D2: OPTIMIZATION

Contents

Books

Bazaraa, M., Jarvis, J. and Sherali, H. *Linear Programming and Network Flows*, second edition, 1990, Wiley.

Luenberger, D. *Introduction to Linear and Non-Linear Programming*, second edition, 1984, Addison-Wesley.

BJS is a paperback and full of lots of examples and reasonably comprehensive discussions of the history, recent results, etc. It is a little heavy-handed about some of the theory. Luenberger is nicer on the theory, but may be harder going.

Notes, examples sheet and overheads

An up-to-date postscript copy of these notes, the examples sheet and any overhead slides used in lectures can be downloaded from the WWW site: `http://www.statslab.cam.ac.uk/~rrw1/opt/index.html`.

Acknowledgement

The original version of these notes were written by Colin Sparrow in 1994.

Schedules

General formulation of constrained problems; the Lagrangian sufficiency theorem. Interpretation of Lagrange multipliers as shadow prices. Examples.

Convexity of feasible region; sufficiency of extreme points. Standardization of problems, slack variables, equivalence of extreme points and basic solutions. The primal simplex algorithm, artificial variables, the two phase method. Practical use of the algorithm; the tableau. Examples. The dual linear problem, duality theorem in a standard case, complementary slackness, dual variables and their interpretation as shadow prices. Relationship of the primal simplex algorithm to dual problem. Two person zero-sum games.

The Ford-Fulkerson algorithm and the max-flow min-cut theorems in the rational case. Network flows with costs, the transportation algorithm, relationship of dual variables with nodes. Examples. Conditions for optimality in more general networks; *the simplex-on-a-graph algorithm*.

Efficiency of algorithms. The formulation of simple practical and combinatorial problems as linear programming or network problems.

Notation

n, m	numbers of decision variables and functional constraints
x	decision variable in a primal problem, $x \in \mathbb{R}^n$
λ	decision variable in the dual problem, $\lambda \in \mathbb{R}^m$
$f(x)$	objective function in a primal problem
$x \in X$	regional constraints, $X \subseteq \mathbb{R}^n$
$g(x) \leq b$, $g(x) = b$	functional constraints, $g : \mathbb{R}^n \to \mathbb{R}^m$
z	slack variable, $z \in \mathbb{R}^m$
$Ax \leq b$, $Ax + z = b$	linear constraints
$c^T x$	linear objective function
$L(x, \lambda)$	Lagrangian, $L(x, \lambda) = f(x) - \lambda^T(g(x) - b)$
$\lambda \in Y$	$Y = \{\lambda : \min_{x \in X} L(x, \lambda) > -\infty\}$.
B, N	sets of indices of basic and non-basic components of x.
A, p, q	pay-off matrix and decision variables in a matrix game
x_{ij}	flow on arc (i, j)
v, $C(S, \bar{S})$	value of a flow, value of cut $(S, \bar{S})$
c_{ij}^-, c_{ij}^+	minimum/maximum allowed flows on arc (i, j)
d_{ij}	costs per unit flow on arc (i, j)
s_i, d_j	source and demands amounts in transportation problem
λ_i, μ_j	node numbers in transportation algorithm

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