

Symbol Index

a	mean arrival rate of service requirement	74, 76
a_i	mean arrival rate at queue j	61
$a(c)$	mean service requirement of a customer of class c	76
\mathcal{A}	subset of the state space \mathcal{S}	8
b_i	normalizing constant for queue j	49, 61
B	normalizing constant for a system	50
B_N	normalizing constant for a closed migration process	41
$B(M(1), M(2), \dots)$	normalizing constant for a closed network of queues	84
c	class of a customer, or factor altering transition rates	65, 200
$c_i(l)$	class of the customer in position l in queue j	60
c_r	clustering process parameter	163
c_j	state of queue j	60
$c(l)$	description of the customer in position l	74
C	simplex of the graph G	186
\mathcal{C}	set of customer classes, or of simplices	65, 186
\mathbf{C}	state of a queueing network	60
$d_i(m, z)$	mean nominal lifetime of a stage	206
$d(c, z)$	mean service requirement of a stage	76
$d(c)$	mean service requirement of a stage	74
f	parameter of a neutral allele model, or of a polymerization process	146, 173
$f_j(x_j)$	reduced description of site j	204
$F_c(x)$	distribution function of service requirement	77
$F_c^*(x)$	distribution function of service effort received	78
$F(\theta)$	expected number of clusters	167
G	graph	8, 184
$h(x)$	strictly concave function	18
H	subset of G	184
$H(t)$	entropy functional	18
I	number of customer types	57
$\mathcal{I}(m)$	subset of the set of customer types	84
J	number of queues	40, 57
$k(r)$	number of units in an r -cluster	163
m	size of a random sample from a population	147
m_r	number of r -clusters	162

m	description of a random sample, or state of a clustering process	147, 161
M	number of individuals in a population	145, 161
M_i	number of alleles represented by i individuals	146
$M(i)$	number of sites with attribute i	195
$M(m)$	number of customers with types in the set $\mathcal{S}(m)$	84
M	state of a neutral allele model, or of a migration process	146, 195
n_j	number in colony j , or attribute of site j	40, 184
n	state of a migration process or of a spatial process	40, 184
\mathbf{n}_H	state of sites in the set H	184
N	number of individuals in a closed migration process	40
$N(i)$	number of type i customers in a network	83
N_j	attribute set for site j	184
\mathbb{N}	set of non-negative integers	49
p_i	equilibrium potential of node j	126
$p_i(m, z)$	probability distribution over \mathcal{Z}	205
$p_j(t)$	potential of node j at time t	127
$p(c, z)$	probability distribution over \mathcal{Z}	76
$p(j, k)$	transition probability of a Markov chain	2
$q(j, k)$	transition rate of a Markov process	3
$q(j)$	transition intensity out of state j	3
$q'(j, k)$	transition rate of the reversed process	28
$q'(j)$	transition intensity out of state j in the reversed process	30
r_{jk}	resistance between nodes j and k	126
$r(i, s)$	route of a customer of type i	57
$r(j, k)$	resistance between nodes j and k	20
R	mean running time of a machine	100
R_u^r	replace an r -cluster and an s -cluster by a u -cluster	162
R_s^u	replace a u -cluster by an r -cluster and an s -cluster	162
R_v^u	replace a u -cluster by a v -cluster	180
R_1^1	remove a one-cluster	164
R_1^1	introduce a one-cluster	164
\mathbb{R}	set of real numbers	1
$s_j(l)$	stage of his route reached by a customer	60
S	mean service time of a machine	100
$S(i)$	number of stages in the route of a customer of type i	57
\mathcal{S}	state space	1
$\mathcal{S}_i(i, s, x_i)$	subset of the state space of queue j	68
$\mathcal{S}(c, \mathbf{x})$	subset of the state space of process \mathbf{x}	66
$\mathcal{S}(M(1), M(2), \dots)$	state space of a closed queueing network	85
$t_i(l)$	type of the customer in position l in queue j	60
T^{im}	introduce a customer of type i	60
T_{jk}	transfer an individual from colony j to colony k	40
T_{jl}	remove the customer in position l in queue j	60
T_{ilm}	move on the customer in position l in queue j	60

T_j^m	change the attribute of site j to m	185
T_j	remove an individual from colony j	48
T_k	introduce an individual at colony k	48
u	mutation probability	145
$u(l)$	stage of service reached	74
$u_j(t)$	distribution over states at time t	17
v_j	payment received on reaching vertex j	126
V	subset of G	126
$w_j(m, z)$	number of stages of a nominal lifetime	206
$w(c, z)$	number of stages of a service requirement	76
$w(c)$	number of stages of a service requirement	74
W	waiting time	11
x_j	detailed description of site j	203
\mathbf{x}	state of a queue, or of a spatial process	65, 203
$X(t)$	stochastic process	1
$X^j(t)$	jump chain	3
\mathcal{X}_j	state space of x_j	203
\mathbf{X}	state of a queuing network	68
z	fine classification or attribute	76, 205
\mathcal{Z}	set of possible values for z	76, 206
\mathbb{Z}	set of integers	1
$\binom{x}{r}$	binomial coefficient	14
α	association parameter in a clustering process	161, 167
α_j	migration process parameter	41, 49
$\alpha_j(i, s)$	mean arrival rate of customers of class (i, s) at queue j	61
$\alpha_j(n)$	spatial process parameter	193
$\alpha(c)$	mean arrival rate of customers of class c	66
β	dissociation parameter in a clustering process	161, 167
γ_{uv}	transmutation rate in a clustering process	180
$\gamma_i(l, n)$	distribution of service effort over positions	58
$\delta_i(l, n)$	distribution over possible positions for an arriving customer	59
∂_j	set of neighbours of site j	184
θ	clustering process parameter	167
κ	break rate of a bond in a polymerization process	173
λ_j	migration process parameter	41, 49
λ_{jk}	factor affecting transfer rate from colony j to colony k	40
λ_{rsu}	association rate in a clustering process	163
μ	death rate in a neutral allele model	145
μ_j	factor affecting emigration rate from colony j	48
μ_{rsu}	dissociation rate in a clustering process	163

ν	parameter of the infinite alleles model	146
ν_k	immigration rate to colony k	48
$\nu(i)$	arrival rate of customers of type i at a network	57
$\nu(c)$	arrival rate of customers of class c	74
$\pi_j(n_j)$	equilibrium distribution for queue j	37, 49
$\pi(j)$	equilibrium distribution for a Markov process	3
$\pi(n_i; \mathbf{n}_{G-i})$	equilibrium distribution for the truncated process n_i	201
$\pi(\mathbf{n})$	equilibrium distribution for \mathbf{n} , or a random field	41, 184
ρ	traffic intensity, or density	15, 177
$\phi_j(n_j)$	service effort supplied at queue j	40, 58
$\phi_C(\mathbf{n}_C)$	factor in a Markov field	186
$\Phi_{G-i}(\mathbf{n}_{G-i})$	function defining the transition rates of a spatial process	193
$\Phi(\mathbf{n})$	function defining the transition rates of a spatial process	193
$\Psi(N(1), N(2), \dots)$	function affecting the arrival rate at a network of queues	90

Subject Index

- Age of an allele, 151–156, 215, 221
- Allele, 145
 - age of an, 151–156, 215, 221
 - oldest, 153–156, 222
 - time to extinction of an, 152–154, 156
- Allocation, optimal, 54, 97–99, 213
- Aperiodic, 2, 24
- Arrival process, 11
 - at a quasi-reversible queue, 66
 - more general, 89–94
 - non-Poisson, 50, 55, 63
- Arrival rate, 11
 - mean, 13, 42, 47, 54, 55, 61, 87
 - more general, 89–94
- Balance,
 - detailed, 5
 - full, 5
 - partial, 27
- Balking, 36, 71
- Birth and death process, 10–17, 24, 35, 44, 100, 221
 - spatial, 191, 192, 197–199, 220
- Birth, death, and immigration process,
 - simple, 14, 16
 - with arbitrarily distributed lifetimes, 115–117
 - with clustering, 182
 - with family sizes, 53, 56, 115, 146, 149, 150, 213
- Birth-illness-death process, 113, 114, 116, 214
- Blocking, 47, 106, 107, 125, 135, 140, 144
- Calculation of the normalizing constant for a closed clustering process, 163, 165, 166
 - migration process, 47, 48
 - queueing network, 87, 212, 213
- Capacity constraint, 47, 106, 107
- Closed migration process, 40–49, 55, 84, 86, 135–145, 193, 195, 203, 212, 213, 222
- Closed network of queues, 82–90, 92, 99–113, 120–124, 183, 207, 217
- Clustering process, 161–183, 215, 222
- Communication network, 95–99, 131, 214
- Compartmental model, 113–117, 214
- Competition model, 209, 210
- Components,
 - in a complex device, 88, 105
 - in a power supply model, 191–193
 - in a renewal process, 79
 - provision of spare, 43–45, 47
- Computer, 96, 105–108, 214, 218
- Conveyor belt inspection, 118, 119, 123
- Counter, electronic, 119, 120, 123, 214
- Customer,
 - class, 65
 - route, 57
 - type, 57
 - typical, 11, 12, 16
- Cyclic queue, 43–47, 218, 220
- Decision making, 134, 190
- Defection, 36, 59
- Departure process, 34–39, 51, 52, 55, 62, 66, 70, 71, 88, 89, 92, 120, 124, 219, 220–221
- Description of a random sample, 147–150
- Detailed balance, 5
- Determining mutation, 158–160
- Disposition, 85–88, 93
- Dynamic reversibility, 31, 32, 39, 75, 144, 182, 212
- Ehrenfest model, 17–21, 212
- Electrical network, 20, 125–134, 157, 212, 214
- Electronic counter, 119, 120, 123, 214
- Entropy, 17, 19, 220
- Equilibrium, 3
 - distribution, 2, 3
 - equations, 2, 3
- Ergodic, 2, 3

228 Subject Index

- Erlang's formula, 13, 79, 111, 123, 219–222
- Exit process, 51, 52
- Family size process, 53, 56, 115, 146, 149, 150, 213
- Fixation times, 156–160, 215
- Flip-flop variable, 35, 37, 71, 73, 120
- Flow model, 125, 128–134, 140–144, 199, 214, 215
- one-dimensional, 132, 142–144
- Forward equations, 18
- Fruit trees, 184
- Full balance conditions, 5
- Garage, 120–124, 183
- Generating function method, 47, 48, 87
- Genetic type, 53, 145
- Geometric distribution, 11
- Grimmett's formula, 188, 215
- Group decision making, 134, 190
- Haulage firm, 88
- Heterozygosity, 148
- Hierarchy, organizational, 114, 142, 143
- Infinite alleles model, 146–160, 162, 169
- Infinite-server queue, 73, 74, 92, 100, 106, 107, 109, 112, 113, 183
- Insensitivity, 79, 87, 120, 124, 184, 198, 200, 203–211, 213, 216, 221
- Invasion model, 125, 132–134, 145, 158, 189, 192, 214
- Irreducible, 1
- Job-shop, 57, 58, 91–93, 97, 218
- Jump chain, 3
- Kac's formula, 21
- Kirchhoff's equations, 20, 126
- Kolmogorov's criteria, 21–25, 31, 92, 137, 165, 190, 212
- Labelled process, 152, 153, 156
- Lifetimes,
- arbitrarily distributed, 137, 149, 196–199
- in a clustering process, 182
- in a migration process, 137
- in a population genetics model, 149
- in a spatial process, 196–199, 203, 205–211
- nominal, 137, 196
- Limiting distribution, 2, 3, 212
- Linear migration process, 41, 52, 53
- Little's result, 13, 16, 54, 64, 82, 116, 123, 140, 212
- Machine interference, 99–105, 108, 214
- Manpower system, 113, 114, 142, 143, 214
- Markov chain, 3, 212
- Markov field, 184–194, 200, 202, 216, 217
- Markov process, 1, 212
- Mean arrival rate, 13, 42, 47, 54, 55, 61, 87
- Method of stages, 72–82, 182, 196, 197, 205, 206, 213, 217
- Migration process,
- closed, 40–49, 55, 84, 86, 135–145, 193, 195, 203, 212, 213, 222
- linear, 41, 52, 53
- open, 48–58, 64, 135–145, 199, 212, 213, 222
- reversible, 125, 135–145, 148, 158, 161, 162, 183, 195, 198, 209, 214, 215
- Mining operation, 44–47, 88, 143, 144
- Mutation, 145
- determining, 158–160
- Negative binomial distribution, 14, 16, 56, 140
- Neutral allele model, 145–151, 215, 221
- Nominal lifetime, 137, 196
- Normalizing constant, calculation of, 47, 48, 87, 163, 165, 166, 212, 213
- Nucleotide, 157–159
- Oldest allele, 153–156, 222
- One-dimensional flow model, 132, 142–144
- Open migration process, 48–58, 64, 135–145, 199, 212, 213, 222
- Open network of queues, 54, 57–72, 95–99, 107, 110–125, 130, 213, 214, 217, 218, 220, 221
- Optimal allocation, 54, 97–99, 213
- Orchard, 184
- Organizational hierarchy, 114, 142, 143
- Output from a simple queue, 34–37
- Partial balance, 27, 33, 42, 50, 67, 93, 182, 184, 193–195, 200–212, 216
- conditions, 27
- equations, 42
- Periodicity, 2, 24, 79

- Plant,
 - birth and death model, 197–199
 - infection model, 191, 192, 198
- Point process, 37
- Pollaczek–Khinchin formula, 81
- Polymerization, 173–180, 215, 222
- Population genetics, 53, 145–160, 215, 218
- Power supply, 191–193, 198
- Priority system, 59, 71, 94, 96, 104
- Probability flux, 8, 10, 12, 13, 15, 16, 33, 42, 46, 52, 55, 63, 67, 68, 70, 85, 87, 140, 195
- Quality control, 118
- Quasi-fixation, 156–158
- Quasi-reversibility, 65–124, 138, 142, 144, 200
- Queue,
 - cyclic, 43–47, 218, 220
 - infinite-server ($M/G/\infty$), 73, 74, 92, 100, 106, 107, 109, 112, 113, 183
 - many-server, 23–25, 29, 30, 33, 35, 36
 - $M/G/1$, 81, 213
 - $M/G/s$, 39
 - $M/M/s$, 14, 35, 39, 41, 52, 63, 64
 - priority, 59, 71, 94, 96, 104
 - server-sharing, 73, 74, 80, 81, 96, 105, 107, 137
 - simple ($M/M/1$), 11–16, 26, 34–40, 44–49, 54, 55, 57, 64, 67, 71, 95–98, 100, 212
 - symmetric, 72–82, 86, 87, 96, 98, 99, 101–103, 105, 109, 112, 121, 122, 213, 216
 - with no waiting room, 73, 74, 79, 80, 81
- Queueing times, 15
 - in a series of queues, 40
- Random field, 184–186, 192, 221
- Random sample, 147–151, 154–156, 215
- Random walk, 8, 41, 125–134, 153, 157, 159, 214, 220
 - symmetric, 126, 129, 131–134
- Renewal process, 37, 79
- Repair shop, 91–93, 111, 214
- Reversed process, 27–39, 46, 51, 52, 61–63, 66–70, 74, 75, 78, 79, 83–86, 91, 121, 122, 132, 144, 153, 198, 200–203, 212, 215
- Reversibility, 5, 125, 212
 - dynamic, 31, 32, 39, 75, 144, 182, 212
 - quasi-, 65–124, 138, 142, 144, 200
- Road traffic, 98, 116–118, 122, 123, 214, 220
- Sampling distribution, 147, 148
- Series of queues, 37–40, 46, 49, 117–119, 212, 214
- Server-sharing queue, 73, 74, 80, 81, 96, 105, 107, 137
- Service,
 - effort, 58, 72
 - rate, 11, 35, 54
 - requirement, 59, 72, 86
 - time, 11, 59
- Service in random order, 59, 64, 95, 100
- Simple birth, death, and immigration process, 14, 16
- Simple queues, 11–16, 26, 71
- network of, 44–48, 54, 55, 57, 64, 67, 95–98
- output from, 34–37
- series of, 37–40, 46, 49, 212
- Social grouping behaviour, 135, 138–140, 161, 214, 215
- Spare components, 43–45, 47
- Spatial process, 184–211, 215, 216
 - definition of a, 189
 - general, 193–200
 - reversible, 189–193
- Species,
 - competition between, 209, 210
 - number trapped, 150
- Stack, 16, 39, 73, 81, 82, 94, 96
- Stages,
 - method of, 72–82, 182, 196, 197, 205, 206, 213, 217
 - of a route, 57
 - of service, 74
- Stationarity, 1, 21, 28, 156
- Stationary distribution, 2, 3
- Stirling's formula, 178
- Stochastic process, 1
- Sufficient statistic, 149, 162, 204
- Switching system, 110–112, 198, 214, 217
- Symmetric queue, 72–82, 86, 87, 96, 98, 99, 101–103, 105, 109, 112, 121, 122, 213, 216

230 *Subject Index*

- Symmetric random walk, 126, 129, 131–134
- Telegraph system, 95, 96
- Telephone exchange,
 - as a symmetric queue, 73, 79
 - basic model, 13, 71, 103, 108, 111
 - calls lost or completed at, 35, 37, 82, 212
 - with a finite source population, 15, 112, 113, 182, 183
 - with unreliable lines, 108, 109, 111, 214
- Teletraffic model, 108–113, 213, 214
- Time homogeneous, 1, 28
- Time to extinction, 152–154, 156
- Time to fixation, 156–160, 215
- Timesharing computer, 105–108
- Traffic,
 - road, 98, 116–118, 122, 123, 214, 220
 - tele-, 108–113, 213, 214
- Traffic intensity, 15
- Transition probability, 2
- Transition rate, 3
- Truncation, 25–27, 33, 147, 200–207, 210, 211
- Typical customer, 11, 12, 16
- Waiting room,
 - joint, 26
 - overflow, 27
 - queue with no, 73, 74, 79, 80, 81
- Waiting times,
 - at a simple queue, 11–13
 - in a network of queues, 52, 55, 63
 - in a series of queues, 38–40, 220
 - virtual, 81, 213