NEW FRONTIERS IN RANDOM GEOMETRY (RaG)
EP/103372X/1
REPORT 1/7/14 – 30/6/15

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1. Management Process

The Management Committee (MC) comprises the three investigators and the four members of the external Advisory Board (AB), namely Yuval Peres, Stanislav Smirnov, Balint Tóth and Wendelin Werner. The local managers have met weekly during term, and more formally about every two months. The advice of the AB has been sought on a variety of matters including the hiring process. Three members of the AB (Peres, Tóth, Werner) have spent periods in Cambridge during the period of this report. A meeting of the Advisory Board took place on 3 October 2014, at which all members were present and the EPSRC was represented. The minutes of that meeting have been agreed by circulation.

2. Personnel

One postdoctoral research fellow was appointed following the advertisement of December 2014.

- Anthoine Dahlqvist[1], PhD (Université de Paris 6), from 1 September 2015 to 31 August 2017.

One postdoc has left the team and another is due to leave later in the summer:

- Zhongyang Li[2], employed from 1 September 2011 to 31 August 2014.
- Laure Dumaz[3], employed from 1 September 2013 to 31 August 2015.

3. Isaac Newton Institute programme

The big event of this year has been the Newton Institute programme on *Random Geometry*, from 12 January to 3 July 2015. Berestycki was the principal organizer with Itai Benjamini, Jean François Le Gall, and Scott Sheffield, and the Scientific Advisory Committee was composed of the RaG investigators and AB. Approximately 115 individuals (of whom 29

Date: July 19, 2015.
http://www.statslab.cam.ac.uk/~grg/rag.html.

[1] http://page.math.tu-berlin.de/~dahlqvis/

[2] http://www.statslab.cam.ac.uk/~zl296/

[3] https://www.dpmms.cam.ac.uk/people/ld437/

were based in the UK) participated in the programme, for visits of lengths between a few days and six months, and there was in addition a substantial number of visitors attending workshops only. There were five workshops totalling six weeks of activity, and a seminar series that spanned the INI and the Statistical Laboratory. Numerous participants from overseas took the opportunity to visit other universities in the UK.

The event was considered generally by the participants to have been an extraordinary success whose full legacy will crystallize over a period of years in the future.

Fuller details of the activities may be found on the INI website at http://www.newton.ac.uk/event/rgm.

4. RaG Research Programme (selected highlights)

4.1. **Liouville quantum gravity and the Brownian map.** Miller and Sheffield have studied and largely resolved the relationship between Liouville quantum gravity (LQG) and the Brownian map (TBM). LQG and TBM have long been considered to be, in some sense, the most natural 'random surface' models, and one of the main accomplishments of this work is a proof that the models are in fact equivalent when the parameter γ is tuned correctly. They also give a more thorough understanding of many aspects of the models taken separately.

4.2. **Oriented random planar maps.** Kenyon and Sheffield have made progress in understanding maps decorated by bipolar orientations (which are acyclic orientations that have only two vertices at which all edges point inward or all edges point outward). They have proved that in the "peanosphere" sense, these models converge to LQG with parameter $\gamma = \sqrt{4/3}$ decorated by SLE with parameter 12. This is interesting for several reasons. For one thing, it brings a new number (namely, 12) into the small canon of special κ values in the SLE theory and suggests a number of conjectures about models on lattices. For another, it presents a model in which the random geometry interpretation arises especially naturally.

4.3. **Hastings–Levitov conformal aggregation and related growth models.** Johansson Viklund, Norris, Sola, and Turner are continuing their investigations into the conformal aggregation processes introduced by Hastings and Levitov, and related growth models.

A Cambridge PhD student V. Silvestri, under the supervision of Norris has shown that the boundary fluctuations for the $\alpha = 0$ Hastings-Levitov model converge to a Gaussian limit, which is an infinite-dimensional Ornstein-Uhlenbeck process. This process has as stationary distribution, and in the large cluster limit, the logarithmically correlated Gaussian field on the circle.

4.4. **Surprise probabilities for Markov chains.** Norris, Peres and Zhai studied robust non-concentration estimates for the probability that a finite state-space Markov chain visits a given state for the first time at a given large time. The specification of transition probabilities for a Markov chain may be considered as a discrete geometry, notably in the reversible case. The aim of this work was to understand to what extent qualitative properties of the chain, such as reversibility, implied upper bounds on these 'first visit probabilities' depending only on the time and the number of states, and not on further quantitative conditions on the chain. Such estimates can be applied for example when the transition probabilities are random and their values are unknown. Sharp estimates were obtained for general chains, for reversible chains, and for random walks on simple graphs.

4.5. **Small-time fluctuations for the bridge of a sub-Riemannian diffusion.** Bailleul, Mesnager and Norris investigated from a geometric point of view the small-time behaviour of diffusion processes conditioned by their initial and final positions. Techniques pioneered by Malliavin, Bismut and Ben Arous can be adapted to prove, under a certain non-cut locus condition, that the process converges to a minimal energy path joining the endpoints, while its fluctuations from this path converge weakly to a Gaussian process whose covariance can be identified. In localizing these results to incomplete manifolds we were led to prove new heat kernel upper bounds for sub-Riemannian Laplacians. In characterizing the limit via its reproducing-kernel Hilbert space, we were led to introduce a notion of second variation for the energy function on paths at its minimum which, while known in Riemannian geometry, was new in the sub-Riemannian context.

4.6. **Self-avoiding walks.** Grimmett and Li have continued their study of self-avoiding walks and the connective constant, with particular attention paid to the locality property of connective constants of transitive graphs. Thye have proved that a sufficient condition for locality is the existence of a linearly increasing harmonic function. Such a harmonic function has been successfully constructed on Cayley graphs of solvable groups. The locality property is also proved for certain classes of transitive graphs beyond Cayley graphs.

4.7. **Mixing time of random graphs.** Berestycki, Lubetzky, Peres and Sly have studied random walks on the giant component of the Erdős-Rényi random graph $G(n, \lambda/n)$. The mixing time from a worst starting point was shown by Fountoulakis and Reed, and independently by Benjamini, Kozma and Wormald, to have order $\log^2 n$ without any cutoff.

The new results show by contrast that starting from a uniform vertex (equivalently, from a fixed vertex conditioned to belong to the giant) both accelerates mixing to $O(\log n)$ and the cutoff phenomenon occurs: the typical mixing is at $(\sigma \dim H)^{-1} \log n \pm (\log n)^{1/2+o(1)}$, where σ and $\dim H$ are the speed of random walk and dimension of harmonic measure on a

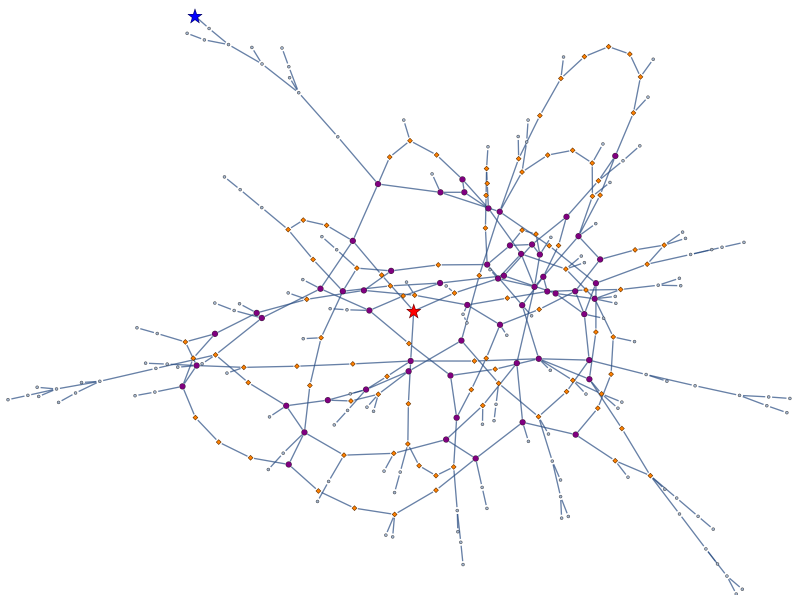


FIGURE 4.1. Structure of the giant component of a random graph. The far end vertex with a blue star is the worst starting point for mixing.

Po(λ) Galton-Watson tree. The results extend to graphs with prescribed degree sequences, where cutoff is shown both for the simple and for the non-backtracking random walk.

4.8. **Equivalence of Liouville measure and Gaussian free field.** Given an instance h of the Gaussian free field on a planar domain D and a constant $\gamma \in (0, 2)$, one can use various regularization procedures to make sense of the *Liouville quantum gravity measure* $\mu := e^{\gamma h(z)} dz$. It is known that the field h a.s. determines the measure μ_h. Berestycki, Sheffield and Sun show that the converse is true: namely, h is measurably determined by μ_h. More generally, given a random closed fractal subset X endowed with a Frostman measure σ_X whose support is X (independent of h), they construct a quantum measure μ_X and ask the following: how much information does μ_X contain about the free field? They conjecture that whenever X is harmonically nontrivial, μ_X always determines h restricted to X, in the sense that it determines its harmonic extension off X. They prove the conjecture in the case where X is an independent SLE_κ curve equipped with its quantum natural time, and in the case where X is Liouville Brownian motion (that is, standard Brownian motion equipped with its quantum clock). The proof in the latter case relies on properties of nonintersecting planar Brownian motion, including the value of some nonintersection exponents.

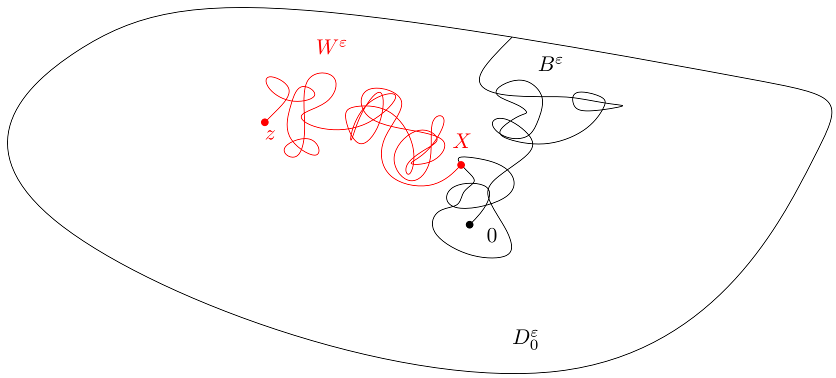


FIGURE 4.2. Probing the harmonic extension of a GFF off a Brownian curve.

4.9. **An elementary approach to Gaussian multiplicative chaos.** Berestycki obtained a new, completely elementary and self-contained proof of convergence of Gaussian multiplicative chaos. The argument shows further that the limiting random measure is nontrivial in the entire subcritical phase ($\gamma < \sqrt{2d}$) and that the limit is universal (i.e., the limiting measure is independent of the regularisation of the underlying field).

4.10. **Existence of self-accelerating fronts for reaction-diffusion equations.** Berestycki, Mouhot and Raoul have studied a family of non local diffusion reaction PDEs. These equations display a surprising acceleration of the speed of propagation observed in certain biological invasions. Prior to this work, nonrigorous WKB scaling had led to predictions that the wavefront position was asymptotically proportional to $t^{3/2}$.

The above is proved rigorously in a work that is essentially completed and uses branching Brownian motion with tools from stochastic calculus.

4.11. **Near-critical spanning forests.** The uniform spanning tree is a fundamental object with important connections to several areas, such as random walks, algorithms, domino tilings, electrical networks, potential theory, percolation etc.

Benoist, Dumaz and Werner describe and study the scaling limit of a 'near-critical' model around the uniform spanning tree. Inspired by earlier work of others, they introduce a dynamical model starting from the critical point, where edges are removed in a Poissonian way, and they prove the convergence of this process when the mesh-size goes to 0 to its continuum counterpart.

It is then possible to describe the evolution of this dynamical model by a natural discrete coalescent Markov process on the state of weighted graphs.

This gives a geometric non-embedded description of the underlying continuous structure and provides a novel discrete approach to some of the renormalization group arguments of theoretical physics in this very particular case. See Figure 4.3 for a representation of the near critical forest.

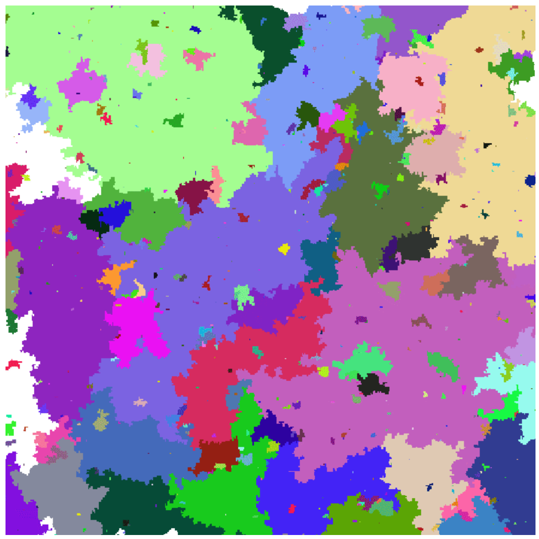


FIGURE 4.3. Representation of the clusters of the near critical spanning forest.

4.12. **Random matrices.** For any $\beta > 0$, consider the probability density function of $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \in \mathbb{R}$ given by:

$$\frac{1}{Z_n^\beta} \exp\left(-\sum_{k=1}^n \lambda_k^2\right) \prod_{j>k} |\lambda_j - \lambda_k|^\beta$$

in which Z_n^β is a normalizing constant. This represents n charged particles in a Coulomb gaz at inverse temperature β. When $\beta = 1, 2$ or 4, this is exactly the joint density of eigenvalues for the Gaussian orthogonal, unitary, or symplectic ensembles, G(O/U/S)E in random matrix theory. For a general β, those point processes are called the β-ensembles and correspond to the eigenvalues of certain tridiagonal random matrices.

It is natural to expect that the Wigner statistics will become Poissonian in the limit as $\beta \to 0$. In a previous paper, Allez and Dumaz investigated the behaviour of the largest particle (edge case) of those β-ensembles in the large n limit and when $\beta \to 0$. They proved its convergence to the Gumbel law, a universal law appearing in extreme value theory. In a new paper, they study the behaviour of the eigenvalues in the bulk, using the Sine-β point process of Valkó and Virág. This point process has been shown to be the scaling limit of the eigenvalues point process in the bulk of β-ensembles and its law is characterized in terms of the winding numbers of the Brownian carousel at different angular speeds (see Figure 4.4). It is proved that the Sine-β point process converges weakly to a Poisson point process on $\mathbb{R}$. Thus, the Sine-β point processes establish a smooth crossover between the rigid clock (or picket fence) process (corresponding to $\beta = \infty$) and the Poisson process.

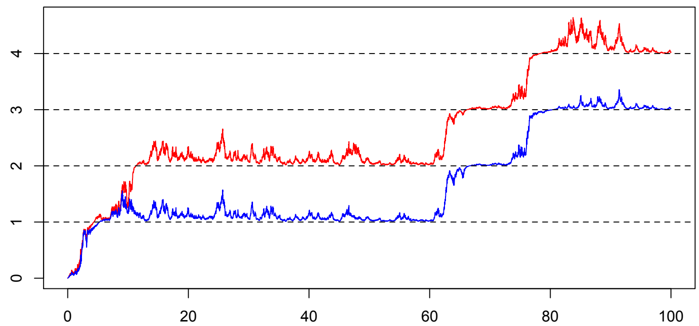


FIGURE 4.4. The coupling of winding numbers of the Brownian carousel at two different angular speeds.

4.13. **Unimodular hyperbolic triangulations: circle packing and random walk.** Angel, Hutchcroft, Nachmias and Ray have worked on understanding the final behaviour of random walk on the circle packing of stationary reversible hyperbolic triangulations (see Figure 4.5). Suppose $f : (G, u, v) \to \mathbb{R}$ is a measurable transport function transporting some mass from u to v. We say a random rooted graph satisfies the mass transport principle if the expected mass out from the root is equal to the expected mass in for every such transport function. A random rooted graph is said to be **unimodular** if it satisfies the mass transport principle. A **circle packing** of a planar graph is a collection of non-crossing circles drawn in the plane, one for each vertex, such that two circles are tangent to each other if and only if the vertices adjacent to each other (see Figure 4.5).

They show first that the circle packing is conformally equivalent to the whole plane if and only if the expected degree of the root is 6. This can be thought of as a discrete version of a uniformization theorem in this setting. They also prove several properties of the random walk in the hyperbolic setting like convergence to the boundary in the hyperbolic disc, speed of convergence, nonatomicity of the exit measure and characterization of the Poisson boundary of such walks.

Angel, Hutchcroft, Nachmias and Ray extend the dichotomy result in the previous work to general unimodular graphs. The basic objective is to show that several notions of parabolicity and hyperbolicity are equivalent for this wide class of random graphs. They show such dichotomy holds when one

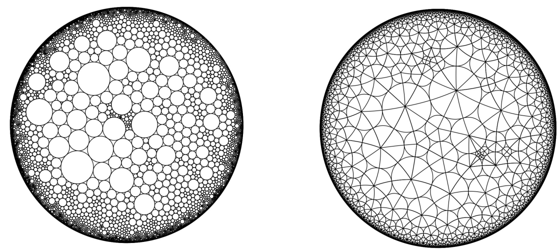


FIGURE 4.5. Left: A circle packing of a random unimodular triangulation. Right: Its circle packing. The circle packing software of Ken Stephenson was used to draw the circle packing.

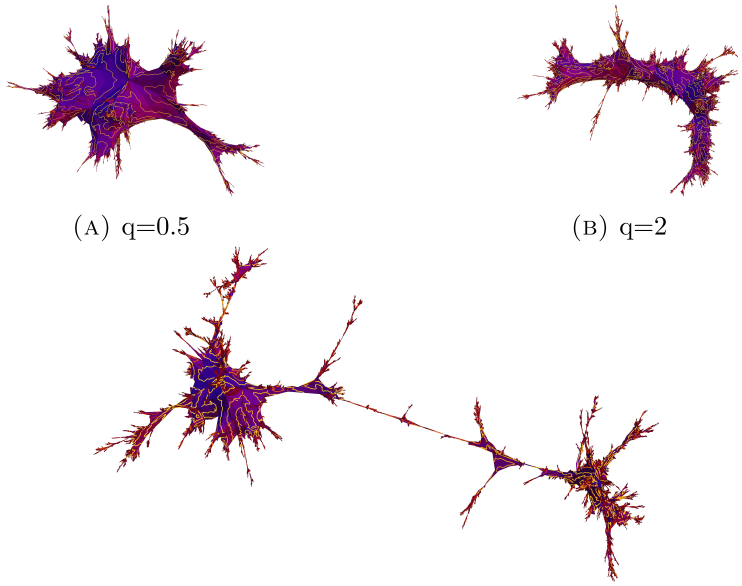
considers properties like the geometry of wired and free uniform spanning forests, the Louiville property, existence of harmonic dirichlet functions, VEL parabolicity, invariant amenability and finally with conformal type for one-ended maps.

4.14. **Recurrence of half plane UIPT.** Angel and Ray extend the result for the recurrence of uniform infinite planar maps in the half plane setting. It was conjectured for about a decade that the uniform infinite random maps are recurrent. This question was resolved in a recent work of Gurel-Gurevich and Asaf Nachmias.

The half-plane uniform infinite planar triangulations are not stationary since the root is not picked uniformly, and thus one cannot directly apply the result of Gurel-Gurevich and Nachmias. Different techniques are required which also involve circle packing and a decomposition of the half-plane maps into layers. A new limiting structure of independent interest emerges which looks like the random map 'seen from infinity'.

4.15. **Critical exponents in FK weighted planar maps.** Berestycki, Laslier and Ray have studied the critical exponents in the FK loop model on random planar maps. Consider a finite planar map M along with a submap T. One can consider interfaces between the vertex clusters of M and the dual vertex clusters which will give us an ensemble of loops such that every edge of the map is adjacent to at least one of the loops. The model we consider gives a weight $q^\ell > 0$ to each such map where ℓ is the number of loops in (M, T). Such a measure is related to the self-dual FK model which is in turn related to the q-state Pott's model. Using a bijection due to Sheffield and a connection to planar Brownian motion in a cone, they obtain the values of critical exponents associated with the length of cluster interfaces,

$$\frac{4}{\pi} \arccos\left(\frac{\sqrt{2-\sqrt{q}}}{2}\right).$$



(A) q=0.5

(B) q=2

(C) q=9

FIGURE 4.6. Simulations of critical FK model on random maps for several values of the parameter q. The loops are shown in several colors which depend on their length. Simulations show that the geometries vary a lot depending on the value of the parameter q.

This matches the known exponents in the percolation cas, as verifies the KPZ predictions.

4.16. **Tilted interface in low temperature 3D Ising model.** At low temperature, the Ising model has two symmetrical distinct translation invariant measures called the two pure phases. In a finite volume, it is possible to force the coexistence of these two phases and in $\mathbb{Z}^3$ an essentially two dimensional interface appear. When this interface is along a lattice direction, the local behaviour of the interface and its fluctuations have been known for a long time but nothing is known when the interface is in a generic position. Coquille and Laslier are investigating the later case via a coupling with the zero temperature case (lozenge tilings).

4.17. **Continuity in the Gaussian free field/uniform spanning tree coupling.** A bijection between uniform spanning tree and dimer height function was discovered by Temperley and generalized by Kenyon et al. In this bijection, the height function at a point is essentially the winding number of the branch of the tree to that point.

A continuous version of that bijection was introduced later by Dubédat and cast by Miller and Sheffield in the more general theory of imaginary geometry as a coupling of a GFF and the UST where each element is measurable with respect to the other.

Berestycki, Laslier and Ray are investigating the relation between these couplings by giving a construction of arbitrarily regularised GFF from a UST with is continuous for the natural topology on trees. In particular this allows to prove convergence (in a weak sense) of the dimer height function to a GFF in new domains.

4.18. **1-2 model.** Grimmett and Li are studying the phase transition of the 1-2 model by observing the different behaviours of the two-edge correlation for different parameters. The exact value of the critical surface is derived rigorously. The proof is based on expressing the two-edge correlation as the Pfaffian of a block Toeplitz matrix, and the holomorphicity of the determinant can be analyzed using Widom's formula.

A similar technique can be applied to the two-point correlation function of the polygon model on the hexagonal lattice, and the critical surface is identified explicitly.

5. Activities

5.1. **Output.** The following publications and preprints have been facilitated by funding through RaG. They are available via

http://www.statslab.cam.ac.uk/~grg/rag-pubs.html

Preprints from this report period

1. Liouville quantum gravity and the Brownian map I: The QLE$(8/3, 0)$ metric, Jason Miller, Scott Sheffield
2. An elementary approach to Gaussian multiplicative chaos, Nathanael Berestycki
3. Critical surface of the 1-2 model, Geoffrey Grimmett and Zhongyang Li
4. An axiomatic characterization of the Brownian map, Jason Miller, Scott Sheffield
5. Liouville quantum gravity spheres as matings of finite-diameter trees, Jason Miller, Scott Sheffield
6. Small-time fluctuations for the bridge of a sub-Riemannian diffusion, Ismael Bailleul, Laurent Mesnager, James Norris
7. Random walks on the random graph, Nathanael Berestycki, Eyal Lubetzky, Yuval Peres, Allan Sly
8. Near-critical spanning forests and renormalization, S. Benoist, L. Dumaz, W. Werner
9. Critical exponents on Fortuin–Kasteleyn weighted planar maps. N. Berestycki, B. Laslier, G. Ray
10. Conformal invariance of dimer heights on isoradial double graphs, Zhongyang Li, *Ann. de l'Instit. Henri Poincaré D*

11. Locality of connective constants, II. Cayley graphs, Geoffrey Grimmett and Zhongyang Li
12. Liouville quantum gravity and the Gaussian free field Nathanael Berestycki, Scott Sheffield, Xin Sun
13. Cutoff for conjugacy-invariant random walks on the permutation group, Nathanael Berestycki, Bati Sengul
14. Locality of connective constants, I. Transitive graphs, Geoffrey Grimmett and Zhongyang Li
15. The Potts and random-cluster models, Geoffrey Grimmett
16. Measure solutions for the Smoluchowski coagulation–diffusion equation, James Norris
17. Cyclic polynomials in two variables, Catherine Bénéteau, Greg Knese, Lukasz Kosiáski, Constanze Liaw, Daniel Seco, Alan Sola, *Transactions of the AMS*
18. Surprise probabilities in Markov chains, James Norris, Yuval Peres, Alex Zhai
19. From Sine kernel to Poisson statistics, Romain Allez, Laure Dumaz, *Electronic J. Probab.* 19 (2014) 1–25

Publications and preprints from previous report periods

1. KPZ formula derived from Liouville heat kernel, N. Berestycki, C. Garban, R. Rhodes, V. Vargas.
2. A consistency estimate for Kac's model of elastic collisions in a dilute gas, J. Norris, *Adv. Appl. Probab.*
3. Random matrices in non-confining potentials, R. Allez, L. Dumaz, *J. Statist. Phys.* 160 (2015) 681–714
4. Tracy–Widom at high temperature, R. Allez, L. Dumaz, *J. Statist. Phys.*
5. Criticality, universality, and isoradiality, G. Grimmett, *Proc. 2014 ICM, Seoul*.
6. Cyclicity in Dirichlet-type spaces and extremal polynomials II: functions on the bidisk, C. Bénéteau, A. Condori, C. Liaw, D. Seco, A. Sola, *Pacific Journal of Mathematics* 276 (2015) 35–58
7. Small-particle limits in a regularized random Laplacian growth model, F. Johansson Viklund, A. Sola, A. Turner, *Commun. Math. Phys.* 334 (2015) 331–366
8. Discrete complex analysis and T-graphs, Z. Li.
9. Conformal invariance of isoradial dimers, Z. Li.
10. Coalescing Brownian flows: a new approach, N. Berestycki, C. Garban, A. Sen.
11. Extendable self-avoiding walks, G. Grimmett, A. Holroyd, Y. Peres, *Ann. Inst. H. Poincaré D* 1 (2014) 61–75
12. Condensation of a two-dimensional random walk and the Wulff crystal, N. Berestycki, A. Yadin

13. The shape of multidimensional Brunet–Derrida particle systems, N. Berestycki, Lee Zhuo Zhao
14. Counting self-avoiding walks, G. Grimmett, Z. Li
15. Percolation of finite clusters and infinite surfaces, G. Grimmett, A. Holroyd, G. Kozma, *Math. Proc. Cam. Phil. Soc. 156* (2014) 263–279
16. Diffusion in planar Liouville quantum gravity, N. Berestycki, *Ann Inst H Poincaré B.*
17. Cyclicity in Dirichlet-type spaces and extremal polynomials, C. Bénéteau, A. Condori, C. Liaw, D. Seco, A. Sola, *Journal d'Analyse Mathématique* 126 (2015) 259–286
18. Expected discrepancy for zeros of random polynomials, I. Pritsker, A. Sola, *Proceedings of the American Mathematical Society* 142 (2014) 4251–4263
19. Elementary examples of Loewner chains generated by densities, A. Sola, *Annales Universitatis Mariae Curie-Sklodowska* A 67 (2013) 83–101.
20. Strict inequalities for connective constants of transitive graphs, G. Grimmett, Z. Li, *SIAM Journal of Discrete Mathematics* 28 (2014), 1306–1333
21. Diffusivity of a random walk on random walks, E. Boissard, S. Cohen, T. Espinasse, J. Norris, *Random Structures & Algorithms.*
22. Uniqueness of infinite homogeneous clusters in 1–2 model, Z. Li, *Electron. Commun. Probab.* 19 (2014), Paper 23, 8 pp.
23. Bounds on connective constants of regular graphs, G. Grimmett, Z. Li, *Combinatorica* 35 (2015) 279–294.
24. Self-avoiding walks and the Fisher transformation, G. Grimmett, Z. Li, *European Journal of Combinatorics* 20 (2013), Paper P47, 14 pp.
25. Influence in product spaces, G. Grimmett, S. Janson, J. Norris, *Advances in Applied Probability* (2016)
26. Critical branching Brownian motion with absorption: particle configurations, J. Berestycki, N. Berestycki, J. Schweinsberg, *Probab. Th. Rel. Fields.*
27. Critical branching Brownian motion with absorption: survival probability, J. Berestycki, N. Berestycki, J. Schweinsberg
28. Three theorems in discrete random geometry, G. Grimmett. *Probability Surveys* 8 (2011) 403–441
29. A small-time coupling between Lambda-coalescents and branching processes, J. Berestycki, N. Berestycki, V. Limic, *Annals of Applied Probability* 24 (2014) 449–475.
30. The genealogy of branching Brownian motion with absorption, J. Berestycki, N. Berestycki, J. Schweinsberg, *Annals of Probability* 41 (2013) 527–618
31. Percolation since Saint-Flour, G. Grimmett, H. Kesten, in *Percolation Theory at Saint-Flour*, Springer, 2012, pages ix–xxvii

32. Cycle structure of the interchange process and representation theory, N. Berestycki, G. Kozma, *Bull. Soc. Math. France.*
33. Galton–Watson trees with vanishing martingale limit, N. Berestycki, N. Gantert, P. Moerters, N. Sidorova, *J. Stat. Phys.* 155 (2014) 737–762.
34. Critical temperature of periodic Ising models, Z. Li, *Communications in Mathematical Physics* 315 (2012) 337–381.
35. Spectral curve of periodic Fisher graphs, Z. Li, *Journal of Mathematical Physics* 55, 123301 (2014)
36. Bond percolation on isoradial graphs, G. Grimmett, I. Manolescu, *Probability Theory and Related Fields* 159 (2014) 273–327.
37. Asymptotic sampling formulae for Lambda-coalescents, J. Berestycki, N. Berestycki, V. Limic, *Ann. Inst. H. Poincaré B*
38. 1–2 model, dimers, and clusters, Z. Li, *Electronic Journal of Probability* 19 (2014) Paper 48.
39. Large scale behaviour of the spatial Lambda–Fleming–Viot process, N. Berestycki, A. M. Etheridge, A. Veber, *Ann. Inst. H. Poincaré B* 49 (2013) 374–401
40. Hastings–Levitov aggregation in the small-particle limit, J. Norris, A. Turner, *Commun. Math. Phys.* (2012) 316, 809–841
41. Weak convergence of the localized disturbance flow to the coalescing Brownian flow, J. Norris, A. Turner, *Annals of Probability* 43 (2015) 935–970
42. Universality for bond percolation in two dimensions, G. Grimmett, I. Manolescu, *Annals of Probability* 41 (2013) 3261–3283.
43. Inhomogeneous bond percolation on square, triangular, and hexagonal lattices, G. Grimmett, I. Manolescu, *Annals of Probability* 41 (2013) 2990–3025.
44. Cluster detection in networks using percolation, G. Grimmett, E. Arias-Castro, *Bernoulli* 19 (2013) 676–719

5.2. **Seminars.** The weekly probability seminar has been lively as always. Details of events may be found at

http://talks.cam.ac.uk/show/archive/9938.

5.3. **Visitors.** Cambridge Probability has received a number of visitors in 2014–15, for short and longer periods, including the many participants at the INI programme described above. The following visitors are connected directly to RaG.

- Wendelin Werner, Dec 2014–Jul 2015
- Scott Sheffield, Jan–Jul 2015
- Bálint Toth, May–Jun 2015
- Yuval Peres, June 2015
- Omer Angel, Jan–Jun 2015
- Bertrand Duplantier, Jan–Jul 2015

- Alan Sola, Jan–Jul 2015
- Zhongyang Li, Apr 2015
- Ander Holroyd, Mar–Apr 2015
- Grégory Miermont, Jan–Feb 2015
- Ofer Zeitouni, Mar–Apr 2015
- Jason Miller, Jan–Jul 2015

5.4. **Visits by members of RaG.** Members of RaG have made numerous visits to other institutions, and have participated in numerous conferences and workshops. Listed here are visits made by *research fellows only*.

5.5. **Scientific visits.**

- November 2014: Research visit to Budapest University of Technology [Ray]
- December 2014: Research visit to ENS, Lyon and Journée Carte. [Ray]
- April 2015: Research visit to ENS, Lyon [Ray].
- Nov 2014: Research visit to Rome [Laslier]
- Apr 2015: Research visit to Bonn [Laslier]
- Sep 2014–May 2015: Several scientific visits to TU and WIAS (Berlin) [Dumaz].
- Feb 2015: Cournot Seminar, École Normale Supérieure [Dumaz].
- Dec 2014: Maths colloquium in Zürich university [Dumaz].

5.6. **Conferences.**

- July 2015: Visit to Oxford for SPA 2015 [Ray].
- Jan–July 2015: Random Geometry programme, Cambridge University [Dumaz].
- June 2015: Conference in honor of Marc Yor, University Paris 6 [Dumaz].
- Dec 2014: *Etats de la recherche en matrices aléatoires*, IHP, Paris [Dumaz].
- Nov 2014: Extrema of Branching Processes and Gaussian Free Fields, TU, Berlin [Dumaz].
- Sep 2014: Clay Research Conference, Oxford university *Advances in Probability: Integrability, Universality and Beyond* [Dumaz].
- September 2014: Conference on *Probabilities on trees and planar graphs*, Banff [Ray]
- September 2014: Research visit to Oxford for conference titled *Advances in Probability: Integrability, universality and beyond.* [Ray]

5.7. **Industrial outreach.** A discussion meeting took place on 19 November 2014 with scientists from IBM.

6. Future activities

Amongst our immediate targets are the following.

- The search for and appointment of postdoctoral fellows within RaG.
- Planning for the next day of industrial outreach.

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