Conspiracy or individualist?

Suppose $x_1, \dots, x_n$ are i.i.d., $Ex_i = 0$. There are two generic ways the event $[x_1 + \cdots + x_n \geq na]$ can happen.

- All r.v.s are about a. This has probability of about $f(a)^n$. We might call this *conspiracy behaviour*.
- One r.v. is about na and others are about 0. This has probability of about $nf(na)$. We might call this *individualist behaviour*.

For normal r.v.s we have $f(x) = e^{-x^2/2}/\sqrt{2\pi}$, and

$$f(a)^n \approx e^{-na^2/2} \;>>\; nf(an) \approx ne^{-n^2a^2/2}.$$

For Cauchy r.v.s we have $f(x) = \pi/2(1+x^2)$, and

$$f(a)^n \approx \frac{1}{1+a^n} \;<<\; nf(an) \approx \frac{n}{1+a^2n^2}.$$

Thus normal r.v.s exhibit conspiracy behaviour, but Cauchy r.v.s exhibit individualist behaviour. The distinction is whether the distribution has a tail that is super or sub-exponential in its decay to 0. (Note that the moment generating function does not exist for the Cauchy.) The exponential distribution sits on the borderline, since given that $S_n = x_1 + \cdots + x_n = na$, $a >> Ex_1$, the distribution of $x_1, \dots, x_n$ is the same as gaps between n points chosen uniformly from $[0, an]$; typically, these gaps are neither all about equal to a, but neither is one gap particularly large relative to the others.