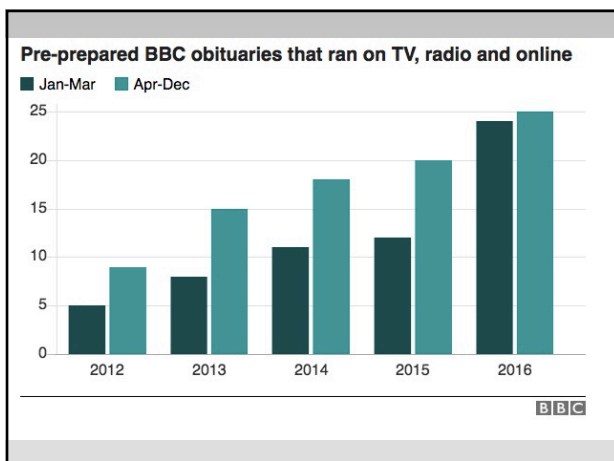


Statistics 1B Interludes

1. Celebrity deaths in 2016



1 Robert Stigwood	26 Burt Kwouk
2 Ed Stewart	27 Carla Lane
3 David Bowie	28 Muhammed Ali
4 Alan Rickman	29 Peter Shaffer
5 Glenn Frey	30 Caroline Aherne
6 Cecil Parkinson	31 Elie Wiesel
7 Frank Finlay	32 Gene Wilder
8 Terry Wogan	33 Pierre Boulez
9 Margaret Forster	34 Arnold Palmer
10 Eric Lubbock	35 Shimon Peres
11 Boutros Boutros Ghali	36 Andrew Sachs
12 Harper Lee	37 Leonard Cohen
13 John Chilton	38 Jimmy Young
14 Tony Warren	39 Robert Vaughn
15 Nancy Reagan	40 Fidel Castro
16 George Martin	41 Peter Vaughan
17 Peter Maxwell Davies	42 Greg Lake
18 Paul Daniels	43 Zsa Zsa Gabor
19 Cliff Michelmore	44 Lionel Blue
20 Barry Hines	45 Liz Smith
21 Garry Shandling	46 Rick Parfitt
22 Ronnie Corbett	47 Richard Adams
23 Zaha Hadid	48 George Michael
24 Victoria Wood	49 Carrie Fisher
25 Prince	

The "birthday paradox"

- 23 random people: 51% chance that 2 share a birthday
- 23 random people: 51% chance that 2 share a deathday
- 49 people: 97% chance that 2 share a death-day

Why does this happen?

- Imagine 49 people in a line
- First deathday can be anything
- 2nd deathday must be different from first: probability $364/365 = 0.997$
- 3rd deathday must be different from 1st and 2nd: probability $363/365 = 0.995$
-
- Probability that all 49 are different = $0.997 \times 0.995 \times \dots \times 0.868 = 0.03$

Useful approximate formulae (Diaconis and Mosteller)

- Suppose a single 'match' between 2 people has a chance $1/K$
eg same birthday: $K = 365$
- Then for a 50% chance of a match, need around $1.2\sqrt{K}$ people (eg $1.2 \times \sqrt{365} = 23$)
- Then for a 95% chance of a match, need around $2.5\sqrt{K}$ people (eg $2.5 \times \sqrt{365} = 48$)

'Proof' of approximation.

Probability of a match between a random pair is $1/K$
Probability that there are no matches in n people denoted $p(n, K)$. Then

$$p(n, K) = \left(1 - \frac{1}{K}\right) \times \left(1 - \frac{2}{K}\right) \times \left(1 - \frac{3}{K}\right) \times \dots \times \left(1 - \frac{(n-1)}{K}\right).$$

If n small compared to K , then $\left(1 - \frac{t}{K}\right) \approx e^{-t/K}$ for $t = 1, 2, \dots, n-1$. So

$$p(n, K) = e^{-\frac{1+2+\dots+(n-1)}{K}} = e^{-\frac{n(n-1)/2}{K}} \approx e^{-\frac{n^2}{2K}}.$$

So $n \approx \sqrt{-2K \ln p}$; setting $p = 0.50, 0.05$ gives approximation.

Alternative 'proof' of approximation

There are $n(n-1)/2 \approx n^2/2$ pairs of people, each with probability $1/K$ of matching.

So expected number of matches $\approx \frac{n^2}{2K}$ [expectation additive for dependent events]

Assuming number of matches is Poisson, then $p(n, K) \approx e^{-\frac{n^2}{2K}}$.

Gap between birthdays	Odds of 2 random people 'matching' 1 in K	Number needed for chance of match = 50%	Number needed for chance of match = 95%
	K	$1.2 \sqrt{K}$	$2.5 \sqrt{K}$
Same day	365	23	48
Within 1 day	122	13	28
Within 3 days	52	9	18
Within 1 week	24	6	12
Within 2 weeks	13	4	9

In 49 celebrity deaths?

We would expect around

$$\frac{n^2}{2K} = \frac{49^2}{2 \cdot 365} = 3.3 \text{ pairs}$$

1 Robert Stigwood	04 January 2016	26 Burt Kwouk	24 May 2016
2 Ed Stewart	09 January 2016	27 Carla Lane	31 May 2016
3 David Bowie	10 January 2016	28 Muhammed Ali	03 June 2016
4 Alan Rickman	14 January 2016	29 Peter Shaffer	06 June 2016
5 Glenn Frey	18 January 2016	30 Caroline Aherne	02 July 2016
6 Cecil Parkinson	22 January 2016	31 Elie Wiesel	02 July 2016
7 Frank Finlay	30 January 2016	32 Gene Wilder	29 August 2016
8 Terry Wogan	31 January 2016	33 Pierre Boulez	05 September 2016
9 Margaret Forster	08 February 2016	34 Arnold Palmer	25 September 2016
10 Eric Lubbock	14 February 2016	35 Shimon Peres	28 September 2016
11 Boutros Boutros Ghali	16 February 2016	36 Andrew Sachs	03 November 2016
12 Harper Lee	19 February 2016	37 Leonard Cohen	07 November 2016
13 John Chilton	25 February 2016	38 Jimmy Young	07 November 2016
14 Tony Warren	01 March 2016	39 Robert Vaughn	11 November 2016
15 Nancy Reagan	06 March 2016	40 Fidel Castro	25 November 2016
16 George Martin	08 March 2016	41 Peter Vaughan	06 December 2016
17 Peter Maxwell Davies	14 March 2016	42 Greg Lake	07 December 2016
18 Paul Daniels	17 March 2016	43 Zsa Zsa Gabor	18 December 2016
19 Cliff Michelmore	17 March 2016	44 Lionel Blue	19 December 2016
20 Barry Hines	18 March 2016	45 Liz Smith	24 December 2016
21 Garry Shandling	24 March 2016	46 Rick Parfitt	24 December 2016
22 Ronnie Corbett	31 March 2016	47 Richard Adams	24 December 2016
23 Zaha Hadid	31 March 2016	48 George Michael	25 December 2016
24 Victoria Wood	20 April 2016	49 Carrie Fisher	27 December 2016
25 Prince	21 April 2016		