A Quantitative Landauer's Principle

Philippe Faist, Frédéric Dupuis, Jonathan Oppenheim*, Renato Renner

Institute for Theoretical Physics, ETH Zurich

*Department of Physics and Astronomy, University College of London

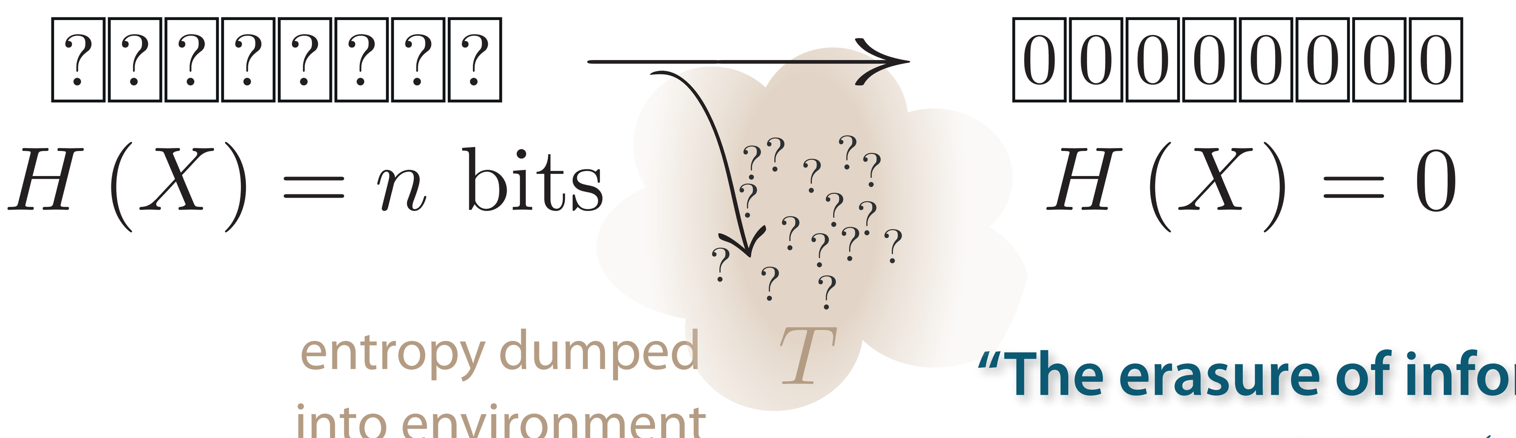
QSIT Quantum Science and Technology — National Centre of Competence in Research

ETH Eidgenössische Technische Hochschule Zürich — Swiss Federal Institute of Technology Zurich

Is there a **fundamental work cost for information processing**?

LANDAUER'S PRINCIPLE

Landauer, IBM J. Res. Dev. (1961)
Bennett, Int. J. Theo. Phys. (1982)



"The erasure of information costs work"

$$W = kT\ln(2)\cdot H(X)$$

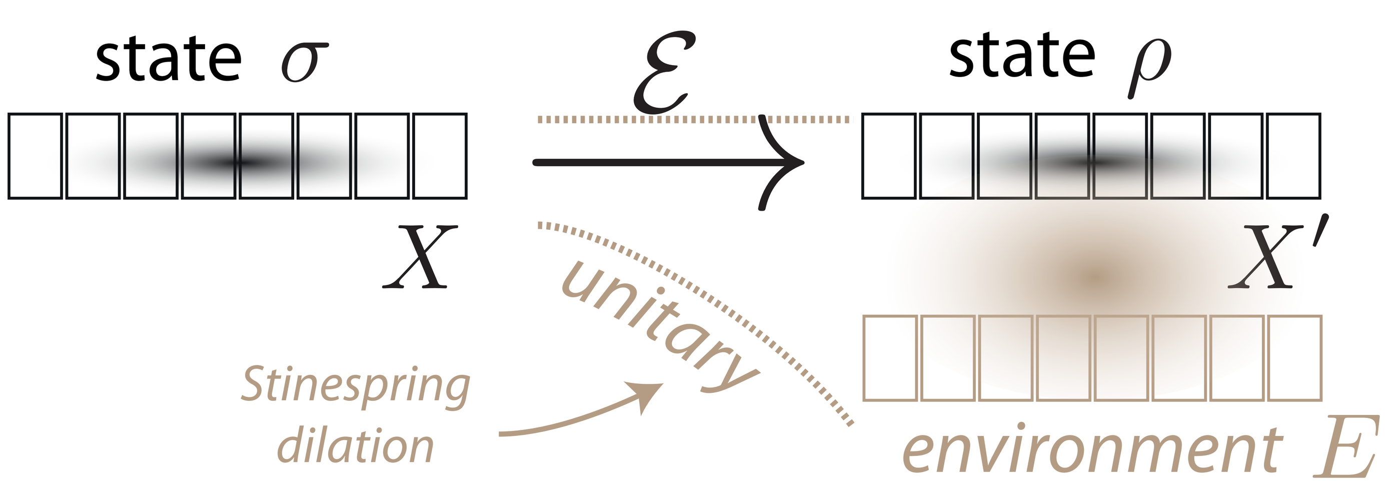
"Logically irreversible processes cost work"

MAIN RESULT, FOR GENERAL PROCESSES

del Rio et al., Nature (2011)
Faist et al., arXiv:1211.1037 (2012)

Given any abstract process or computation $\mathcal{E}$ ...

(trace-preserving, completely positive map)



... its physical implementation will have to dump entropy into the environment ...

... inducing a corresponding work cost

$$W \geqslant kT\ln(2)\cdot H^{\varepsilon}_{\max}(E|X')$$

(as would require the second law of thermodynamics !)

✔ Optimal

✔ Achievable

(up to $\log\varepsilon$ terms)

i.i.d. only: $= [H(\text{initial}) - H(\text{final})]\ kT\ln 2$

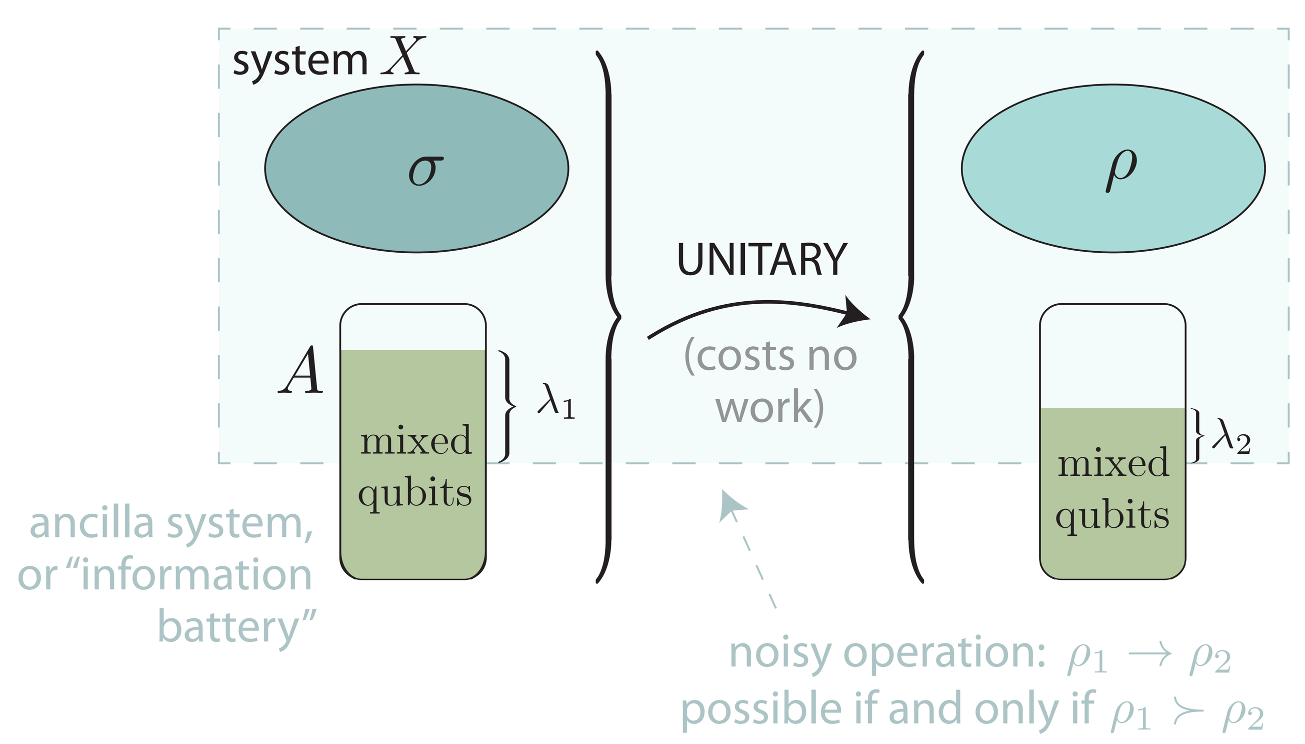
In general, correlations between input and output are important!

OUR MODEL

Oppenheim et al., PRA (2003)
Åberg, arXiv:1110.6121 (2011)
Horodecki et al., arXiv:1111.3834 (2011)
Egloff et al., arXiv:1207.0434 (2012)
Faist et al., arXiv:1211.1037 (2012)

What is **work** in a quantum system?

use **lambda-majorization**: $\sigma \xrightarrow{\lambda} \rho$



"absorption" of λ mixed qubits
($\lambda = \lambda_1 - \lambda_2$)

Restoring A with thermal operations extracts $\lambda \cdot kT\ln 2$ work.

(in any reasonable framework)

Mixed qubits "absorbed" = work extracted

Got rid of Hamiltonians! Accounting for "randomness" instead.

LAMBDA-MAJORIZATION

$$\sigma \xrightarrow{\lambda} \rho \ :\quad 2^{-\lambda_1}\mathbb{1}_{2^{\lambda_1}}\otimes\sigma \succ 2^{-\lambda_2}\mathbb{1}_{2^{\lambda_2}}\otimes\rho$$

extracts $\lambda\, kT\ln(2)$ work.

ALLOWED OPERATION SET

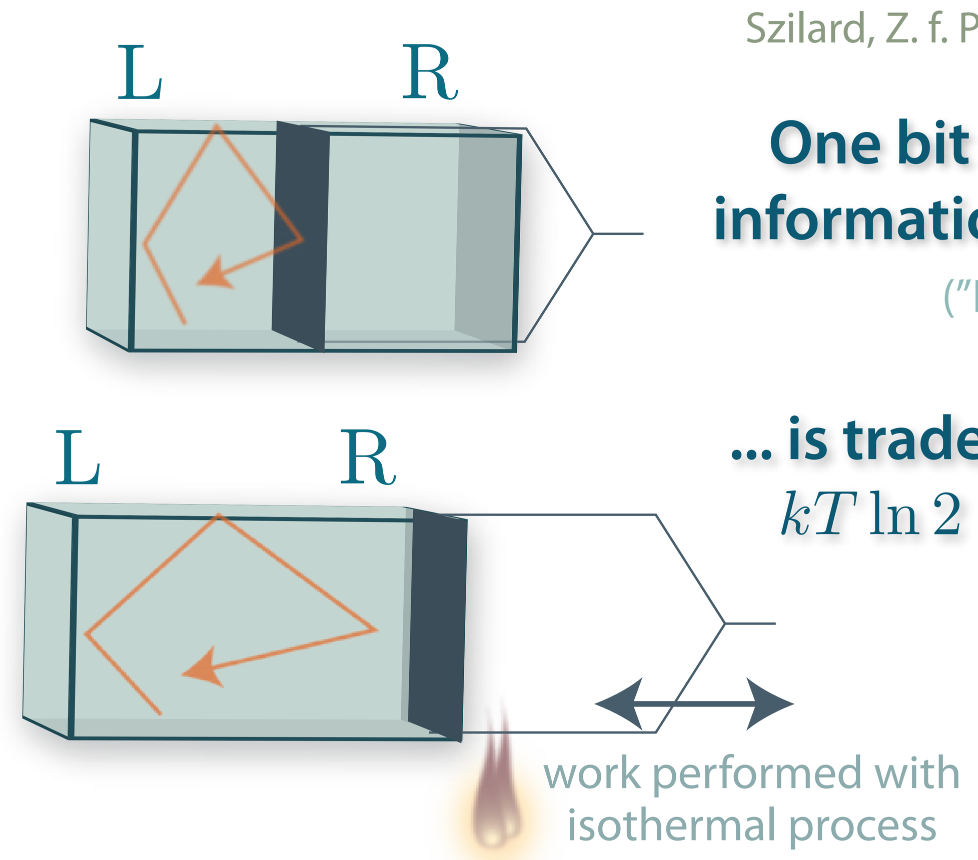
(a) $|0\rangle \to \mathbb{1}$ $\quad (W = kT\ln 2)$
(b) $\mathbb{1} \to |0\rangle$ $\quad (W = -kT\ln 2)$
(c) add pure ancillas, restore
(d) global unitaries

postulate lambda-majorization as only allowed operation, motivated by the second law of thermodynamics

... and then: formulate in terms of a class of channels, write as semidefinite problem, solve optimally.

THE SZILARD BOX

Szilard, Z. f. Phys. (1927)

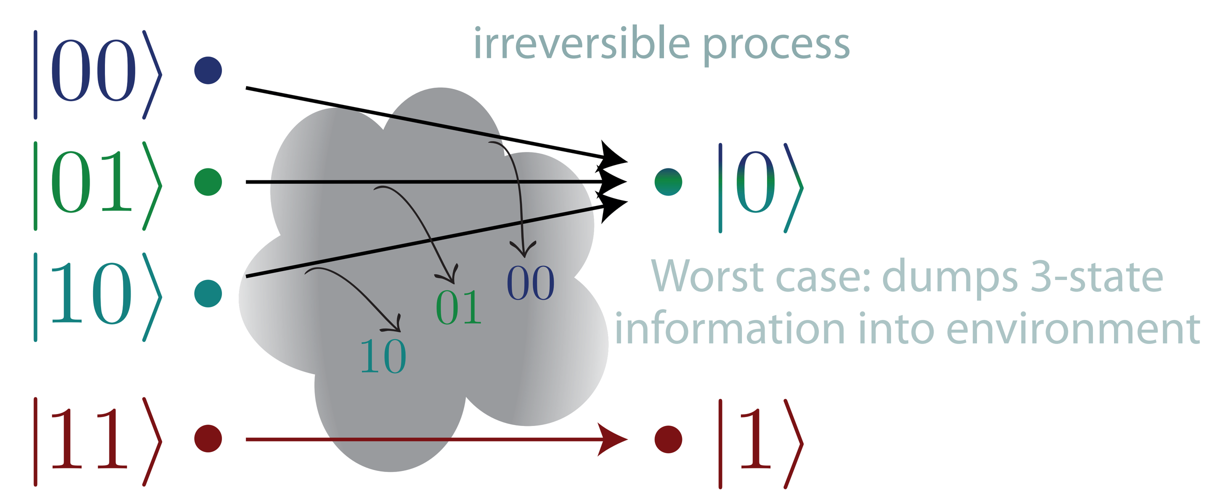


One bit of information ...

("L" or "R")

... is traded for $kT\ln 2$ work.

EXAMPLE: THE "AND" GATE

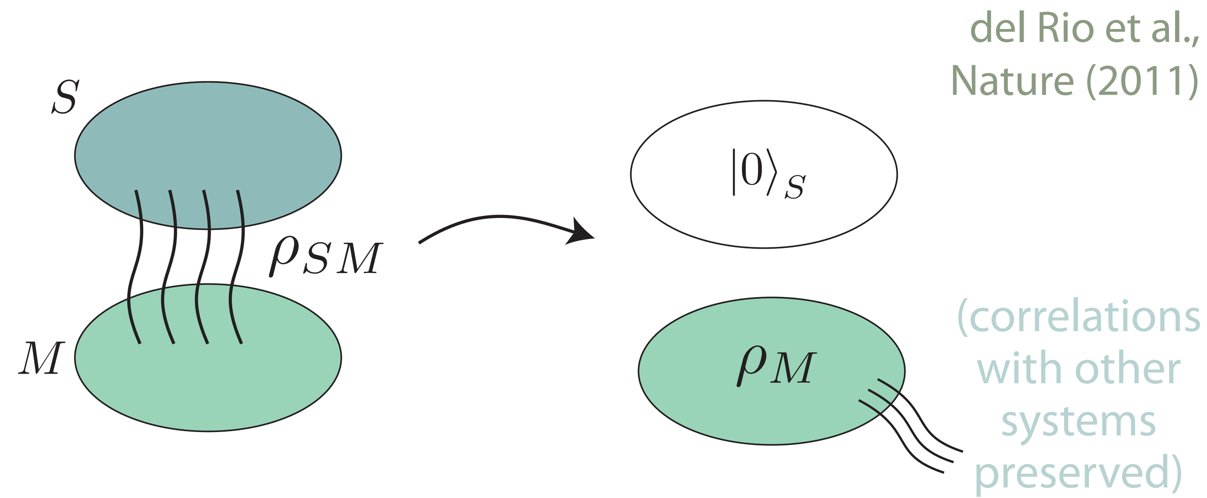


A physical implementation of the "AND" gate will have a work cost of at least

$$W_{\text{AND}} = kT\ln(2)\cdot H_{\max}(E|X')$$
$$\approx 1.6\, kT\ln(2)$$

ERASURE WITH MEMORY

del Rio et al., Nature (2011)



Erasure of S, given access to M, costs

$$W \approx kT\ln(2)\cdot H^{\varepsilon}_{\max}(S|M)$$

✔ Now proven optimal as corollary

WHICH ENTROPY MEASURE?

Main result derived without probability of failure $(\varepsilon = 0)$

$$W \geqslant kT\ln(2)\cdot H_0(E|X')_\rho$$

Rényi-zero entropy

Smoothing to allow a probability of error $(\varepsilon \neq 0)$

$$W \geqslant kT\ln(2)\cdot H^{\varepsilon}_{\max}(E|X')_\rho$$

Smooth max-entropy

"Operational smoothing": connection to D^{ε}_{H} measure?

Buscemi et al., IEEE TIT (2010)
Brandão et al., IEEE TIT (2011)
Wang et al., PRL (2012)
Tomamichel et al., arXiv:1208.1478 (2012)
Dupuis et al., arXiv:1211.3141 (2012)

... more in **arXiv:1211.1037**