1. The Poisson distribution

- Limit of Binomial as $n \to \infty$ and $np$ stays fixed
- Useful whenever we can estimate expected number of rare events
- $X$ has a Poisson distribution
  - $P(X=x) = \frac{r^x e^{-r}}{x!}$, $x = 0, 1, 2, 3, ...$
- In particular, the chance of no events ($x=0$) is $e^{-r}$

What is the approximate probability of a rollover in the lottery? [i.e. nobody win the jackpot?]

- Jackpot if pick correct 6 numbers out of 49
- Around 14,000,000 different choices
- They sell around 30,000,000 tickets each draw
- So expect around 2 winners each draw
- Assume number of winners is Poisson
- So Probability(no winners) $\sim e^{-2} = 0.13$
Was there something special about 10th July 2008?


Homicides: Metropolitan Police, April 2004 – March 2007

- **On average**: 160 per year, 13 per month, 3 per week, 0.44 per day
- **Just knowing this overall rate means we can predict how often ‘rare events’ will happen**

![Predicted clustering of 483 homicides over 1095 days, 2004-7](image_url)

- Expected outcomes over 1095 days assuming Poisson distribution with mean 0.44
  - Predict 702 days with no homicides (64%), 10 days with 3 and 1 day with 4

![Number of homicides each day, 2004 -2007](image_url)
Observe 713 days with no homicides (65%), 16 days with 3 and 1 day with 4.

In London

- Homicides currently follow a stable ‘random’ pattern with average gap of 54 hours
- Over 3 years, expect one day with 4 independent homicides in London
- Just what was observed

Expected outcomes over 1095 days assuming Poisson distribution with mean 0.44:
Predict 702 days with no homicides (64%), 10 days with 3 and 1 day with 4.
Poisson also fits data at a national level

Figure 14: Observed and expected number of homicide incidents recorded on a day, combined years 2009-10 to 2011-12

Can put intervals which estimate the underlying ‘true’ rate

Figure 15: Homicide incident trend analysis, 1997-98 to 2011-12