

Magnetization field at criticality in the Ising model

Abstract :

If one considers an $N \times N$ grid with independent coin flips $\sigma_x \in -1, 1$ at each vertex, it is well known that the renormalized field $N^{-1} \sum_x \sum_x \delta_{x/N}$ converges as N goes to infinity to a Gaussian white noise in the square $[0, 1]^2$. More precisely for each “nice” subset A of this square, the field measured in A is a Gaussian random variable with variance the area of A .

The aim of this talk is to study what happens when the coin flips are no longer independent of each other. This situation has been considered in various contexts and one cannot hope for a “universality” result as in the iid case. In particular, one has to precise what type of dependency structure one is interested in. In this talk, I will focus on some famous distributions which arise in statistical mechanics and in particular on the case where the coin flips σ_x are defined to be the spins of an Ising model on the $N \times N$ grid. In this context, the sum over the spins corresponds to the so called magnetization field. Away from the critical point, it is known that this magnetization field (properly renormalized) converges also towards a Gaussian white noise.

It remains to understand the magnetization field at criticality. In a joint work with Federico Camia and Chuck Newman, we prove the following facts:

- (i) at $T = T_c$, the discrete magnetization fields have a unique scaling limit.
- (ii) This limit is non-Gaussian.
- (iii) The limit has an explicit conformally covariant structure.