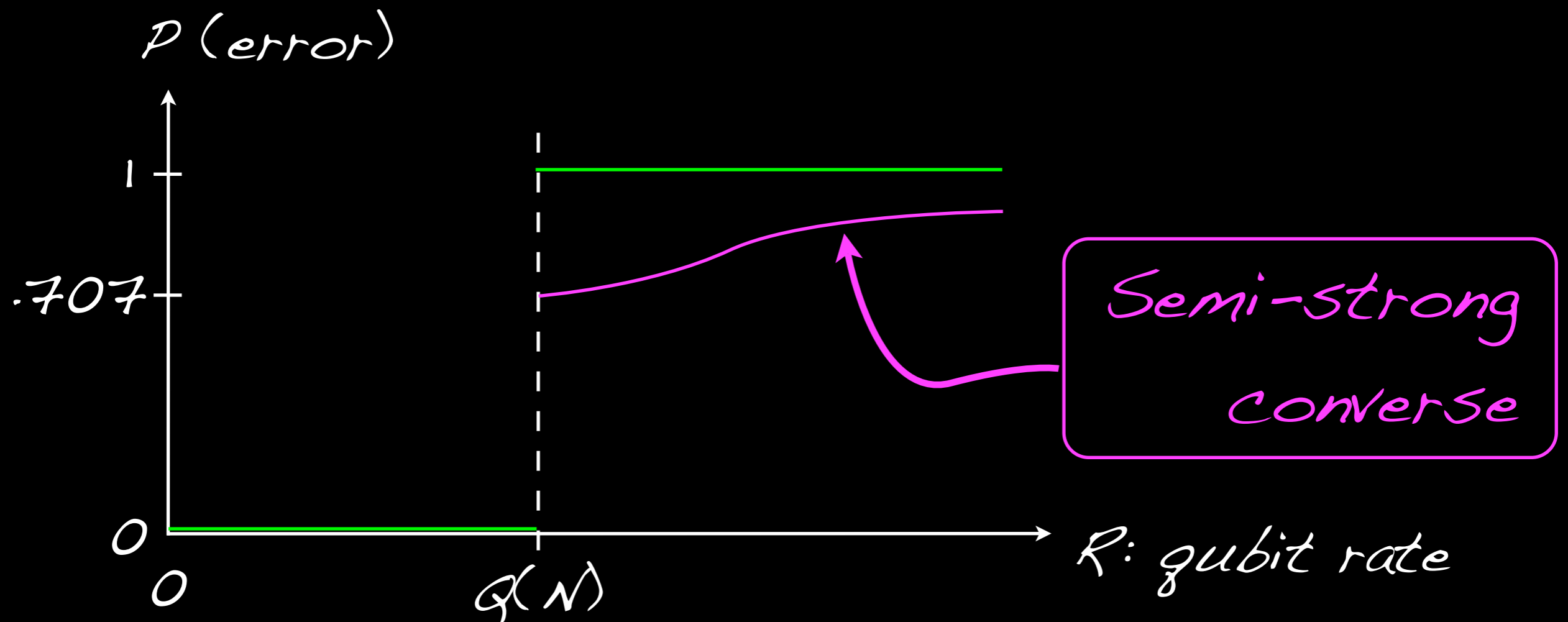
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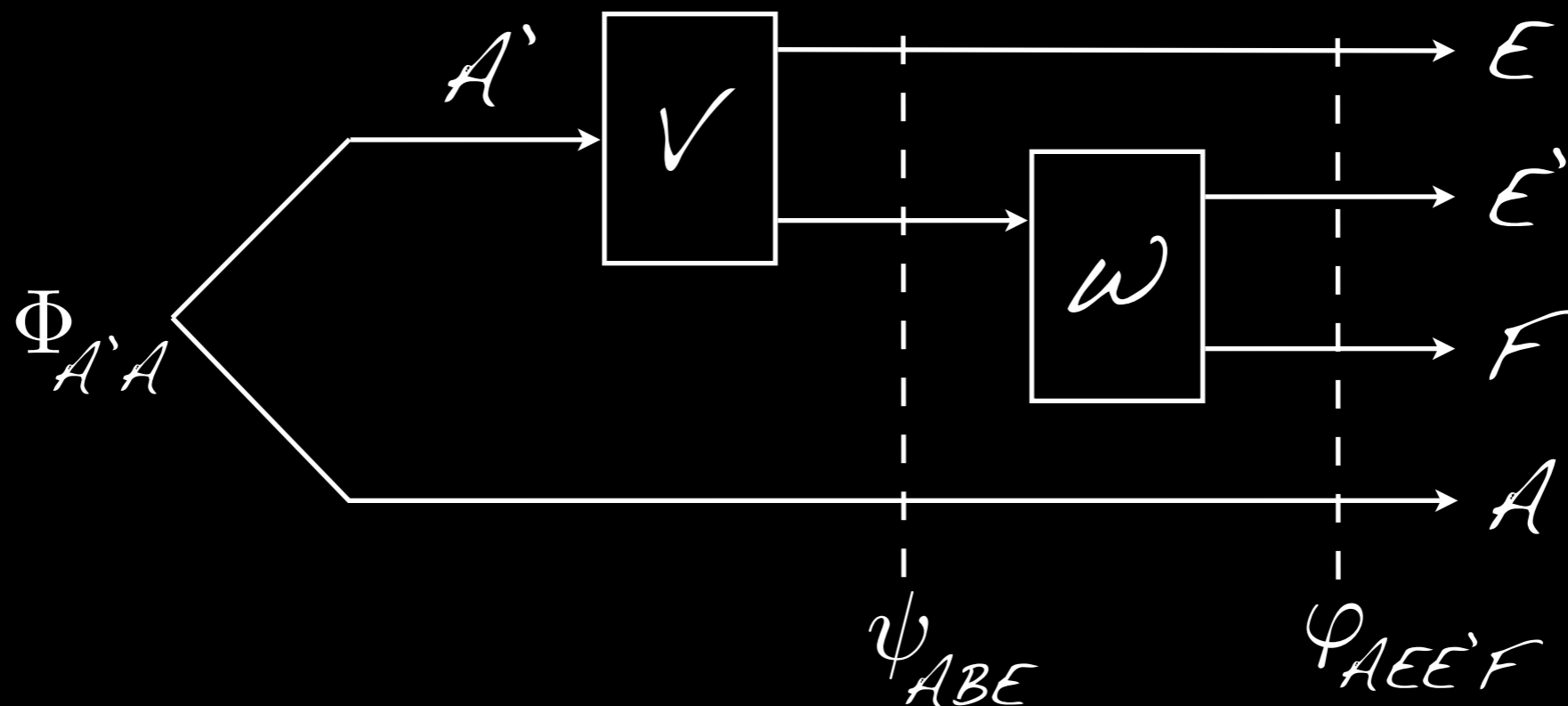
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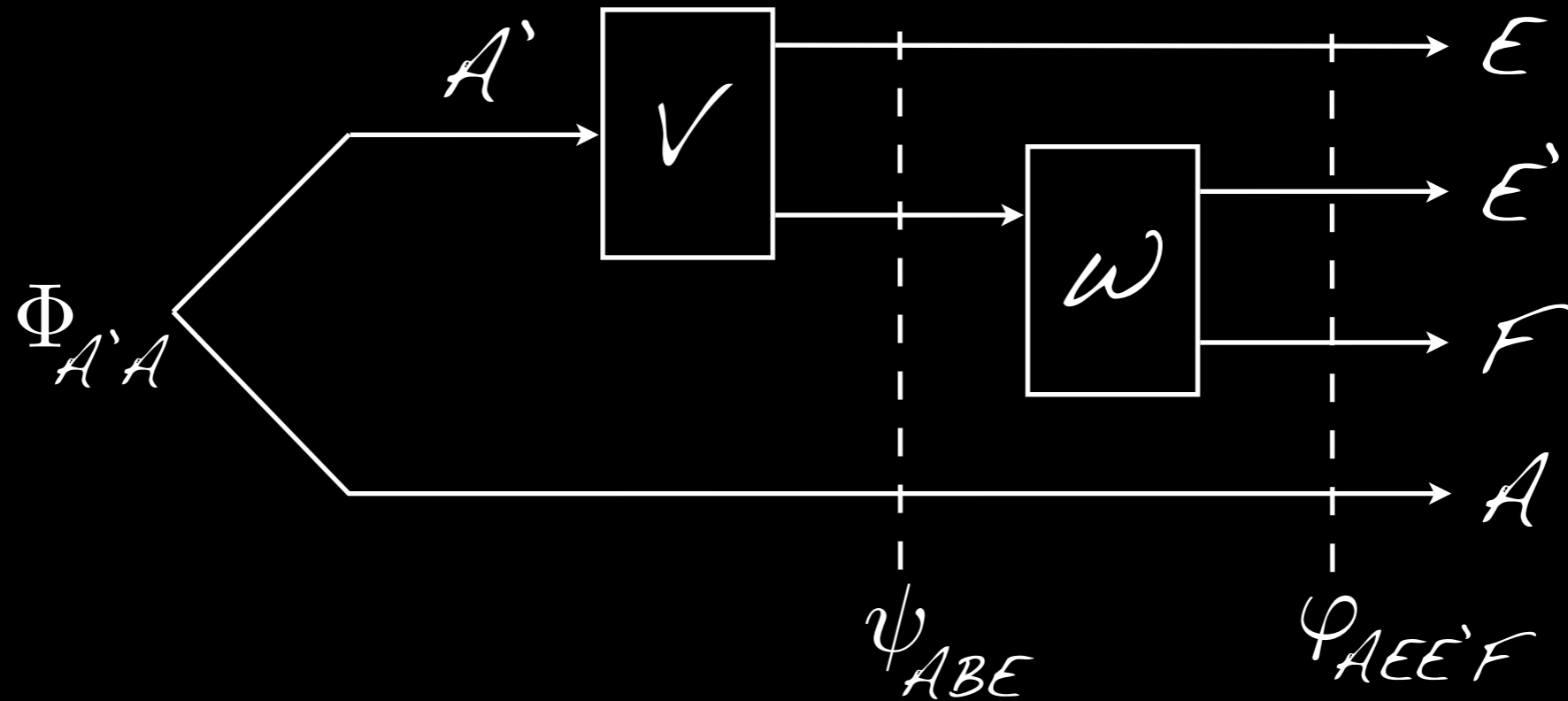
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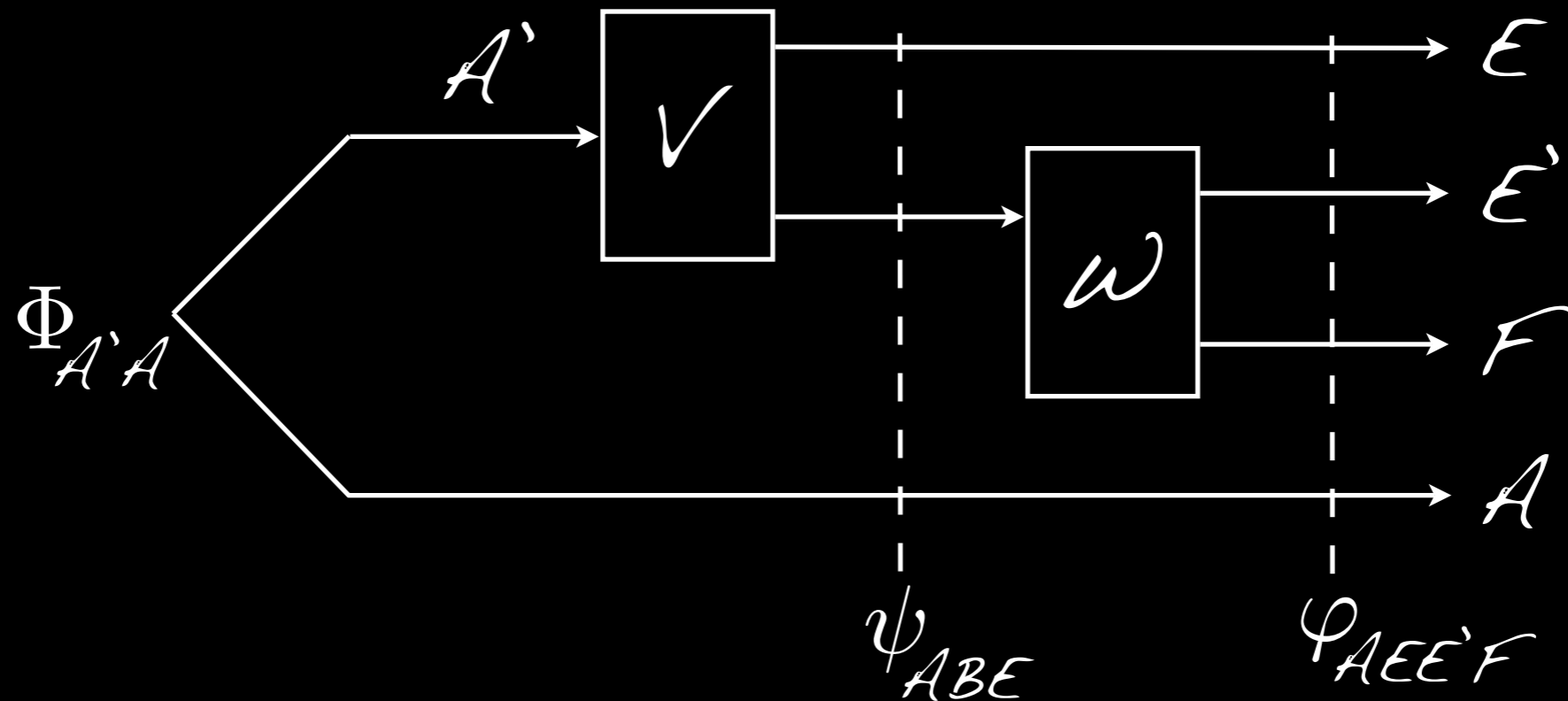
1) Use code - for simplicity subspace -
 with maximally entangled state Φ of k
 qubits:



Maximally entangled state Φ of k qubits:

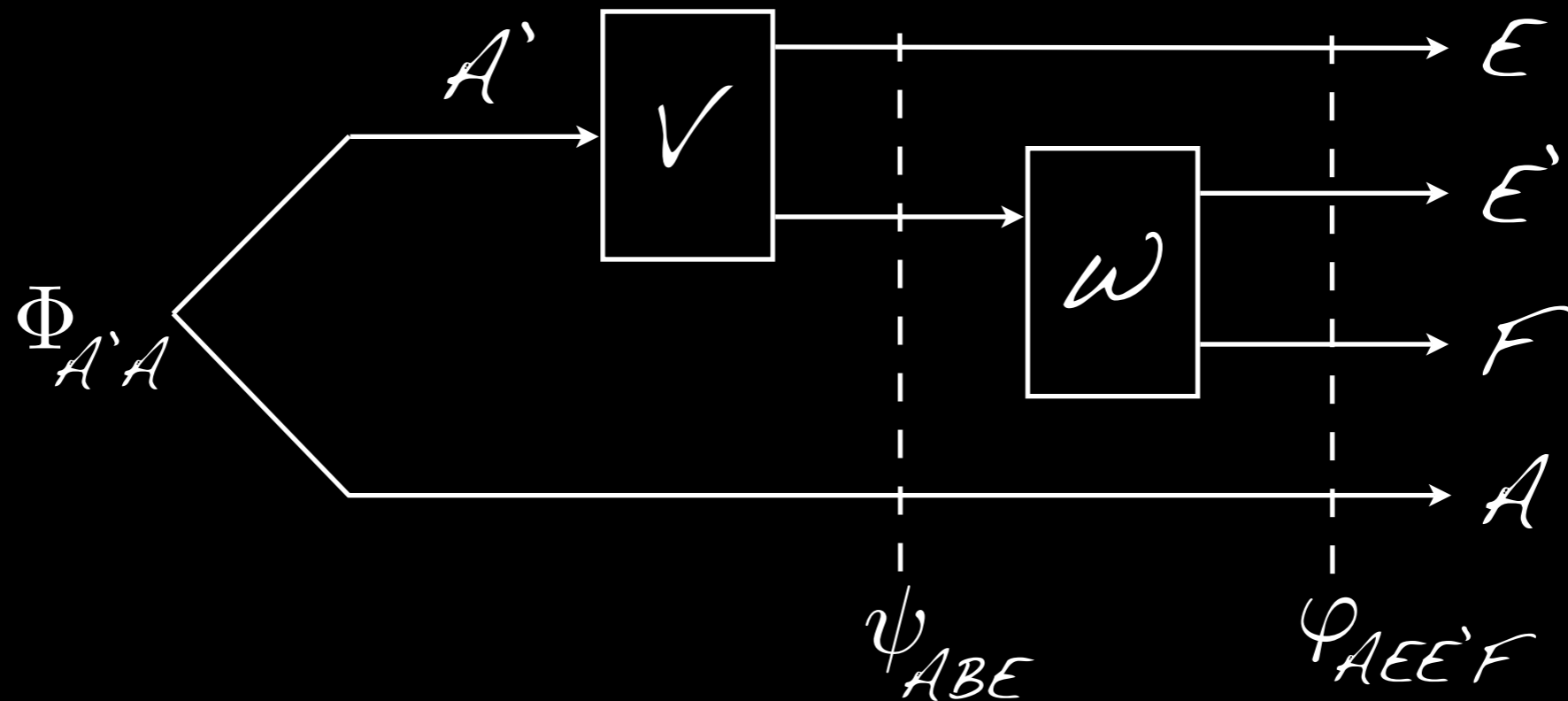


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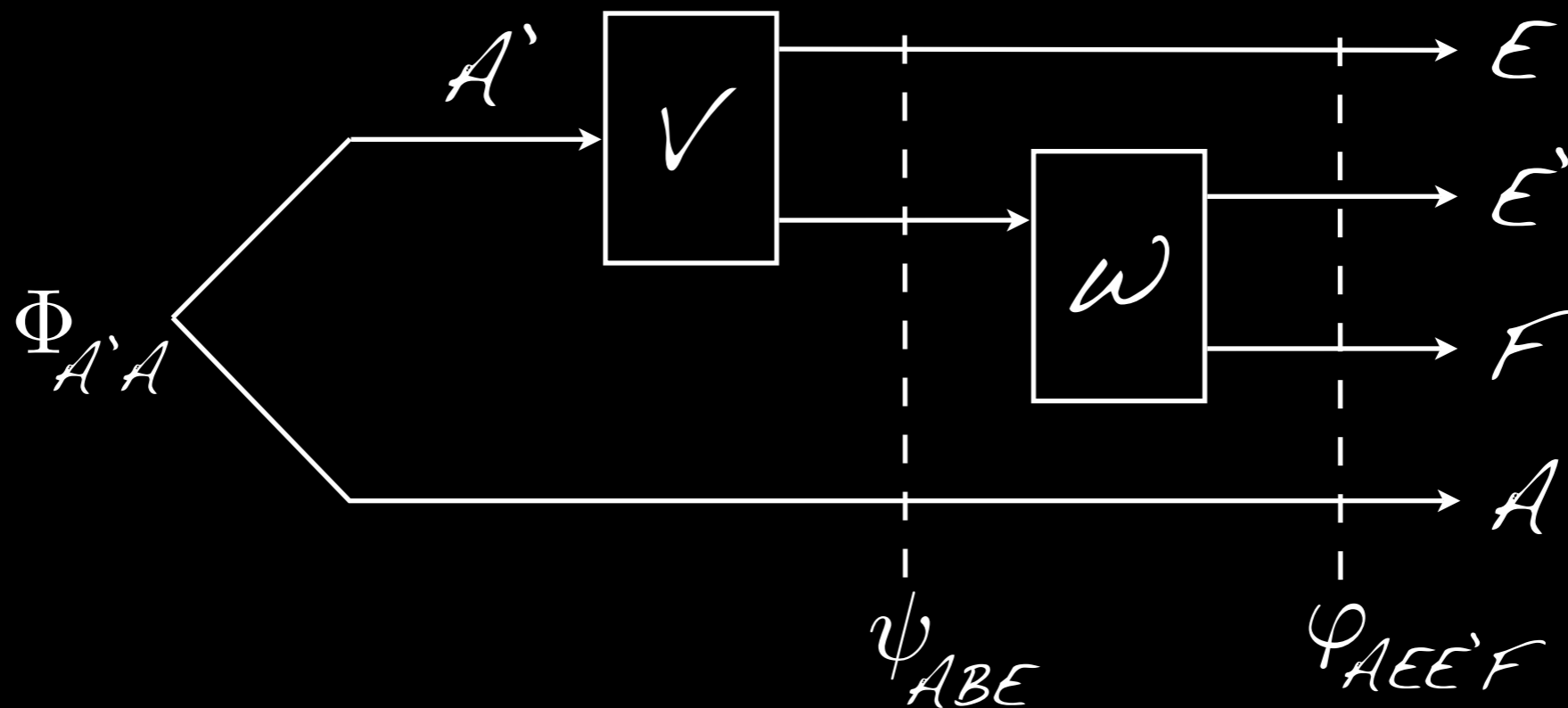
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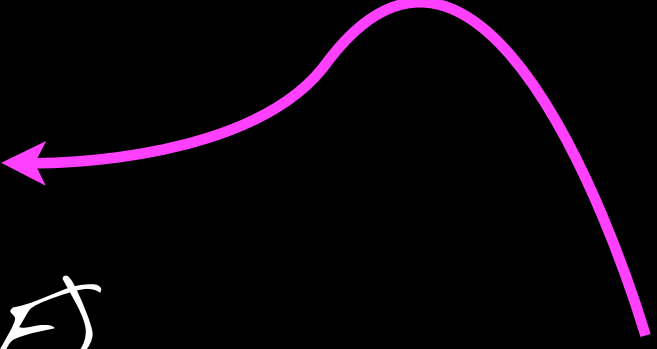
[For min-entropy calculus, consult
 R. Renner, PhD thesis, quant-ph/0512258
 & M. Tomamichel, PhD thesis, arXiv:1203.2142]

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$$K \leq H_{\min}^{\epsilon}(A|E) \\ = -H_{\max}^{\epsilon}(A|E^*F)$$

[Cf. also Buscemi/Datta,
IEEE-IT 56(3), 2010;
Datta/Hsieh, 1103.1135]

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Note: If we knew that for n channel uses, the maximum min-entropy is attained on a tensor product input, we'd be done by AEP (= asymptotic equipartition property)...

$$\begin{aligned} K &\leq \mathcal{H}_{\min}^{\epsilon}(A|E) \\ &= -\mathcal{H}_{\max}^{\epsilon}(A|E'F) \\ &\leq \mathcal{H}_{\max}^{\lambda}(F|E') - \mathcal{H}_{\max}^{\delta}(AF|E') + O(1) \end{aligned}$$

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 Chain rule, $\delta = \epsilon + 3\lambda$.

$$\begin{aligned}
k &\leq \Psi_{\min}^{\epsilon}(A|E) \\
&= -\Psi_{\max}^{\epsilon}(A|E'F) \\
&\leq \Psi_{\max}^{\lambda}(F|E') - \Psi_{\max}^{\delta}(AF|E') + O(1)
\end{aligned}$$


 Chain rule, $\delta = \epsilon + 3\lambda$.

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...if $\delta < 0.707$, by inequality Ψ_{\min} vs. Ψ_{\max} , and using symmetry between E and E' ...

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3) By exponential de Finetti theorem

[R. Renner, PhD thesis, quant-ph/0512258]:

$$K \leq \max_{\rho_A} \mathcal{H}_{\max}^{\lambda''}(F^n | E^n)_{\rho^{\otimes n}} + o(n)$$

4) By AEP (asymptotic equipartition property) [M. Tomamichel, arXiv:1203.2142]:

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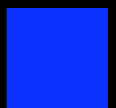
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- Proved "semi-strong" converse for the quantum capacity of degradable channels; similar statement possible for private classical capacity...
- Can we get an honest strong converse? (Bottleneck are symmetric channels: the current error threshold is that of a 50% erasure channel.)
- Indeed: Strong converse for degradable channels if it holds for symmetric ch.

• Sharma/Warsi (arXiv:1205.1712) propose a different methodology, based on certain generalised divergences. They can't prove a strong converse, but if an additivity problem could be solved for some channels (degradable?), it would follow..

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- How to prove strong converses without additivity? Note that neither Q nor $Q^{(1)}$ are additive! Similar for other capacities...