Probability Theory and Statistics in High and Infinite Dimensions Empirical Process Theory and Beyond

On the occasion of Evarist Giné's 70th birthday

23rd - 25th June 2014,
Centre for Mathematical Sciences,
University of Cambridge, UK

Abstracts

Stein's method, logarithmic Sobolev and transport inequalities

Michel Ledoux

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Abstract

Logarithmic Sobolev and transportation cost inequalities have proved to be relevant tools in the study of concentration inequalities for various models of statistical interests. Stein’s method is a powerful technique for proving central limit theorems. In this talk, we develop new connections between Stein's approximation method and logarithmic Sobolev and transport inequalities by introducing a class of functional inequalities involving the relative entropy, the Stein (factor or) matrix, the relative Fisher information and the Wasserstein distance. For the Gaussian model, these results improve upon the classical logarithmic Sobolev inequality and the Talagrand quadratic transportation cost inequality. The new inequalities produce bounds for normal entropic convergence expressed in terms of the Stein discrepancy applicable to various examples of multidimensional functionals.

Joint work with Ivan Nourdin and Giovanni Peccati.

- Presentation 30 Minutes

Suprema of Bernoulli processes

Rafal Latala

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Abstract

We discuss a positive solution to the so-called Bernoulli Conjecture concerning two-sided bounds for suprema of Bernoulli processes. We also review some applications and related open problems.

Joint work with Witold Bednorz.

- Presentation 30 Minutes

Asymptotic expansions for entropy distances in limit theorems

Friedrich Götze

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Abstract

We investigate the convergence of sums of random variables to Gaussian and stable laws in Entropy resp. Fischer-Information distances. In particular we show asymptotic expansions of such distances in terms of semi-invariants (under minimal assumptions) in the context of classical and free probability.

Joint work with Sergey Bobkov and Gennadiy Chistyakov.

- Presentation 30 Minutes

Sequential complexities and uniform martingale LLN

Alexander Rakhlin

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Abstract

Uniform laws of large numbers play a key role in statistics and learning theory. In this talk, we describe martingale analogues of the uniform laws and introduce new notions of ``sequential complexities''. Extending the symmetrization technique to sequences of dependent random variables leads us to a notion of a tree. We then introduce a definition of a tree covering number, extend the chaining analysis, introduce an analogue of the VC dimension, and prove a counterpart to the classical combinatorial result of Vapnik-Chervonenkis-Sauer-Shelah. Our definitions and results can be seen as non-i.i.d. extensions of some of the key notions in empirical process theory.

- Presentation 30 Minutes

Cauchy-Schwarz Principles for uniform entropy

Ramon van Handel

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Abstract

Giné and Zinn have given a Gaussian characterization of classes of functions for which the empirical process satisfies the central limit theorem uniformly over all distributions of the underlying variables. A central object that arises in this characterization is the uniform Gaussian width, which can be upper bounded by the Koltchinskii-Pollard uniform entropy integral. In simple examples, however, the uniform Gaussian width proves to behave in a strictly better manner than might be expected from such computations. This phenomenon is not due to the inefficiency of classical chaining arguments, as might be expected in view of the majorizing measure theory, but rather due to the fact that the uniform entropy can grow at a strictly faster rate than the entropy with respect to any fixed distribution. In this talk I will aim to explain this phenomenon, its connection with combinatorial parameters, and whatever understanding I have at the present time (which is very limited) about the occurence of such behavior in general settings.

- Presentation 30 Minutes

Adaptive nonparametric credible balls

Aad van der Vaart

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Abstract

Credible sets are central sets in the support of a posterior probability distribution, of a prescribed posterior probability. They are widely used as means of uncertainty quantification in a Bayesian analysis. We investigate the frequentist coverage of such sets in a nonparametric Bayesian setup. We show by example that credible sets can be much too narrow and misleading, and next introduce a concept of `polished tail' parameters for which credible sets are of the correct order. The latter concept can be seen as a generalisation of self-similar functions as considered in a recent paper by Giné.

Joint work with Botond Szabó and Harry van Zanten.

- Presentation 30 Minutes

On the nonparametric Bernstein-von Mises phenomenon

Ismaël Castillo

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Abstract

In this talk I will discuss some limiting shape results for Bayesian posterior distributions in canonical nonparametric settings (white noise, density estimation). In particular, we will see that it is possible to obtain nonparametric Bernstein-von Mises theorems by stating the convergence in well-chosen large enough multiscale spaces. I will discuss applications to Donsker-type theorems for the posterior distribution function, as well as to (confident) credible sets.

Joint work with Richard Nickl.

- Presentation 30 Minutes

Constructing shrinkage prior distributions in high dimensions

Natesh Pillai

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Abstract

In this talk we outline a theory for constructing shrinkage priors in high dimensional problems. These prior distributions are popular because the corresponding MCMC algorithms mix very quickly. However, nothing much is known about their statistical efficiency. We present some results in this direction and also give a new prior which is both statistically and computationally efficient. We also present examples in which commonly used continuous shrinkage priors do not give optimal performance.

Joint work with Anirban Bhattacharya, Debdeep Pati and David Dunson.

- Presentation 30 Minutes

On the structure of random coordinate projections

Shahar Mendelson

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and
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Abstract

The Gine-Zinn symmetrization theorem shows that the suprema of natural empirical processes indexed by a class is determined by the structure of a typical coordinate projection of the class, namely, $\{ (f(X_i))_{i=1}^N : f \in F\}$.

I will survey some results on the geometric structure of these random sets, their connection with Dvoretzky type theorems, and the way the structure exhibits various probabilistic phenomena, like the uniform law of large numbers and the central limit theorem with weak boundedness assumptions.

- Presentation 30 Minutes

Adaptation in some shape-constrained regression problems

Adityanand Guntuboyina

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Abstract

We consider the problem of estimating a normal mean constrained to be in a convex polyhedral cone in Euclidean space. We say that the true mean is sparse if it belongs to a low dimensional face of the cone. We show that, in a certain natural subclass of these problems, the maximum likelihood estimator automatically adapts to sparsity in the underlying true mean. We discuss the problems of convex regression and univariate and bivariate isotonic regression as examples.

- Presentation 30 Minutes

A robust and adaptive estimator for regression

Yannick Baraud and **Lucien Birgé**

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Abstract

Our purpose is to present a new method for adaptively estimating a regression function when little is known about the shape and scale of the errors. For instance, it can cope with error distributions as different as Gaussian, Uniform, Cauchy or even with unimodal unbounded densities. In favorable cases and when the true distribution belongs to the model, the estimator is asymptotically equivalent to the M.L.E. and, nevertheless, still behaves reasonably well when the model is wrong, even in cases for which the least-squares do not work. The assumptions that are needed to get our results are rather weak, in particular no moment condition is required on the errors, and this is why the method can adapt to both the regression function, the shape of the errors and their scale. Moreover, it appears that the practical results obtained by simulation are surprisingly good as compared to more specific estimators. The corresponding paper is available on arXiv at http://arxiv.org/abs/1403.6057

Joint work with Mathieu Sart.

- Presentation 60 Minutes

Estimator selection: methods and calibration

Pascal Massart

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Abstract

There is a huge literature devoted to the topic of estimator selection, both from a theoretical and a practical view point. Estimator selection methods typically involve tuning parameters which drastically influence the behaviour of the methods. Therefore the choice of these tuning parameters is an important issue. It is often the case that performance bounds for estimator selection methods are over pessimistic or not precise enough to provide a valuable guide-line for practitioners. Fortunately computer intensive simulations can be used to understand how to calibrate these tuning parameters in practice but nevertheless from a mathematical view point, it is desirable to better understand the issue of calibrating selection methods. We shall see that in a number of situations that include penalized model selection, it is possible to reduce the gap between theory and practice by providing data-driven rules for choosing tuning parameters which are based on sharp upper and lower performance bounds.

- Presentation 30 Minutes

Exponential inequalities and local independence graphs to unravel functional neuronal connectivity

Patricia Reynaud-Bouret

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Abstract

The starting theoretical point is to provide "data-driven" exponential inequalities for counting processes with as few assumptions as possible on the underlying conditional intensity. This allows us to present a weighted Lasso method in the abstract set-up of linear counting processes. Thanks to this method, it is possible to approximate real neuronal data, called spike trains, by Hawkes models even if this model is not true. This leads to an estimation of local independence graphs for which we are currently testing the adequation with the biological notion of "functional connectivity".

- Presentation 30 Minutes

Confidence bands for distribution functions: A new look at the law of the iterated logarithm

Lutz Dümbgen

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Abstract

We present a general law of the iterated logarithm for stochastic processes on the open unit interval having subexponential tails in a locally uniform fashion. It applies to standard Brownian bridge but also to suitably standardized empirical distribution functions. This leads to new goodness-of-fit tests and confidence bands which refine the procedures of Berk and Jones (1979) and Owen (1995). Roughly speaking, the high power and accuracy of the latter procedures in the tail regions of distributions are essentially preserved while gaining considerably in the central region.

Joint work with Jon Wellner.

- Presentation 30 Minutes

Regularity of Gaussian processes in a geometric framework

Gerard Kerkyacharian

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Abstract

Recently, T. Coulhon, P. Petrushev and G.K. have proposed to consider a Littlewood-Paley theory in the framework of Dirichlet spaces with a suitable behavior of the associated heat kernel. This covers the case of compact Riemannian spaces. One can then construct regularity spaces and an associated wavelet theory. In this talk we will study how this could be used to analyze the regularity of Gaussian processes.

- Presentation 30 Minutes

Estimation of the Lévy measure: statistical inverse problem and Donsker Theorem

Markus Reiß

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Abstract

We consider estimation of the characteristics of a Lévy process from discrete time observations. When the distance in observation time does not tend to zero, we face a nonlinear statistical inverse problem (that of decompounding in the compound Poisson case) with the Lévy measure as a nonparametric quantity. We shall concentrate on the generalized distribution function of the Lévy measure and show that a Donsker theorem or uniform CLT is feasible provided the characteristic functions does not decay too fast. Interestingly, for both, the pointwise CLT and the tightness part, we need advanced Fourier analysis (multiplier theorems, Besov spaces). Smoothed empirical processes, as introduced by Giné and Nickl (2008), will prove to be an essential ingredient.

Joint work with Richard Nickl.

- Presentation 30 Minutes

Gaussian Approximations and Bootstrap with p >> n.

Victor Chernozhukov

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Abstract

We show that central limit theorems hold for high-dimensional normalized means hitting high dimensional rectangles (and, more generally, convex sets formed as sparse deformations of rectangles). These results apply even when p>> n. These theorems provide Gaussian distributional approximations that are not pivotal, but they can be consistently estimated via Gaussian multiplier methods (Gine and Zinn) and the empirical bootstrap. These results generalize to the suprema of empirical processes indexed by function sets with diverging complexity, and are useful for building confidence bands in modern high-dimensional and nonparametric problems (Gine and Nickl) and for multiple testing via the step-down methods. This is joint work with Denis Chetverikov (UCLA) and Kengo Kato (Tokyo). Refs: arxiv 1212.6906, 1212.6885,1303.7152.

- Presentation 30 Minutes

Functional depth and quantiles

Joel Zinn

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Abstract

We will discuss some recent work with J. Kuelbs on Empirical Quantiles for time dependent data and connections to work on Functional Depth

- Presentation 30 Minutes

Some historical remarks on the Giné-Zinn work on empirical processes

Richard Dudley

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Abstract

I will talk about bootstrap limit theorems with something about techniques of symmetrization and randomization used in their proofs. Another topic will be uniform Donsker classes.

- Presentation 30 Minutes

The Hurst Phenomenon and the Rescaled Range Statistic

David M. Mason
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ABSTRACT

In 1950 H. E. Hurst published the results of his investigations of water outflow from the great lakes of the Nile basin. Hurst wanted to determine the reservoir capacity that would be needed to develop the irrigation along the Nile to its fullest extent. His work motivated the notion of long range dependence through the application of a statistic that he developed for his study. This is the rescaled range statistic-the R/S statistic.

Given data X_i, $i = 1, \ldots, n$, set $\mu_n = \frac{1}{n}\sum_{i=1}^n X_i$, and let $S_i^* = \sum_{j=1}^i (X_j - \mu_n)$, for $i = 1, \ldots, j$, $M_n^* = \max(0, S_1^*, \ldots, S_n^*)$ and $m_n^* = \min(0, S_1^*, \ldots, S_n^*)$. Define the *adjusted range* $R_n^* = M_n^* - m_n^*$. The *rescaled range statistic* (the R/S statistic) is $R_n^* = M_n^* - m_n^*$. where D_n is the sample standard deviation $D_n = \sqrt{\sum_{i=1}^n (X_i - \mu_n)^2 / n}$, for $n \geq 1$.

Hurst argued via a small simulation study that if X_i, $i = 1, \ldots, n$, are i.i.d. normal then $(R/S)_n$ should grow in the order of $\sqrt{n}$. (Hurst was later proved correct by Feller.) However, Hurst found that for the Nile River data, $(R/S)_n$ increased not in the order of $\sqrt{n}$, but in the order n^H, where H ranged between .68 and .80 with a mean of .75. For annual tree ring data H ranged between .79 and .86 with a mean of .80, for sunspots and wheat prices an average H of .69 was obtained, and data on the thickness of annual layers of lake mud deposits gave an average of $H = .69$. All of the above data had normal-like histograms, yet all gave estimates of H consistently greater than 1/2, which an i.i.d. normal model would give. This is now called the *Hurst phenomenon.*

We shall discuss some unexpected universal asymptotic properties of the R/S statistic, which show conclusively that the Hurst phenomenon can never appear for i.i.d. data.

Adaptive estimation of the copula correlation matrix for semiparametic elliptical copulas

Marten H. Wegkamp

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Abstract

In this joint work with Yue Zhao (Cornell University), we study the adaptive estimation of copula correlation matrix Σ for elliptical copulas. In this context, the correlations are connected to Kendalls tau through a sine function transformation. Hence, a natural estimate for Σ is the plug-in estimator $\widehat{\Sigma}$ with Kendalls tau statistic. We first obtain a sharp bound for the operator norm of $\widehat{\Sigma} - \Sigma$. Then, we study a factor model for Σ, for which we propose a refined estimator $\widetilde{\Sigma}$ by fitting a low-rank matrix plus a diagonal matrix to $\widehat{\Sigma}$ using least squares with a nuclear norm penalty on the low-rank matrix. The bound on the operator norm $\widehat{\Sigma} - \Sigma$ serves to scale the penalty term, and we obtain finite sample oracle inequalities for $\widetilde{\Sigma}$. We also consider an elementary factor model of Σ, for which we propose closed-form estimators. We provide data-driven versions for all our estimation procedures and performance bounds.

Presentation 30 minutes

Precise moment and tail estimates for Rademacher series in terms of weak parameters

Krzysztof Oleszkiewicz

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Abstract

We establish a new deviation inequality for Rademacher sums and use it to obtain tight moment and tail bounds for Rademacher series in terms of weak parameters. In particular, we prove an asymptotic equality of the optimal constants in the Khinchine in Khinchine-Kahane inequalities

- Presentation 30 Minutes

Moment estimates implied by modified log-Sobolev inequalities

Radoslaw Adamczak

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Abstract

I will present connections between modified log-Sobolev inequalities and Poincare inequalities for the p-th moments in which the Euclidean norm of the gradient is replaced by a certain Orlicz type norm related to the energy form in the log-Sobolev inequality. In special cases, using estimates of moments of linear combinations of independent random variables with log-concave tails due to Gluskin and Kwapien, this Poincare inequality can be rewritten in terms of moments of auxiliary independent random variables which allows to obtain a weak decoupling principle for functions with bounded derivatives of higher order, relating their moments to moments of tetrahedral polynomials in independent random variables. In the case of the classical log-Sobolev inequality this leads to an extension of Latala's inequalities for Gaussian chaos to more general non-Lipschitz functions and non-product measures. If time permits I will also discuss counterparts of such inequalities for polynomials in arbitrary independent subgaussian random variables (to which concentration inequalities for general smooth functions do not apply).

Joint work with Witold Bednorz and Pawel Wolff.

- Presentation 30 Minutes

General Strassen type results for partial sum processes in Euclidean space

Professor Uwe Einmahl

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Abstract

One of the classical results of probability for sums of i.i.d. random variables is the functional LIL of Strassen (1964) who under the classical assumptions that the second moment is finite and the expectation of the underlying distribution is equal to zero showed that with probability one, the sequence $\{S_{(n)}/\sqrt{2n \log\log n}\}$ where $S_{(n)} : \Omega \to C[0,1]$ is the partial sum process of order n, is relatively compact in $C[0,1]$ and moreover that the (random) set of limit points of this sequence is equal to a certain deterministic subset of $C[0,1]$ which we call the cluster set of the sequence $\{S_{(n)}/\sqrt{2n \log\log n}\}$.

This result extends to higher dimensions and there are versions in the infinite variance case where one has to use different normalizing sequences $\{c_n\}$. In the 1-dimensional case it turned out that one still gets the standard cluster set as in the Strassen LIL provided that the normalizing sequence satisfies some mild regularity assumptions. This is no longer the case if one looks at this problem in the multidimensional setting.

The purpose of this talk is to give a survey of some recent work in this direction. Among other things, we are able to determine all possible cluster sets in the independent component case. In the general case we can identify minimal and maximal sets for the functional cluster sets in terms of the cluster sets of the normalized sums.

Presentation 30 minutes

Alexandre (Sasha) Tsybakov

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and
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Abstract

To be confirmed.

- Presentation 30 Minutes

On tensor completion via nuclear norm minimization

Cun-Hui Zhang

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Abstract

Many problems can be formulated as recovering a low-rank tensor. Although an increasingly common task, tensor recovery remains a challenging problem because of the delicacy associated with the decomposition of higher order tensors. To overcome these difficulties, existing approaches often proceed by unfolding tensors into matrices and then apply techniques for matrix completion. We show here that such matricization fails to exploit the tensor structure and may lead to suboptimal procedure. More specifically, we investigate a convex optimization approach to tensor completion by directly minimizing a tensor nuclear norm and prove that this leads to an improved sample size requirement. To establish our results, we develop a series of algebraic and probabilistic techniques such as characterization of subdifferetial for tensor nuclear norm and concentration inequalities for tensor martingales, which may be of independent interests and could be useful in other tensor related problems.

Joint work with Ming Yuan.

- Presentation 30 Minutes

Empirical risk minimization for heavy-tailed losses

Gábor Lugosi

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Abstract

In this talk we discuss empirical risk minimization when the losses are not necessarily bounded and may have a distribution with heavy tails. In such situations usual empirical averages may fail to give accurate risk estimates. However, some robust mean estimators proposed in the literature may be used to replace empirical means. We pay particular attention to empirical risk minimization based on a robust estimate proposed by Olivier Catoni. We develop performance bounds based on a chaining argument tailored to Catoni's mean estimator. The results are illustrated on examples of regression function estimation and k-means clustering.

Joint work with Christian Brownlees and Emilien Joly.

- Presentation 30 Minutes

Estimating a directional trend from noisy directional data

<u>Rudy Beran</u>

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<u>Abstract</u>

Consider measured positions of the paleomagnetic north pole over time. Each measured position may be viewed as a direction, expressed as a unit vector in three dimensions. The abstract problem is to estimate an underlying trend from an observed sequence of unit vectors in q-dimensions, each indexed by an ordinal covariate and measured with random error. In this sequence, mean directions are expected to be close to one another at nearby covariate values. A simple trend estimator that respects the geometry of the sphere is to compute a running average over the covariate-ordered observed direction vectors, then normalize these average vectors to unit length. This talk treats a considerably richer class of competing directional trend estimators that respect spherical geometry. The analysis relies on a nonparametric error model for directional data that makes no symmetry or other shape assumptions. Good trend estimators are selected through calculations of estimated risk under the error model. Empirical process theory underlies claims that the estimated risks are trustworthy surrogates for the unknown risks of competing directional trend estimators.

- Presentation 30 Minutes