On the Number of Wavelengths in Arbitrarily-Connected Wavelength-Routed Optical Networks

Stefano Baroni*, Polina Bayvel*, and Richard J. Gibbens†

* Department of Electronic & Electrical Engineering, University College London, Torrington Place, London, WC1E 7JE, UK
† Statistical Laboratory, University of Cambridge, 16 Mill Lane, Cambridge CB2 1SB, UK

Abstract

Wavelength-routed optical networks (WRONs) represent the most promising solution for the next generation high-capacity transport layer. Their feasibility critically depends on the number of wavelengths $N_\lambda$ required to satisfy a given traffic demand. This paper studies the allocation of active and restoration lightpaths in arbitrarily-connected WRONs and addresses the role played by the physical topology on $N_\lambda$. Wavelength interchange functionality in the optical cross-connects is also addressed. The results can be used in the analysis and optimisation of WRON design.

Keywords

WDM, Wavelength Routing, Optical Transport Networks, Optical Restoration.

Introduction

Wavelength division multiplexed (WDM) optical networks using wavelength routing (WRONs) [1] are key for telecom operators to deploy the next generation optical transport layer, to satisfy the ever increasing traffic demand [2]. The greatest operational advantage of WRONs is achieved when the network node-pairs are assigned high-capacity optical channels, known as lightpaths [3], resulting in a single-hop logical topology [4]. The intermediate optical cross-connects (OXCs) route the lightpaths within the network from sources to destinations [5], simplifying network management and processing compared to the routing in digital cross-connected systems. Similar operational advantages are expected from link failure restoration within the WDM transport layer [6].

The feasibility of this approach strongly depends on the number of wavelengths $N_\lambda$ required to satisfy a given traffic demand. Several near-optimal lightpath allocation algorithms have recently been reported aimed at minimising the network wavelength requirement, the wavelengths being a limited resource [3], [7]-[9]. However, insufficient attention has been paid to the importance of the physical topology. This is by far the most critical parameter, as it directly determines lightpath allocation and wavelength requirement, and hence the size and complexity of OXCs. Where physical topologies have been investigated such as in [10], [11] for finite blocking probability and in [12], [13] for zero-blocking, this has been done only in networks with regular topologies. Considered as theoretical limits, these results are difficult to apply to real transport networks whose physical topologies, determined by cost and operational constraints, are neither fully nor regularly connected.

This paper studies the allocation of active and restoration lightpaths in arbitrarily-connected WRONs. The wavelength requirement $N_\lambda$ is derived as a function of the physical topology, characterised by the physical connectivity $\alpha$. The relative merits of wavelength interchanging functionality in the OXCs is also addressed.

An integer linear program (ILP) formulation is proposed for the exact solution of the routing and wavelength allocation (RWA) problem. Lower bounds are discussed and new lightpath allocation heuristic algorithms described.

Several existing or planned fibre network infrastructures are analysed first. The results demonstrate the computational complexity of the ILP formulation and the good solutions obtained by the heuristics. A systematic analysis of a large number of arbitrarily-connected randomly generated WRONs is then performed. The results confirm the limited effect of wavelength conversion on $N_\lambda$, and identify the relationship between $N_\lambda$ and $\alpha$, which is shown to be in excellent agreement with analytical lower bounds. By considering restoration of single link failures, the importance of the number of links in key network cuts is highlighted.

Network model

The analysed network architecture consists of $N$ nodes, arbitrarily-connected by $L$ links (see Fig. 1). It is assumed that each link is a single bidirectional fibre. This is the worst case scenario for the wavelength requirement, as multiple fibres per link result in a larger number of disjoint physical paths and, therefore, a smaller $N_\lambda$.

It is assumed that (CI) any two subsets of the network nodes are connected by at least two links. This is crucial for the network reliability, so that in the case of a single link failure, the network remains connected and restoration lightpaths can be established along alternative physical paths.
assigned a bidirectional lightpath. In transport applications (now a wavelength blocking is allowed, and in the case of link failure all the interrupted lightpaths must be restored, without disturbing live traffic.

The network physical connectivity $\alpha$ is defined as the normalised number of bidirectional links with respect to a physically fully-connected network of the same size [14]:

$$\alpha = \frac{l_{PC}}{L} = \frac{3l}{N(N-1)} \quad (1)$$

Each node consists of an end-node directly connected to an OXC (see Fig. 1). The end-nodes emit and terminate the lightpaths, whilst the OXCs route the lightpaths from sources to destinations. The OXCs are assumed strictly non-blocking in the spatial domain and reconfigurable. Both OXC configurations with and without wavelength conversion are considered, denoted as wavelength interchange and wavelength selective cross-connect, WIXC and WSXC, respectively. In the WSXC case, each lightpath is assigned a fixed end-to-end wavelength (as in Fig. 1). In this case, the transmitters/receivers within the end-nodes are assumed wavelength functional. Since the interconnected OXCs stop receiving power from the failed fibres, a fault-signal is transmitted to the NMS, which sends a command to all the end-nodes and OXCs to switch to the restoration mode for that particular link failure, resulting in a new lightpaths allocation.

**Lightpaths allocation: ILP formulation**

In this section an integer linear program (ILP) formulation is developed for the exact solution of the routing and wavelength allocation problem. In [16] a similar analysis was carried out for the WIXC case only. However, the ILP, based on the flow formulation, was shown to be computationally expensive and pruning techniques were proposed. As described in [17], the path formulation allows to impose constraints on the set of possible paths for any node-pair, and therefore is used to limit the complexity of the problem.

In this work, a unified framework of ILPs based on path formulation is presented. These allow to address all possible network configurations, including the use of WIXC and WSXC, and link failure restoration.

Let $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ be the network graph consisting of arcs (links), $j \in \mathcal{A}$, with $|\mathcal{A}| = L$ and nodes, $\mathcal{N} = \{1, 2, \ldots, |\mathcal{N}|\}$, with $|\mathcal{N}| = N$. A path $p \subseteq \mathcal{A}$ is a connected series of arcs, written $p : s(p) \rightarrow d(p)$, from source node $s(p)$ to destination node $d(p)$ not including any cycles. Let $\ell(p)$ be the length of the path as measured by the number of arcs. Define $I(j \in p) = 1$ if $j$ is an arc of path $p$ and $I(j \in p) = 0$ otherwise.

Represent the set of node-pairs in the graph $\mathcal{G}(\mathcal{N}, \mathcal{A})$ by $\Xi = \{(z_1, z_2) \in \mathcal{N} \times \mathcal{N} | z_1 < z_2\}$.

Define $\forall z = (z_1, z_2) \in \Xi$, $MNH(z_1, z_2, \mathcal{G}(\mathcal{N}, \mathcal{A})) = m(z)$ to be the minimum number of hops distance, and for $e = 0, 1, \ldots$

$$\mathcal{A}_{z,e} = \{p : z_1 \rightarrow z_2 | \ell(p) \leq MNH(z_1, z_2, \mathcal{G}(\mathcal{N}, \mathcal{A})) + e = m(z) + e\} \quad (2)$$

to be the set of paths connecting the node-pair $z$ with length at most the minimum length $m(z)$ plus a constant $e$ which is fixed at several values. Define $\forall j \in \mathcal{A}$

$$\mathcal{F}_j = \{p | \exists z \in \Xi, p \in \mathcal{A}_{z,e}, j \in p\} \quad (3)$$

as the set of active lightpaths that might potentially use arc $j \in \mathcal{A}$, and $\forall p : z_1 \rightarrow z_2 \in \mathcal{F}_j$ let for $a = 0, 1, \ldots$

$$\mathcal{R}_{p,j,a} = \{r : z_1 \rightarrow z_2 | \ell(r) \leq MNH(z_1, z_2, \mathcal{G}(\mathcal{N}, \mathcal{A} \setminus \{j\})) + a = m^d(z) + a\} \quad (4)$$

be the set of potential restoration paths for active lightpath $p$ when link $j$ fails, with length at most the new MNH length $m^d(z)$ plus a constant $a$.

In the following sections, the constraints corresponding to the specific problems are given. The number of variables and constraints determines the computational complexity.

**WIXC case**

In the WIXC case, a lightpath may be identified only by the path $p$, as the wavelengths can be locally assigned in each link. Therefore $\mathcal{N}_A$ is determined by the number of lightpaths in the most loaded link for both cases without and with link failure restoration. Set $\forall z \in \Xi$ and $\forall p \in \mathcal{A}_{z,e}$

$$\delta^{A}_{p,z} = \begin{cases} 1 & \text{if } p \text{ is selected as active lightpath for } z \text{ otherwise.} 
0 & \end{cases} \quad (5)$$
Given that one lightpath is assigned to each node-pair \( z \), the following must be satisfied:

\[
\sum_{p \in \mathcal{A}_z} \delta^A_{p,z} = 1, \quad \forall z \in \mathcal{Z} \quad (6)
\]

Furthermore, let \( \forall r \in \mathcal{R}_{p,j,a} \)

\[
\delta^R_{r,p,j} = \begin{cases} 1 & \text{if active lightpath } p \text{ is restored to lightpath } r \text{ when } j \text{ fails} \\ 0 & \text{otherwise} \end{cases} \quad (7)
\]

To ensure that each selected active lightpath \( p \) is restored by precisely one restoration lightpath, it is required that

\[
\sum_{r \in \mathcal{R}_{p,j,a}} \delta^R_{r,p,j} = \delta^A_{p,(s(p),d(p))}, \quad \forall j \in \mathcal{A}, \quad \forall p \in \mathcal{F}_j \quad (8)
\]

Wavelength routing

This problem minimises the number of wavelengths \( N_\lambda \) within the network subject to there being an active lightpath for each node-pair; each lightpath requiring any one wavelength with at most \( N_\lambda \) wavelengths per fibre:

\[
\min \quad N_\lambda
\]

subject to eq.(6) and

\[
\delta^A_{p,z} \geq 0, \text{ integer,} \quad \forall z \in \mathcal{Z}, \quad \forall p \in \mathcal{A}_z \quad (9)
\]

\[
\sum_{z \in \mathcal{Z}} \sum_{p \in \mathcal{A}_z} \delta^A_{p,z} \sigma(j \in p) \leq N_\lambda, \quad \forall j \in \mathcal{A} \quad (10)
\]

Wavelength routing and restoration

This problem assigns the minimum number of wavelengths \( N_\lambda' \) within the network subject to there being an active lightpath for each node-pair and a restoration lightpath for every active lightpath interrupted by any link failure; each lightpath requiring any one wavelength with at most \( N_\lambda' \) wavelengths per fibre:

\[
\min \quad N_\lambda'
\]

subject to eqs.(6), (8)-(10), and

\[
\delta^R_{r,p,j} \geq 0, \text{ integer,} \quad \forall j \in \mathcal{A}, \quad \forall p \in \mathcal{F}_j, \quad \forall r \in \mathcal{R}_{p,j,a} \quad (11)
\]

\[
\sum_{z \in \mathcal{Z}} \sum_{p \in \mathcal{A}_z} \sum_{r \in \mathcal{F}_j} \delta^R_{r,p,j} \sigma(j \in r) + \sum_{p \in \mathcal{F}_j} \sum_{r \in \mathcal{R}_{p,j,a}} \delta^R_{r,p,j} \sigma(j \in r) \leq N_\lambda', \quad \forall j \in \mathcal{A}, \quad \forall j' \neq j \in \mathcal{A} \quad (12)
\]

As shown, the complexity of the WIXC formulations depends only on the network size and connectivity.

WSXC case

In the WSXC case, the absence of wavelength conversion results in any lightpath being identified by the path \( p \) and the wavelength \( w \), which is fixed end-to-end. Therefore, the wavelength requirement \( N_\lambda \) is determined by the total number of distinct wavelengths utilised within the network by at least one lightpath.

Assume that \( W \) wavelengths are available on each fibre and \( \forall z \in \mathcal{Z}, \forall p \in \mathcal{A}_z, \) and \( \forall \sigma = 1, \ldots, W \) set

\[
\delta^A_{p,w,z} = \begin{cases} 1 & \text{if } \sigma(p) \text{ is selected as active lightpath for } z \\ 0 & \text{otherwise} \end{cases} \quad (13)
\]

One lightpath is assigned to each node-pair, hence

\[
\sum_{w=1}^{W} \sum_{p \in \mathcal{A}_z} \delta^A_{p,w,z} = 1, \quad \forall z \in \mathcal{Z} \quad (14)
\]

Furthermore, let \( \forall r \in \mathcal{R}_{p,j,a} \) and \( \forall \lambda = 1, \ldots, W \)

\[
\delta^R_{r,p,w,j} = \begin{cases} 1 & \text{if active lightpath } (p, \sigma) \text{ is restored to lightpath } (r, \lambda) \text{ when } j \text{ fails} \\ 0 & \text{otherwise} \end{cases} \quad (15)
\]

To ensure that each selected active lightpath \( (p, \sigma) \) is restored by precisely one restoration path, it is required that

\[
\sum_{\lambda=1}^{W} \sum_{r \in \mathcal{R}_{p,j,a}} \delta^R_{r,p,w,j} = \delta^A_{p,(s(p),d(p))}, \quad \forall j \in \mathcal{A}, \quad \forall p \in \mathcal{F}_j, \quad \forall \sigma = 1, \ldots, W \quad (16)
\]

In this case there is no restriction on the restoration wavelength which can be any of the \( W \) available wavelengths, as wavelength agility in the end-nodes is assumed. Define a variable \( u_w \) which is set to 1 if wavelength \( w \) is used by at least one lightpath within the network, 0 otherwise. Thus,

\[
u_w \geq \delta^A_{p,w,z}, \quad \forall z \in \mathcal{Z}, \quad \forall p \in \mathcal{A}_z, \quad \forall \sigma = 1, \ldots, W \quad (17)
\]

\[
u_w \geq \delta^R_{r,p,w,j}, \quad \forall j \in \mathcal{A}, \quad \forall p \in \mathcal{F}_j, \quad \forall r \in \mathcal{R}_{p,j,a}, \quad \forall \sigma = 1, \ldots, W, \quad \forall \lambda = 1, \ldots, W \quad (18)
\]

and

\[
u_w \leq 1, \text{ integer,} \quad \forall \sigma = 1, \ldots, W \quad (19)
\]

Wavelength routing

This problem assigns the minimum number of wavelengths \( N_\lambda \) within the network subject to there being an active lightpath for each node-pair; each lightpath requiring the same wavelength along the path with at most \( N_\lambda \) wavelengths per fibre:

\[
\min \quad N_\lambda = \sum_{w=1}^{W} u_w
\]
subject to eqs. (14), (17), (19), and

\[
\delta^A_{p.w.z} \geq 0, \text{ integer,}
\]

\[
\forall z \in Z, \quad \forall \lambda \in \mathcal{A}, \quad \forall w = 1, \ldots, W
\]

\[
\sum_{z \in Z} \sum_{p \in \mathcal{A}} \delta^A_{p.w.z} I(j = p) \leq 1, \quad \forall j \in \mathcal{A},
\]

\[
\forall w = 1, \ldots, W
\]

(20)

Wavelength routing and restoration

This problem assigns the minimum number of wavelengths \( N^\lambda \) within the network subject to there being an active lightpath for each node-pair and a restoration lightpath for every active lightpath interrupted by any link failure; each lightpath requiring the same wavelength along the path with at most \( N^\lambda \) wavelengths per fibre:

\[
m \in \text{min } N^\lambda = \sum_{w=1}^{W} u_w
\]

subject to eqs. (14), (16)-(21), and

\[
\delta^R_{\lambda.p.w.j} \geq 0, \text{ integer,}
\]

\[
\forall j \in \mathcal{A}, \quad \forall p \in \mathcal{F}_j, \quad \forall r \in \mathcal{R}_{p,j,a},
\]

\[
\forall w = 1, \ldots, W, \quad \forall \lambda = 1, \ldots, W
\]

\[
\sum_{z \in Z} \sum_{p \in \mathcal{A}} \sum_{j \in \mathcal{R}^p} \delta^R_{p.w.z} I(j = p) +
\]

\[
+ \sum_{\lambda=1}^{W} \sum_{j \in \mathcal{R}^p} \sum_{r \in \mathcal{R}^p} \delta^R_{r.w.p.\lambda,j} I(j = r) \leq 1, \quad \forall j \in \mathcal{A},
\]

\[
\forall w = 1, \ldots, W
\]

(22)

Lightpaths allocation: lower bounds

In this work, lower bounds on the optimal solution are also developed. These have the advantage of reduced computational complexity. Two lower bounds on the wavelength requirement can be defined for a given network topology. Since in calculating these, no constraints on wavelength continuity are imposed, these limits define lower bounds for the WIXC case, and used for comparison with the WSXC case.

Distance bound

Wavelength routing

The total length, in number of links, of all the network lightpaths is \( L_T = \sum_{z \in Z} m(z) \), and the average internodal distance is \( \bar{H} = \frac{L_T}{|Z|} = \frac{2 L_T}{N(N-1)} \). The optimal solution is to evenly distribute the lightpaths among the \( L \) links, leading to a lower bound equal to [12]:

\[
W_{DB} = \left[ \frac{L_T}{T} \right]
\]

(24)

where \([x]\) represents the lowest integer greater than or equal to \( x \). This lower limit will be referred to as \textit{distance bound} [18]. \( W_{DB} \) can be easily derived once the MNH distance for all the node-pairs is obtained, for example by using Dijkstra algorithm [19].

Wavelength routing and restoration

Given the original network topology, eliminate link \( j \in \mathcal{A} \), and by using MNH routing calculate the new minimum distance (in number of links) for each node-pair \( m^j(z) \). The total number of links occupied by all the lightpaths is \( L^j_T = \sum_{z \in Z} m^j(z) \) and therefore at least \( \left[ \frac{L^j_T}{T} \right] \) wavelengths are required. The distance bound can be written as follow:

\[
W'_{DB} = \max_{j \in \mathcal{A}} \left[ \frac{L^j_T}{T} \right]
\]

(25)

Partition bound

Wavelength routing

Consider a network cut \( C \), that is a set of links \( j \in C \subset \mathcal{A} \) \((C \neq \emptyset, \mathcal{A})\), whose elimination results in two disjoint subsets of nodes \( \mathcal{S} \) and \( \mathcal{N} \setminus \mathcal{S} \). The number of lightpaths traversing the cut is \( D_C = \sum_{z \in \gamma(C)} d(z) \), where \( \gamma(C) = \{(z_1, z_2) | z_1 < z_2, z_1, z_2 \in \mathcal{S}, z_2 \in \mathcal{N} \setminus \mathcal{S}\} \), and \( d(z) \) is the demand, in number of bidirectional lightpaths, for the node-pair \( z \). In the case of uniform traffic considered here, it follows that \( D_C = |\mathcal{S}| \cdot |\mathcal{N} \setminus \mathcal{S}| \). The minimum number of distinct wavelengths necessary to satisfy the traffic demand across the cut \( C \) is \( W_C = \left[ \frac{D_C}{T} \right] \), where \( |C| \) is the number of links in the cut. The different cuts \( C \) within the network result in different values of \( W_C \), with the largest one determining the lower bound \( W_{PB} \) [8], [14]:

\[
W_{PB} = \max_{C \subset A} W_C = \max_{C \subset A} \left[ \frac{|\mathcal{S}| \cdot |\mathcal{N} \setminus \mathcal{S}|}{T} \right]
\]

(26)

This bound will be referred to as \textit{partition bound} [18], and the network cut which determines \( W_{PB} \) is referred to as the \textit{limiting cut}.

For a network topology with \( N \) nodes, enumerating all the network cuts to find \( W_{PB} \) is \( O(2^{N-1}) \). Therefore this approach is practical only for small size networks.

For a given topology, the largest value between \( W_{DB} \) and \( W_{PB} \) determines the actual lower bound without restoration, that is \( W_{LB} = \max(W_{DB}, W_{PB}) \). It will be shown that in real networks the partition bound sets the lower limit on \( N^\lambda \). Conversely, in random networks with size \( N \to \infty \), the lower limit is governed by \( W_{DB} \), as proved in [18], and discussed later.
Wavelength routing and restoration
Consider a network cut $C$. In the case of single link failure restoration, the minimum wavelength requirement increases, as the same number of lightpaths $D_C$ must be routed along $|C| - 1$ links. Therefore, the partition bound with restoration $W'_{PB}$ can be derived from eq.(26) by replacing $|C|$ with $(|C| - 1)$.

The lower bound with restoration $W_{LB}'$ is determined by the maximum between $W_{DB}'$ and $W_{PB}'$, the latter being the limiting factor for real networks. If the new partition bound $W_{PB}'$ results from the same network cut which determines $W_{PB}$, it follows that $W'_{PB} = W_{PB} \left(1 + \frac{1}{|C|-1}\right)$, and an increment

$$\Delta N_{\lambda} = \frac{100}{|C|-1} \%$$

(27)

in the wavelength requirement is expected, strongly driven by the number of links $|C|$ in the limiting cut. As a consequence, the limiting cut and all the network cuts $C$ whose $W_C \simeq W_{PB}$ must consist of as many links as possible to minimise the number of additional wavelengths required for restoration [20].

The calculation of the lower bounds does not provide any information on the routing of the lightpaths to achieve these limits. $W_{LB}$ and $W_{LB}'$ may not be achieved if routing rules (such as constraints on the path length and wavelength continuity) are imposed. However, these bounds are useful for verifying the accuracy of the lightpath allocation algorithms described below.

Lightpaths allocation: heuristic algorithms
As will be shown in the result section, the ILP formulations are computationally expensive, and can only be used to analyse relatively small networks. For the case of large networks, heuristic algorithms are developed to construct good (although not necessarily optimal) solutions.

Wavelength routing
The algorithm developed in this work for the allocation of the active lightpath solves the routing and wavelength assignment subproblems separately [14].

First, the physical paths are assigned to all node-pairs. The minimum-number-of-hops (MNH) algorithm is considered [19], as in this case, each lightpath utilises the minimum number of physical links and OXCs, minimising the total and average transit traffic, and hence the OXC size. This is also key to minimising the crosstalk penalties associated with the physical limitations of the OXCs. However, if the lower bound is not achieved, longer paths are also considered ($e > 0$ in the eq.(2)). In a network with $N$ nodes, there exist $P$ node-pairs and therefore $P!$ different ways they can be ordered and assigned paths. In the proposed algorithm, the node-pairs with the largest MNH are assigned paths first. Since usually the sets $A_{\lambda}$ consist of more than one path, a certain degree of freedom is available to allocate, as evenly as possible, the lightpaths among the network links. This helps to minimise the number of lightpaths (congestion) in the network links.

In the WSXC case, the wavelengths are then assigned to the paths. There exist $P!$ different ways in which the paths can be ordered and assigned wavelengths. Here the paths are ranked for decreasing length, and the longest ones are assigned wavelength first, as, intuitively, long paths are harder to allocate because a unique free wavelength must be found on more links [3]. The highest wavelength assigned amongst all node-pairs determines the network wavelength requirement $N_{\lambda}$.

For the WIXC case, no wavelength assignment is performed, since the wavelengths can be allocated link by link, and the wavelength requirement $N_{\lambda}$ is equal to the number of lightpaths in the most congested link.

Wavelength routing and restoration
The aim is to provide full restoration for single link failures with the minimum number of extra wavelengths [15].

The network is assumed to be in the normal operation state determined by the previously described lightpaths allocation algorithm. Each link $j \in A$ is randomly eliminated in turn and the node-pairs whose active lightpaths have been interrupted are ranked in order of decreasing length of the new MNH path $m^j(r)$. For each of those node-pairs, the restoration path (and wavelength in the WSXC case) are assigned to minimise the number of wavelengths required for restoration: among all of the possible restoration paths $r \in R_{p,a}$, the one which has the lowest maximum congestion (WIXC case) or which requires the lowest wavelength (WSXC case) is assigned. This is repeated for all the node-pairs whose lightpaths have been interrupted. For each link failure, the highest congestion among the network links or the highest wavelength assigned among all the restored node-pairs determines the wavelength requirement for that link failure ($N^j_{\lambda}$). The largest $N^j_{\lambda}$ among all the network link failures determines the new wavelength requirement $N'_{\lambda}$.

Results: wavelength routing

Real Networks
Several existing or planned fibre network infrastructures were analysed as possible topologies for WRON applications to evaluate their topological parameters (see Table 1) [20]. They are examples of US and pan-European core networks, as well as a UK topology approximating the current BT network.

As shown, the networks’ sizes vary from 11 to 46 nodes. Since the maximum nodal degree is relatively constant and does not scale with $N$, $\alpha$ increases as $N$ decreases, ranging between 0.07 and 0.45. This is the range which is considered in this analysis, as most of the real transport networks...
Table 1: Topological parameters for existing or planned network topologies. The dotted lines represent the limiting cuts.

<table>
<thead>
<tr>
<th>Network</th>
<th>N</th>
<th>L</th>
<th>α</th>
<th>b</th>
<th>D</th>
<th>l[l]</th>
<th>l[l']</th>
</tr>
</thead>
<tbody>
<tr>
<td>USNet</td>
<td>46</td>
<td>76</td>
<td>0.07</td>
<td>4.4</td>
<td>11</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>EURO-Large</td>
<td>43</td>
<td>90</td>
<td>0.10</td>
<td>3.6</td>
<td>8</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>ARPANet</td>
<td>20</td>
<td>31</td>
<td>0.16</td>
<td>2.81</td>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>UKNet</td>
<td>21</td>
<td>39</td>
<td>0.19</td>
<td>2.51</td>
<td>5</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>EON</td>
<td>20</td>
<td>39</td>
<td>0.21</td>
<td>2.38</td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>NSFNet</td>
<td>14</td>
<td>21</td>
<td>0.23</td>
<td>2.14</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>EURO-Core</td>
<td>11</td>
<td>25</td>
<td>0.45</td>
<td>1.58</td>
<td>3</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

have comparable values of α. An increase in α leads to a more connected network, and a decrease in the average internodal distance $\overline{H}$ and diameter $D$ (defined as the longest path within the network). For the analysed topologies, $\overline{H}$ varies between 1.58 and 4.4, and $D$ is between 3 and 11, typical for real transport networks.

The dotted lines in the graphs identify the limiting cuts which determine $W_{PB}$ and $W_{P'B}$. In the UKNet the central and upper cuts determine the partition bounds $W_{PB}$ and $W_{P'B}$, respectively. The number of links $|\mathcal{C}|$ in the limiting cut is also given. It can be seen that for all the networks, except for the UKNet, the same cut sets the limit for both configuration without and with link failure restoration, i.e. $|\mathcal{C}'| = |\mathcal{C}| - 1$.

Table 2: Results for real network topologies. $W_{PB}$ obtained by inspection are marked by 1. A dash is shown where the ILP failed in giving any result in one day. The wavelength requirements $N_{\lambda}$ were obtained with MNH paths. The results with lightpaths one hop longer than MNH ($c = 1$ in eq.(2)) are in parenthesis. The results in bold reached the lower bounds.

<table>
<thead>
<tr>
<th>Network</th>
<th>P</th>
<th>$W_{DB}$</th>
<th>$W_{PB}$</th>
<th>$W_{PB}$</th>
<th>$W_{PB}$</th>
<th>$W_{PB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$W_{DS}$</td>
<td>$W_{PS}$</td>
<td>$W_{PS}$</td>
<td>$W_{PS}$</td>
<td>$W_{PS}$</td>
</tr>
<tr>
<td>USNet</td>
<td>1035</td>
<td>60</td>
<td>103'</td>
<td>-</td>
<td>-</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(108)</td>
</tr>
<tr>
<td>EURO-Large</td>
<td>903</td>
<td>37</td>
<td>66</td>
<td>-</td>
<td>-</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(66)</td>
</tr>
<tr>
<td>ARPANet</td>
<td>190</td>
<td>18</td>
<td>33</td>
<td>33</td>
<td>-</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(33)</td>
</tr>
<tr>
<td>UKNet</td>
<td>210</td>
<td>14</td>
<td>19</td>
<td>21</td>
<td>-</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(19)</td>
</tr>
<tr>
<td>EON</td>
<td>190</td>
<td>12</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(18)</td>
</tr>
<tr>
<td>NSFNet</td>
<td>91</td>
<td>10</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(13)</td>
</tr>
<tr>
<td>EURO-Core</td>
<td>55</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

The results, including lower bounds, are presented in Table 2. As expected, an increase in $\alpha$ leads to a reduction of the distance bound $W_{DB}$. The values of the partition bound $W_{PB}$ were determined using eq.(26) except for the two largest topologies. In these cases, $W_{PB}$ was derived by inspection, and the values were then confirmed by the results achieved by the heuristic algorithms.

As shown, for all the networks, excluding the EURO-Core, the partition bound $W_{PB}$ is much larger than the distance bound $W_{DB}$, and therefore determines the ultimate lower bound $W_{LB}$. There are two operational reasons for this: the distribution of nodes over a given geographical area, and the cost of deploying the fibre infrastructure which determines that each node is most likely to be connected to its neighbouring nodes, result in elongated topologies, where a particular cut becomes significant. However, in more optimally and richly connected topologies (see for example the EURO-Core), $W_{LB}$ can be equal or even larger than $W_{PB}$, setting the network lower limit. This is the case for random networks with very large size, as demonstrated in [18] and discussed later.

Table 2 also shows the results for $N_{\lambda}$ calculated with both the ILP formulations and heuristic algorithms. For the EURO-Core, NSFNet, and EON topologies, the lower bound was achieved by both ILPs and heuristics utilising MNH paths. The same results were obtained for the WIXC and WSXC cases, confirming that wavelength conversion within the OXCs does not lead to any reduction in $N_{\lambda}$. In fact, the wavelength requirement is ultimately determined by the physical connectivity and topology, i.e. the limiting cut.

The number of variables and constraints in the ILP formulations increases with an increase in the network size $N$, resulting in longer running time, particularly in the WSXC
case. For the UKNet and ARPANet, only the WIXC ILP formulation was feasible, whereas the WSXC case failed to yield any results after at least one day of computation on a Unix workstation (dashes in Table 2).

The UKNet, EURO-Large, and USNet were also analysed with lightpaths one hop longer than MNHs (shown in parenthesis in Table 2). For the UKNet, this allowed to achieve the lower bound with both the ILP and heuristic WIXC case, whereas it was not reached by the heuristic WSXC case (even with \( e \geq 2 \)). For the EURO-Large and USNet, both the ILP formulations become prohibitive, given the very large size. As shown, the heuristic WIXC case achieved the lower bound, whereas a few extra wavelengths are required for the WSXC case, although the difference is negligible. This confirms that the reduction achievable by the use of wavelength conversion is very small even in large topologies.

As expected, the wavelength requirement \( N_\lambda \) increases as the physical connectivity \( \alpha \) decreases. However, it is worth noting that, for each network, the wavelength requirement \( N_\lambda \) is much smaller than the number of bidirectional lightpaths \( P \), highlighting the large wavelength reuse achievable in WRONs (of a factor of 10 in most of the topologies). Thus, a relatively small \( N_\lambda \) can satisfy a very large traffic demand, even in weekly connected topologies.

Finally, it is worth noting the accuracy of the heuristic algorithms, which were observed to produce results always equal or very close to the exact solution of the ILPs or lower bounds.

**Randomly Connected Networks**

To study the relationship between the wavelength requirement \( N_\lambda \) and the physical connectivity \( \alpha \), a systematic investigation of a large number of randomly generated, arbitrarily-connected networks was performed. These topologies, referred to as Randomly Connected Networks (RCNs), were generated for 0.1 \( \leq \alpha \leq 0.4 \) as follows.

Given \( N \) and \( \alpha \), a randomly selected link was added at a time until the value of \( \alpha \) was achieved. A uniform probability distribution was considered, such that all the \( P \) links have the same probability to be selected. A new link was accepted only if it was not already present and the nodal degree of both the interconnecting nodes did not exceed a maximum degree, determined by values of \( N \) and \( \alpha \) [20]. To verify that this random process did not result in an unconnected network, a following step was performed to ascertain the constraint (C1), and only the connected networks were analysed.

Networks with the same \( (N, \alpha) \) but different physical topologies, may have different wavelength requirements \( N_\lambda \). By studying a large number of distinct topologies, a distribution of \( N_\lambda \) was obtained.

Figs. 2 shows the distributions for the heuristic WSXC case (with MNH paths) for RCNs with \( N = 14 \) and \( \alpha = 0.23 \). The position of the NSFNet (\( N_\lambda = 13 \)) is also presented.

As shown, the distribution assumes a wide range of values, and is bimodal with peaks centred respectively around \( N_\lambda = 14 \) and 17. The average value is \( N_\lambda = 14.2 \) and the range \( 11 \leq N_\lambda \leq 20 \) contains 95% of the results. The analysis of the topologies within the distribution (not reported here) showed that the generated networks are good representations of real fibre network infrastructures. The diameter and the nodal degree distribution appeared to have little impact on \( N_\lambda \), which is governed by the number of links \( |C| \) in the limiting cut, and its position within the network.

The results obtained with both the WIXC and WSXC heuristic algorithms were always equal or very close to the lower bound, and no significant reduction in \( N_\lambda \) was achieved by the introduction of wavelength conversion.

In Fig. 3 the distribution of \( N_\lambda \) for RCNs with \( N = 14 \) are plotted for different values of \( \alpha \). As expected, an in-
increase in $\alpha$ leads to a decrease in $N_\lambda$, since the lightpaths can be mapped onto a larger number of links. Consequently, the distribution shifts towards lower values and becomes narrower. In Fig. 4 the mean values and the ranges containing 95% of the results for RCNs with $N = 14$ are plotted versus the physical connectivity $\alpha$. As discussed, both the mean values and the ranges decrease with increasing connectivity.

The same analysis was performed for RCNs with different network sizes in the range $20 \leq N \leq 50$, and the results showed similar behaviour to the case with $N = 14$, i.e. wavelength requirements driven by the limiting cuts and limited improvement achievable with wavelength conversion.

In Fig. 5 the mean values of the distributions for all the analysed RCNs are plotted versus $\alpha$. It is interesting to note that the mean values of the wavelength requirements are independent of the network size $N$. [Similarly, for a given $\alpha$, the 95% ranges for the different $N$ were found comparable [20].] A clear trade-off exists between the mean values of $N_\lambda$ and the connectivity $\alpha$, and their relationship is quantified by the results of Fig. 5. It is shown that on average RCNs can satisfy the uniform traffic demand with a moderate number of wavelengths. For example, on average no more than 32, 16 and 8 wavelengths are necessary for $\alpha \geq 0.15, 0.2, \text{and } 0.3$, respectively.

The results of several real networks are also shown. It can be seen that UKNet, EON, NSFNet, and EURO-Core match well the average wavelength requirements of the RCNs, whereas the ARPANet requires a larger $N_\lambda$, given its suboptimal topology resulting from its limiting cut.

A complete analysis of all the possible topologies was performed for networks with $N = 5$ and 6. The range of possible values for the physical connectivity is $\alpha \geq 0.5$ and 0.4, respectively, resulting in narrow $N_\lambda$ distributions. The mean values, also plotted in Fig. 5, show that as $\alpha$ increases the wavelength requirement decreases reaching $N_\lambda = 1$ for $\alpha = 1$, as expected. It is worth noting that these results correspond well with those obtained for the RCNs, confirming the significance of the RCNs modelling results.

Asymptotic lower bounds on the wavelength requirement $N_\lambda$ were analytically derived in [18] for random networks with size $N \rightarrow \infty$ and diameter $D \leq 2$. It was shown that the partition bound is $W_{DB}^\star = 1/\alpha$ and the distance bound is $W_{DB}^\star = 2/\alpha - 1$, which determines the networks lower bound. Several optimal finite-size topologies reaching the lower bound $W_{DB}^\star$ were proposed in [21]. However, only selected values of $N$ were feasible, and for all of them $D = 2$, as required.

In Fig. 6 the minimum values of $N_\lambda$ for RCNs and analytical lower bound $W_{DB}^\star$ are plotted against the physical connectivity $\alpha$. As shown, the results are in very good agreement for $\alpha \geq 0.4$, with a slight difference for $0.1 \leq \alpha \leq 0.4$. This difference mainly results from the suboptimality of the generated RCNs, whose diameters were always $D > 2$, condition required to achieve the theoretical lower bound $W_{DB}^\star$. Taking into account this main topological difference, the analytical lower bound confirms the validity of the results obtained in this analysis.

**Results: wavelength routing and restoration**

The real network topologies were analysed to evaluate the extra wavelengths required to fully restore the uniform traf-
Table 3: Results for real network topologies. \( W'_{PB} \) obtained by inspection are marked by \(^1\). For each case, the smallest \( N_\alpha \) obtained is presented, and the corresponding value of \( \alpha \) in restoration sets \( R_{p,j,a} \) is in parenthesis. A dash is shown where the ILP failed in giving any result in one day. The results in bold reached the lower bounds.

<table>
<thead>
<tr>
<th>Network</th>
<th>( W'_{DB} )</th>
<th>( W'_{PB} )</th>
<th>( W_{LB} )</th>
<th>( W_{ILP} )</th>
<th>( W_\alpha )</th>
<th>( N_\alpha )</th>
<th>Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>USNet</td>
<td>64</td>
<td>129(^1)</td>
<td>129</td>
<td>156</td>
<td>(4)</td>
<td>(6)</td>
<td></td>
</tr>
<tr>
<td>EURO-Large</td>
<td>38</td>
<td>77(^1)</td>
<td>91</td>
<td>95</td>
<td>(4)</td>
<td>(3)</td>
<td></td>
</tr>
<tr>
<td>ARPA-Net</td>
<td>20</td>
<td>59</td>
<td>59</td>
<td>20</td>
<td>(2)</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td>UKNet</td>
<td>15</td>
<td>27</td>
<td>27</td>
<td>29</td>
<td>(2)</td>
<td>(3)</td>
<td></td>
</tr>
<tr>
<td>EON</td>
<td>13</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>(0)</td>
<td>(0)</td>
<td></td>
</tr>
<tr>
<td>NSFNet</td>
<td>11</td>
<td>17</td>
<td>17</td>
<td>18</td>
<td>(2)</td>
<td>(2)</td>
<td></td>
</tr>
<tr>
<td>EURO-Core</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>(2)</td>
<td>(2)</td>
<td></td>
</tr>
</tbody>
</table>

The ILP formulation for the WSXC case was found to be intractable given the large number of constraints and variables generated. Similarly the WIXC case was feasible only for small topologies.

The lower bounds and the best results obtained are shown in Table 3. Consider the NSFNet topology. The lower bound is set by the partition bound to the value \( W'_{LB} = W_{PB} = 17 \). This determines an increase in the expected wavelength requirement of about 30\%, from eq.(27). With the ILP formulation for the WIXC case, the lower bound was achieved by allowing the restoration lightpaths to be one hop longer than MNH, i.e. \( \alpha = 1 \) in the restoration sets \( R_{p,j,a} \) defined in eq.(4). The best result obtained with the heuristic algorithms was \( N_{\alpha} = 18 \) (with \( \alpha = 2 \)), and 19 (with \( \alpha = 4 \)), respectively, for the WIXC and WSXC, implying that a limited reduction can be achieved with the introduction of the wavelength interchange within the OXCs.

The results for the other networks are also reported. As shown, the lower bound is achieved or approached in most cases, with slight differences for the EURO-Large and US-Net (WSXC case only). The difference between the WIXC and WSXC cases is still small: the reduction in \( N_{\alpha} \) achievable with WIXCs is about 20\% for the USNet, whereas it is less than 10\% for all the other topologies. This confirms that the benefit achieved with wavelength interchange in the OXCs is very small, even with restoration, when wavelength agility is available within the end-nodes.

Figs. 7-8 show the extra wavelengths required to provide for restoration as a function of the number of links \(|C|\) in the network limiting cut. The results obtained with the WIXC and WSXC heuristic algorithms are reported. The \( LB\) variation curve is the expected increment in the wavelength requirement as derived from eq.(27).

As shown in Fig. 7, in the WIXC case the availability of restoration paths longer than MNH (\( \alpha \) as large as 6) allows better sharing of restoration capacity and therefore reduce the extra wavelengths required for restoration. However, in the WSXC case (Fig. 8), increasing \( \alpha \) beyond 4 does not lead to any significant improvement for any of the analysed topologies, since it is harder to find a unique free wavelength over longer paths. Therefore a trade-off exists between \( \alpha \) and the wavelength continuity constraint.

The results highlight the importance of increasing the number of links in critical network cuts to minimise the extra wavelengths required for restoration.

Conclusions

This paper studied the wavelength requirements of arbitrarily-connected WRONs as a function of the physical topology. The results showed that \( N_{\alpha} \) strongly depends on the physical connectivity \( \alpha \), and the number of links \(|C|\) in the limiting cut, whereas it is almost independent of the net-
work size $N$. In addition it is shown that WRONs allow a large wavelength reuse, resulting in high transport capability with a moderate number of wavelengths. The benefit achievable by the availability of wavelength interchange within the OXC was found to be negligible. The results have been shown to be in excellent agreement with analytical lower bounds.

Finally single link failure restoration was considered. A clear trade-off between the number of links $|C|$ in the network limiting cut and the maximum increment in the number of wavelengths was identified. It was shown that wavelength interchange in the OXC allows little benefit even with restoration when wavelength agility is provided within the end-nodes.

These algorithms and results can be used in the WRON analysis, design, and optimisation.

Acknowledgements
The authors would like to thank J. E. Midwinter, F. P. Kelly and R. Barry for useful comments. Financial support from Nortel Technology Ltd, the Royal Society, EC (DAWRON project), and Fotobaroni (Italy) is gratefully acknowledged.

References


