Statistics 1B Interludes

6. Epistemic uncertainty

Bayes' paper - 1763

LII. *An Eſſay towards ſolving a Problem in the Doctrine of Chances. By the late Rev. Mr.* Bayes, *F. R. S. communicated by Mr.* Price, *in a Letter to* John Canton, *A. M. F. R. S.*

Bayes' probability - 1763

5. The *probability of any event* is the ratio between the value at which an expectation depending on the happening of the event ought to be computed, and the value of the thing expected upon it's happening.
6. By *chance* I mean the ſame as probability.

- Suppose I get £1 if X occurs
- I am willing to bet 60p on X
- Then *my probability* is 60/100 = 0.6
- Nothing to do with 'frequency' or 'randomness' – reasonable betting odds

Flipping coins

Two types of uncertainty

Aleatory
– chance, unpredictable
(can't know)

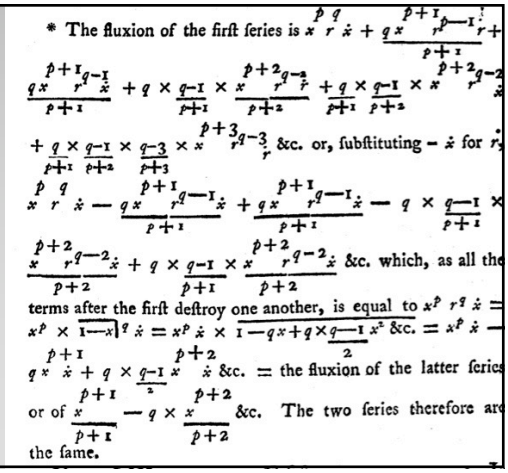
Epistemic
– lack of knowledge, ignorance
(don't know)

(rather simplistic)

Aleatory or epistemic?

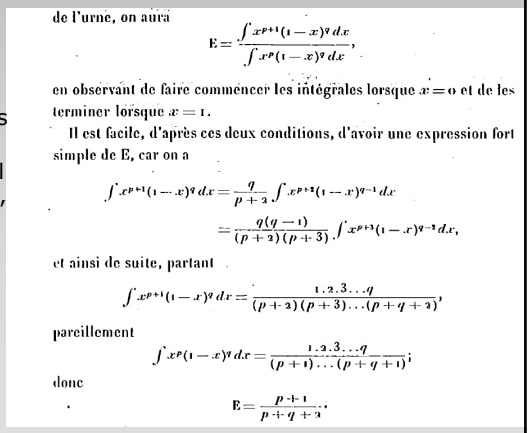
- Lottery
- Scratchcards
- Gender of a foetus
- Current sexual behaviour
- Whether you will live to 100

Bayes' horrible notation



* The fluxion of the firſt ſeries is $x^p r^q \dot{x} + q x^{p+1} r^{q-1} \dot{r} +$ $\frac{p+1}{p+1}$ $+ q \times \frac{q-1}{p+1} \times x^{p+2} r^{q-2} \dot{r} + q \times \frac{q-1}{p+1} \times x^{p+2} r^{q-2} \dot{x}$ $+ q \times \frac{q-1}{p+1} \times \frac{q-2}{p+2} \times x^{p+3} r^{q-3} \dot{r}$ &c. or, ſubſtituting $-\dot{x}$ for $\dot{r}$, $x^p r^q \dot{x} - q x^{p+1} r^{q-1} \dot{x} + q x^{p+1} r^{q-1} \dot{x} - q \times \frac{q-1}{p+1} \times$ $\frac{p+2}{p+2}$ $x^{p+2} r^{q-2} \dot{x} + q \times \frac{q-1}{p+1} \times x^{p+2} r^{q-2} \dot{x}$ &c. which, as all the terms after the firſt deſtroy one another, is equal to $x^p r^q \dot{x} = x^p \times \overline{1-x}^q \dot{x} = x^p \dot{x} \times 1 - qx + q \times \frac{q-1}{2} x^2$ &c. $= x^p \dot{x} -$ $q x^{p+1} \dot{x} + q \times \frac{q-1}{2} x^{p+2} \dot{x}$ &c. = the fluxion of the latter ſeries or of $\frac{x^{p+1}}{p+1} - q \times \frac{x^{p+2}}{p+2}$ &c. The two ſeries therefore are the ſame.

Laplace's 1774 beautiful notation, almost identical to now



de l'urne, on aura

$$E = \frac{\int x^{p+1}(1-x)^q dx}{\int x^p (1-x)^q dx},$$

en observant de faire commencer les intégrales lorsque $x = 0$ et de les terminer lorsque $x = 1$.

Il est facile, d'après ces deux conditions, d'avoir une expression fort simple de E, car on a

$$\int x^{p+1}(1-x)^q dx = \frac{q}{p+2} \int x^{p+2}(1-x)^{q-1} dx$$

$$= \frac{q(q-1)}{(p+2)(p+3)} \int x^{p+3}(1-x)^{q-2} dx,$$

et ainsi de suite, partant

$$\int x^{p+1}(1-x)^q dx = \frac{1.2.3 \ldots q}{(p+2)(p+3) \ldots (p+q+2)},$$

pareillement

$$\int x^p (1-x)^q dx = \frac{1.2.3 \ldots q}{(p+1) \ldots (p+q+1)};$$

donc

$$E = \frac{p+1}{p+q+2}.$$

Probabilities can be applied to 'facts'

Quantifying your ignorance

- Think whether you prefer (A) or (B) for each question
- Then think of how confident you are with your answer
- Give your confidence a number 5 to 10
- Score yourself when you hear the correct answer
- The 'scoring rule' is designed to encourage you to be honest

Your 'confidence' in your answer	***5***	***6***	***7***	***8***	***9***	***10***
Score if you are ***right***	0	9	16	21	24	25
Score if you are ***wrong***	0	-11	-24	-39	-56	-75

1. Which university was founded first A) Cambridge or B) Oxford?
B (1209 vs 1096)

2. Which is nearest to Cambridge (by road on Google maps), A) Edinburgh or B) Paris?
B (352m vs 332m)

3. Which has the bigger population, A) Luxembourg, B) Iceland?
A (523,000 vs 328,000)

4 . Who is older, A) Angela Merkel B) Me (DJS)?
B (Born 1954 vs 1953

5. Which college came higher in the Tompkins Table 2016? A) St Johns or B) Christ's?
B (5th vs 3rd)