PERCOLATION AND DISORDERED SYSTEMS

ERRATA

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This document contains corrections and additions to the paper [2].

Proof of Theorem 8.13 (13 Jan 1998)

Roberto Schonmann has kindly pointed out that the proof of part of Theorem 8.13 has a snag, namely in the demonstration of equation (8.20). The statement and proof of Lemma 8.18 are correct, but there is a difficulty with the 'simplified' version of the proof beginning at the bottom of page 224. The caption of Figure 8.1 is incorrect. Papers [1, 3] provide the following corrected version of the proof of (8.20). Start after the end of the proof of Lemma 8.18.

For $A \subseteq \mathbb{Z}^d$ and $x, y \in \mathbb{Z}^d$, let $\tau_p^A(x,y) = P_p(x \leftrightarrow y \text{ off } A)$. Now,

$$\tau_p(x,y) = \tau_p^A(x,y) + P_p(x \leftrightarrow y, \text{ but } x \not\leftrightarrow y \text{ off } A) \tag{8.20a}$$
$$\leq \tau_p^A(x,y) + \sum_{a\in A} P_p\big(\{x \leftrightarrow a\} \circ \{y \leftrightarrow a\}\big)$$
$$\leq \tau_p^A(x,y) + \sum_{a\in A} \tau_p(x,a)\tau_p(y,a)$$

by the BK inequality. This equation is valid for all sets A, and we are free to choose A to be a random set.

By (8.17) and Lemma 8.18,

$$\frac{d\chi}{dp} \geq \alpha(p) \sum_{x,y} \sum_{|u|=1} P_p\big(0 \leftrightarrow x,\ u \leftrightarrow y \text{ off } C_B(x)\big). \tag{8.20b}$$

Next, we condition on the random set $C_B(x)$. For given $C \subseteq \mathbb{Z}^d$, the event $\{C_B(x) = C\}$ depends only on the states of edges in $\mathbb{Z}^d \setminus B$ having at least one end-point in C; in particular, we have no information about the states of edges which either touch no vertex of C, or touch at least one vertex of B. We may therefore apply the FKG inequality to obtain that

$$P_p\big(0 \leftrightarrow x,\ u \leftrightarrow y \text{ off } C_B(x) \,\big|\, C_B(x)\big) \geq P_p(0 \leftrightarrow x \mid C_B(x))\tau_p^{C_B(x)}(u,y).$$

Hence,

$$P_p\big(0 \leftrightarrow x,\ u \leftrightarrow y \text{ off } C_B(x)\big) \geq E_p\Big(E_p\big(1_{\{0\leftrightarrow x\}}\tau_p^{C_B(x)}(u,y) \,\big|\, C_B(x)\big)\Big)$$
$$= E_p\big(1_{\{0\leftrightarrow x\}}\tau_p^{C_B(x)}(u,y)\big).$$

We have proved that

$$P_p\big(0 \leftrightarrow x,\ u \leftrightarrow y \text{ off } C_B(x)\big) \geq \tau_p(0,x)\tau_p(u,y) - \Big[E_p\big(1_{\{0\leftrightarrow x\}}\tau_p(u,y)\big) - E_p\big(1_{\{0\leftrightarrow x\}}\tau_p^{C_B(x)}(u,y)\big)\Big].$$

Applying (8.20a), we have that

$$\tau_p(u,y) - \tau_p^{C_B(x)}(u,y) \leq \sum_{w\in C_B(x)} \tau_p(u,w)\tau_p(y,w),$$

whence

$$\text{(8.20c)}\quad P_p\big(0 \leftrightarrow x,\ u \leftrightarrow y \text{ off } C_B(x)\big) \geq \tau_p(0,x)\tau_p(u,y) - \sum_{w\in\mathbb{Z}^d\setminus B} P_p\big(0\leftrightarrow x,\, w\leftrightarrow x \text{ off } B\big)\tau_p(u,w)\tau_p(y,w).$$

Finally, using the BK inequality,

$$P_p\big(0 \leftrightarrow x,\ w \leftrightarrow x \text{ off } B\big) \leq \sum_{v\in\mathbb{Z}^d\setminus B} \tau_p(0,v)\tau_p(w,v)\tau_p(x,v).$$

We insert this into (8.20c), and deduce (8.20). The proof may be continued as in [2].

References

1. Aizenman, M. and Newman, C. M., *Tree graph inequalities and critical behavior in percolation models*, Journal of Statistical Physics **36** (1984), 107–143.
2. Grimmett, G. R., *Percolation and disordered systems*, Lectures on Probability Theory and Statistics, Ecole d'Eté de Probabilités de Saint Flour XXVI–1996 (P. Bernard, ed.), Lecture Notes in Mathematics, vol. 1665, Springer, Berlin, 1997, pp. 153–300.
3. Hara, T. and Slade, G., *Mean-field behaviour and the lace expansion*, Probability and Phase Transition (G. R. Grimmett, ed.), Kluwer, Dordrecht, 1994, pp. 87–122.

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