NEW FRONTIERS IN RANDOM GEOMETRY (RaG) EP/103372X/1 REPORT 1/7/13 – 30/6/14

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1. Management Process

The Management Committee (MC) comprises the three investigators and the four members of the external Advisory Board (AB), namely Yuval Peres, Stanislav Smirnov, and Wendelin Werner, plus Balint Tóth who joined the AB last year. The local managers have met weekly during term, and more formally about every two months. The advice of the AB has been sought on a variety of matters including the hiring process. Two members of the AB (Peres, Werner) have spent periods in Cambridge during the period of this report. A meeting of the AB is planned for 3 October 2014.

2. Personnel

Two postdoctoral research fellows were appointed following the advertisement of December 2013, and will take up post on 1 September 2014.

- Benoît Laslier[1], PhD (Université Claude Bernard Lyon I, France), from 1 September 2014 to 31 August 2016.
- Gourab Ray[2], PhD (UBC, Vancouver, Canada), from 1 September 2014 to 31 August 2016.

One postdoc has left the team and another is due to leave later in the summer.

- Alan Sola[3], employed from 1 January 2012 to 31 December 2013.
- Zhongyang Li[4], employed from 1 September 2011 to 31 August 2014.

3. Research Programme (selected)

3.1. **Phase transition in the 1-2 model.** A 1-2 model is a probability measure on the edge-subsets of a hexagonal lattice satisfying the condition that each vertex is incident to 1 or 2 edges. The connected components of a

Date: August 6, 2014.
http://www.statslab.cam.ac.uk/~grg/rag.html.

[1]http://math.univ-lyon1.fr/homes-www/laslier/

[2]http://www.math.ubc.ca/~gourab/

[3]http://www.statslab.cam.ac.uk/~as2221/

[4]http://www.statslab.cam.ac.uk/~zl296/

1-2 model configuration can only be either cycles or self-avoiding path. See Fig. 3.1.

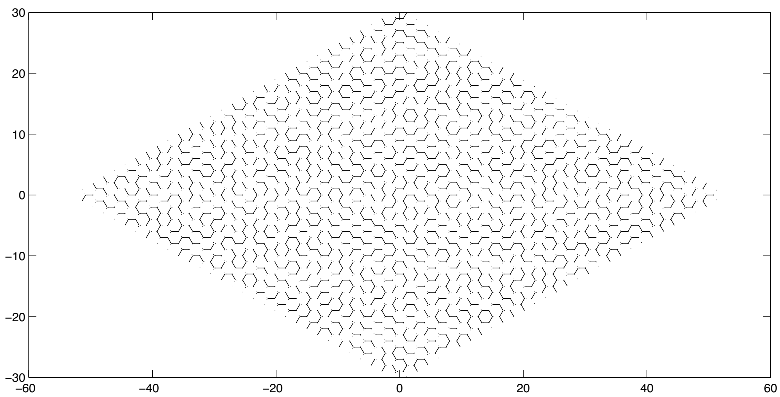

FIGURE 3.1. A 1-2 model configuration

Li has constructed a measure-preserving correspondence between configurations on the hexagonal lattice and dimer configurations on a decorated graph, and has proved a closed form to compute the probability that a finite path appears in the 1-2 model. With the help of the mass transport principle, she proves that almost surely there are no infinite paths in a 1-2 model configuration for any translation-invariant Gibbs measure.

A homogeneous cluster is a connected set of vertices such that each vertex in the set has exactly the same configuration. She has proved that almost surely there is at most 1 infinite homogeneous cluster in a 1-2 model configuration under any translation-invariant Gibbs measure.

She is also able to find a closed form for the edge–edge correlation of the 1-2 model. By investigating the behaviour of such correlations with changing parameters, she expects to prove that there is a sharp phase transition in the 1-2 model, with critical parameter given by the condition that the spectral curve intersect the unit torus at a unique real point.

3.2. **Hastings–Levitov conformal aggregation and related growth models.** Johansson Viklund, Norris, Sola, and Turner are continuing their investigations into the conformal aggregation processes introduced by Hastings and Levitov, and related growth models. Simulations as well as heuristic arguments suggest that, as the feedback parameter α of the model increases, the number of branches present in the random clusters decreases, collapsing into one single branch when $\alpha \to \infty$; one of the current goals is to prove rigorously that this is the case.

Another direction of this project is to study boundary fluctuations of clusters converging to growing disks, as is the case when $\alpha = 0$ or the process is regularized in a strong sense. Comprehensive numerical studies in

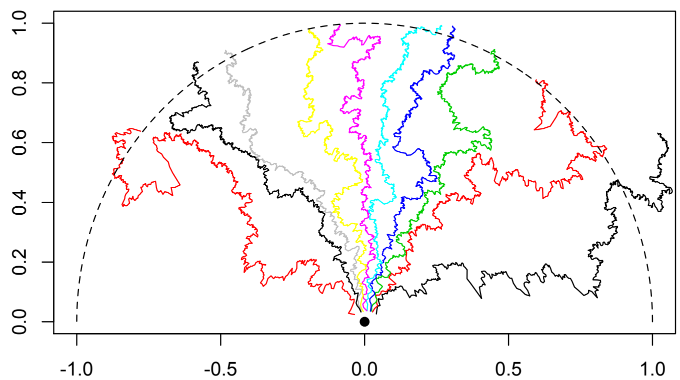

FIGURE 3.2. Loewner curves generated by Dyson's process and semi-circle scaling limit.

the discrete setting lend credence to the idea that, at the phase transition point $\alpha = 1$, the boundary fluctuations in Hastings–Levitov clusters should fall within the Kardar–Parisi–Zhang (KPZ) universality class. As a first step, numerical studies will be carried out to determine whether this is indeed the case, and also what kind of fluctuations should be expected for other values of α.

3.3. **SLE and the Dyson process.** As is well-known in the theory of random matrices, the eigenvalues associated with Brownian motion in certain spaces of $n \times n$-matrices satisfy a system of SDEs of the form

$$d\lambda_k(t) = \sum_{j \neq k} \frac{\beta}{\lambda_j - \lambda_k} dt + dB_k(t), \quad k = 1, \dots, n,$$

for suitably chosen constants $\beta > 0$. By driving Loewner's differential equation by point masses at the positions of the eigenvalues on the real line, one obtains n random interacting curves in the upper half-plane, providing a natural generalization, proposed by Cardy and others, of the usual Schramm–Loewner evolutions.

Allez, Dumaz, and Sola are studying Loewner chains generated by Dyson's Brownian motions, and the geometric properties of the associated curves. In a suitable scaling, the empirical measures associated with Dyson's Brownian motion are known to converge to a deterministic density. One can ask for a similar scaling limit for the collection of Loewner curves, including a detailed analysis of bulk and edge curves and their fluctuations, as the number of particles tends to infinity. It is natural to allow for variable repulsion terms, where $\beta_k \neq \beta_j$, and there are additional connections between such models, and the Laplacian path models of Carleson and Makarov, where the coefficients β in the repulsion terms are determined by the evolution in a manner reminiscent of the Hastings-Levitov model of random aggregation.

3.4. **Dirichlet-type spaces in higher dimensions.** Continuing earlier work concerning Hilbert spaces of analytic functions on the unit disk in $\mathbb{C}$, Bénéteau, Condori, Liaw, Seco, and Sola have investigated Dirichlet spaces in several complex variables, mostly in the setting of the bidisk. The interest here is to determine when a function is cyclic with respect to the coordinate shifts, that is, when the linear span of the polynomial multiples $\{z_1^k z_2^l f\}$ of a function f is dense in the whole space. The higher-dimensional setting exhibits several surprising features: there exist polynomials that do not vanish inside the domain in question but are nevertheless non-cyclic, and it is possible to distinguish between different degrees of cyclicity depending on the sizes and shapes of boundary zero sets.

The ultimate goal of this project is to obtain a complete characterization of cyclic polynomials in Dirichlet-type spaces in higher dimensions in terms of the geometry of their boundary zero sets.

3.5. **Dynamical uniform spanning tree.** Benoist, Dumaz and Werner are currently working on a dynamical model of the scaling limit of the uniform spanning tree on the plane. The uniform spanning tree (see Fig. 3.3) is a fundamental object with important connections to several areas, such as random walks, algorithms, domino tilings, electrical networks, potential theory, percolation etc.

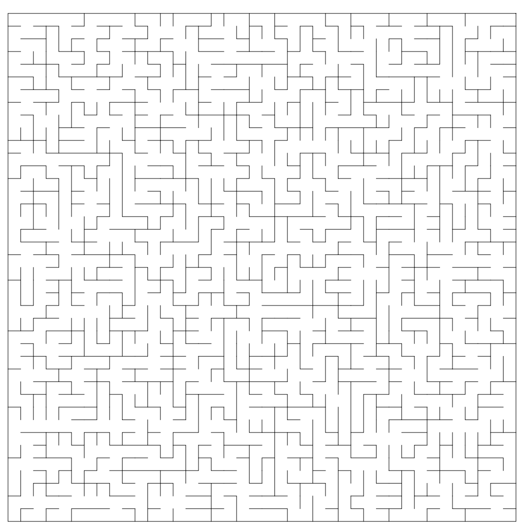

FIGURE 3.3. Realization of a uniform spanning tree in a box of size $n = 20$ with wired boundary conditions.

The uniform spanning tree is one particular instance where one can describe and understand the scaling limit of the "near-critical" model. It is indeed possible to describe the evolution of a dynamical model around the critical point by a natural discrete coalescent Markov process on the state of weighted graph. This should give a geometric non-embedded description of the underlying continuous structure and provide a novel discrete approach to some of the renormalization group arguments of theoretical physics in this very particular case.

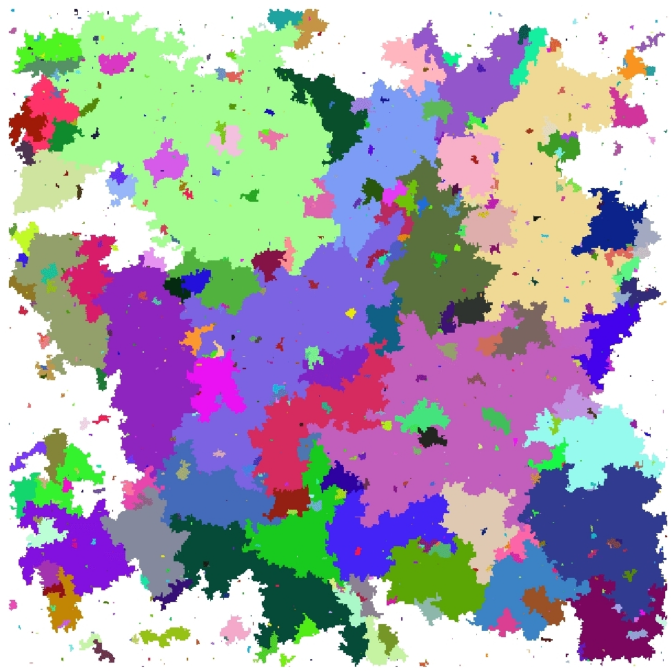

FIGURE 3.4. The near critical spanning tree at a given positive time.

3.6. **Random matrices.** For any $\beta > 0$, consider the probability density function of $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \in \mathbb{R}$ given by:

$$\frac{1}{Z_n^\beta} \exp\left(-\sum_{k=1}^{n} V(\lambda_k)\right) \prod_{j>k} |\lambda_j - \lambda_k|^\beta$$

in which Z_n^β is a normalizing constant. Notice that it corresponds to the joint density of independent random variables in the potential energy V to which a repulsion term coming from $|\lambda_j - \lambda_k|^\beta$ is added. When $\beta = 1, 2$ or 4 and V is the quadratic potential, this is exactly the joint density of eigenvalues for the Gaussian orthogonal, unitary, or symplectic ensembles, G(O/U/S)E in random matrix theory.

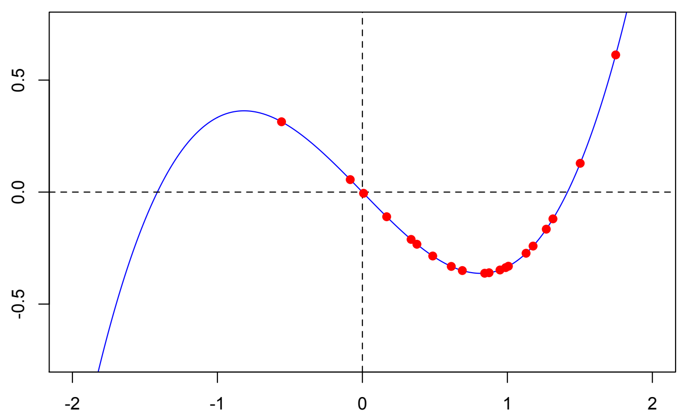

FIGURE 3.5. Simulation of the eigenvalues in a cubic potential.

An important question is to understand how the model behaves when the repulsion term decreases towards zero (i.e., $\beta \to 0$). It is natural to expect that the Wigner statistics will become Poissonian in this limit. In the article "Tracy–Widom at high temperature", Allez and Dumaz investigate the behaviour of the largest eigenvalues (edge case) of famous models of random matrices (containing the Gaussian case) when the temperature goes to infinity. They prove its convergence (when properly rescaled and centered) towards the Gumbel law, an universal law appearing in extreme value theory. In an ongoing project, Allez and Dumaz are looking at the behaviour of the eigenvalues in the bulk. Using the Brownian carousel representation of the eigenvalues in the bulk discovered by Valkó and Virág, they should obtain the convergence of the eigenvalues in the bulk towards a Poisson point process.

In the paper "random matrices in non-confining potentials", Allez and Dumaz introduce and study a new model of random matrices, evolving in non-confining potentials of the type $V(x) = x^3/3 - ax$ instead of the usual confining case which prevents explosions of the eigenvalues (see Fig. 3.5).

Thanks to a dynamical model where the exploding eigenvalues immediately restart, one can define the matrix evolution for general potentials which are non-confining. Moreover one can exhibit an interesting sharp phase transition for the empirical density depending on the parameter a for the limiting spectral density (see Fig. 3.6).

3.7. **Liouville Brownian motion and diffusions evolving in the 2-d quantum gravity.** Liouville quantum gravity, introduced first by Polyakov

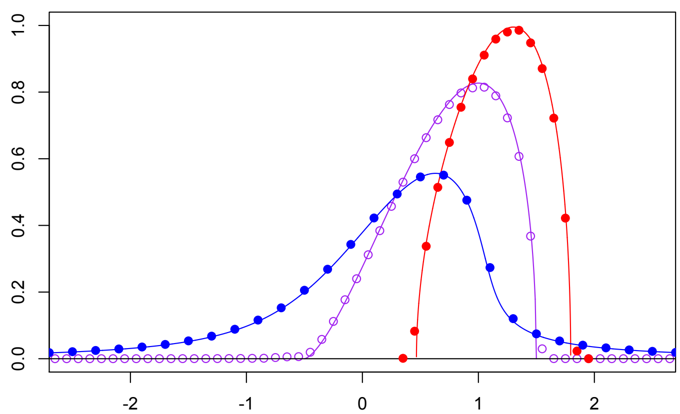

FIGURE 3.6. The sharp transition for the empirical density of the eigenvalues in a cubic potential.

in 1981, has been the object of numerous very recent mathematical studies. It is defined using the celebrated Gaussian free field (GFF), a random surface on a 2-d domain which is the 2-d analogous of the 1-d Brownian motion. Informally, the Liouville measure is given by $\exp(\gamma h(z))dz$ where h represents the Gaussian free field.

In 2013, Berestycki and independently Garban, Rhodes and Vargas were able to define a Brownian motion evolving according to this geometry, called Liouville Brownian motion (LBM). The trace of the LBM is simply given by a 2-d Brownian motion and a time change is performed according to the Liouville measure of the visited points so that the invariant measure of this process corresponds to the Liouville measure.

Berestycki, Dumaz and Jackson are now investigating another diffusion, evolving according to Langevin dynamics in a GFF potential. The invariant measure should be the Liouville measure as in the LBM case, although the trace of the process should be more complex than a 2-d Brownian motion. The trajectories of this process appear to be more natural for geometric perspectives and it should permit one to derive new estimates related to the random Liouville metric.

3.8. **KPZ formula derived from Liouville heat kernel.** In Liouville quantum gravity, the KPZ (for Knizhnik–Polyakov–Zamolodchikov) formula is a far-reaching identity, relating the 'size' (dimension) of a given set $A \subset \mathbb{R}^2$ from the point of view of standard Euclidean geometry, to its counterpart from the point of view of the geometry induced by the "metric tensor"

$$e^{\gamma X(x)} dx^2,$$

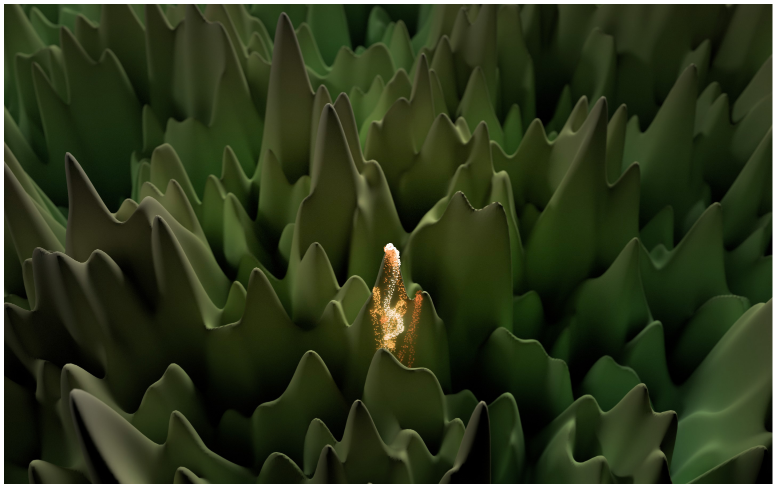

FIGURE 3.7. Trace of Liouville Brownian motion in the Gaussian Free Field landscape

where $X(x)$ is a Gaussian Free Field (GFF) in $\mathbb{R}^2$. The original (physics) formulation of the KPZ formula was made in the context of the light-cone gauge. Rigorous versions of the KPZ formula appeared in a celebrated 2010 paper by Duplantier and Sheffield as well as a paper by Rhodes and Vargas. However, both results rely on an ad hoc formulation of the notion of Hausdorff dimensions, which rely implicitly on Euclidean structure. (The problem is that the usual notion of Hausdorff dimension requires a metric space to work with, and the metric space associated with the above Riemann tensor has not yet been constructed).

Berestycki, Garban, Rhodes and Vargas have proposed a new approach to the KPZ formula which uses the recently introduced Liouville Brownian motion. This is a geometrically intrinsic object, which does not use the underlying Euclidean structure, and so is conceptually more satisfying. The idea is to introduce the Mellin transform

$$M_s^\gamma(x,y) = \int_0^\infty t^{-s} p_t^\gamma(x,y)dt.$$

where $p_t^\gamma(x,y)$ is the Liouville heat kernel. The capacity of a set A is then defined as

$$C_s^\gamma(A) = \sup\Big\{\Big(\int_{A\times A} M_s(x,y)\,\mu(dx)\mu(dy)\Big)^{-1}\Big\}.$$

and in turn the Hausdorff dimension can be defined, as usual, as:

$$\dim_\gamma(A) = \inf\{s \geq 0; C_s^\gamma(A) = 0\}.$$

We are then able to rigorously derive the relation

$$\dim_0(A) = \big(1 + \frac{\gamma^2}{4}\big)\dim_\gamma(A) - \frac{\gamma^2}{4}\dim_\gamma(A)^2$$

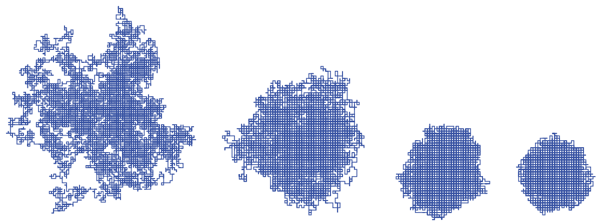

Figure 3.8. Wulff crystal random walk simulations for various values of β.

which is the KPZ identity. This validates (and clarifies) an approach first suggested nonrigorously by David and Bauer. There are interesting heuristic implications for the regularity of the Liouville heat kernel, which we also discuss.

3.9. **Random walk model for Wulff crystal.** Berestycki and Yadin considered a Gibbs probability distribution on random paths in $\mathbb{Z}^d$ where the weight of each path $\omega = (\omega_1, \ldots, \omega_t)$ is proportional to $\exp(-\beta|\partial R_t|)$, where $\beta > 0$ and ∂R_t denote the set of boundary vertices of ω (i.e., those vertices which have been visited by the walk but which have one neighbour that the walk has never visited). This gives a natural random walk construction of the Wulff crystal. A year ago, we proved that for β sufficiently small, the diameter of the path is of order $t^{1/3}$ in dimension $d = 2$, up to logarithmic corrections. With additional work, we have move proved the following much stronger result: with high probability as $t \to \infty$,

(1) In dimension $d = 2$, the diameter or R_t is $t^{1/3}$, up to *constants*, and for all $\beta > 0$.
(2) In higher dimensions, the diameter or R_t is at least $c_1 t^{1/(d+1)}$ and the volume of R_t is at most $c_2 t^{d/(d+1)}$, again for all $\beta > 0$.

3.10. **Mixing time and Ricci curvature of random walks on symmetric group.** Let $n \geq 1$ and fix $C = C_n$ a conjugacy class of the symmetric group. Consider the random walk on S_n induced by C: that is, the random process $X_t = \gamma_1 \ldots \gamma_t$ where γ_i are iid uniform on C and $t = 0, 1, \ldots$. An old question, going back to Diaconis and Shahshahani, concerns the mixing time of this process. The conjecture is that the mixing time is $(1/|C|)n \log n$, where $|C|$ is the number of non-fixed points of any permutation $\gamma \in C$, as soon as $|C| = o(n)$. This is a conjecture with a long history. It was first established by Diaconis and Shahshahani (1981) for $|C| = 2$, and subsequently for k-cycles with $k \leq 7$ by Roussel (2000), and then more recently by Berestycki, Schramm and Zeitouni (2011) for k-cycles with k finite and fixed. Bob Hough, using representation theory, extended this result to arbitrary $k = k(n)$ with $k = o(n)$.

Berestycki and Sengul have established the full conjecture. The proof goes via an interesting discrete Ricci curvature argument (which is simply a reformulation of Bubley and Dyer's path coupling method). Recall the following definitions. Let $t > 0$. The curvature between two points $x \neq x' \in S$ is given by

$$\kappa_t(x, x') := 1 - \frac{W_1(X_t^x, X_t^{x'})}{d(x, x')}$$

where X_t^x and $X_t^{x'}$ denote Markov chains started from x and x' respectively. The curvature of the random walk is by definition equal to

$$\kappa_t := \inf_{x \neq x'} \kappa_t(x, x').$$

We are able to show that (say in the case of random transpositions or $|C| = 2$), if $t = cn/2$ for some constant $c > 0$, then asymptotically the curvature is zero for $c \leq 1$, while it is strictly positive for $c > 1$. What is crucial for the argument is that for c large, the asymptotic curvature grows very close to 1, sufficiently quickly.

3.11. **Self-avoiding walks.** Grimmett and Li have continued their project to understand properties of connective constants for counts of self-avoiding walks (SAWs). Particular attention has been given to the problem of 'locality': if two graphs agree on a ball of large radius, then are their two connective constants close in value? Progress has been made and an article is in preparation.

Grimmett, Holroyd, and Peres have completed their project on extendable SAWs. They prove that the connective constants for regular SAWs, and also for forward-extendable, backward-extendable, and doubly-extendable walks are equal for any quasi-transitive, doubly-connected, directed graph.

4. Activities

4.1. **Output.** The following publications and preprints have been facilitated by funding through RaG. They are available via

http://www.statslab.cam.ac.uk/~grg/rag-pubs.html

Preprints from this report period

1. KPZ formula derived from Liouville heat kernel, N. Berestycki, C. Garban, R. Rhodes, V. Vargas.
2. A consistency estimate for Kac's model of elastic collisions in a dilute gas, J. Norris.
3. Random matrices in non-confining potentials, R. Allez, L. Dumaz.
4. Tracy–Widom at high temperature, R. Allez, L. Dumaz, *J. Statist. Phys.*
5. Criticality, universality, and isoradiality, G. Grimmett, *Proc. 2014 ICM, Seoul.*
6. Cyclicity in Dirichlet-type spaces and extremal polynomials II: functions on the bidisk, C. Bénéteau, A. Condori, C. Liaw, D. Seco, A. Sola.

7. Small-particle limits in a regularized random Laplacian growth model, F. Johansson Viklund, A. Sola, A. Turner, *Commun. Math. Phys.*
8. Locality of the connective constant on Cayley graphs, Z. Li.
9. Discrete complex analysis and T-graphs, Z. Li.
10. Conformal invariance of isoradial dimers, Z. Li.
11. Coalescing Brownian flows: a new approach, N. Berestycki, C. Garban, A. Sen.
12. Extendable self-avoiding walks, G. Grimmett, A. Holroyd, Y. Peres, *Ann. Inst. H. Poincaré D* 1 (2014) 61–75

Publications and preprints from previous report periods

1. Condensation of a two-dimensional random walk and the Wulff crystal, N. Berestycki, A. Yadin
2. The shape of multidimensional Brunet–Derrida particle systems, N. Berestycki, Lee Zhuo Zhao
3. Counting self-avoiding walks, G. Grimmett, Z. Li
4. Percolation of finite clusters and infinite surfaces, G. Grimmett, A. Holroyd, G. Kozma, *Math. Proc. Cam. Phil. Soc. 156* (2014) 263–279
5. Diffusion in planar Liouville quantum gravity, N. Berestycki, *Ann Inst H Poincaré B.*
6. Cyclicity in Dirichlet-type spaces and extremal polynomials, C. Bénéteau, A. Condori, C. Liaw, D. Seco, A. Sola, *Journal d'Analyse Mathématique*
7. Expected discrepancy for zeros of random polynomials, I. Pritsker, A. Sola, *Proceedings of the American Mathematical Society*
8. Elementary examples of Loewner chains generated by densities, A. Sola, *Annales Universitatis Mariae Curie-Sklodowska* A 67 (2013) 83–101.
9. Strict inequalities for connective constants of transitive graphs, G. Grimmett, Z. Li, *SIAM Journal of Discrete Mathematics.*
10. Diffusivity of a random walk on random walks, E. Boissard, S. Cohen, T. Espinasse, J. Norris, *Random Structures & Algorithms.*
11. Uniqueness of infinite homogeneous clusters in 1–2 model, Z. Li
12. Bounds on connective constants of regular graphs, G. Grimmett, Z. Li, *Combinatorica.*
13. Self-avoiding walks and the Fisher transformation, G. Grimmett, Z. Li, *European Journal of Combinatorics* 20 (2013), Paper P47, 14 pp.
14. Influences in product spaces: BKKKL re-revisited, G. Grimmett, S. Janson, J. Norris, *arXiv*:1207.1780
15. Critical branching Brownian motion with absorption: particle configurations, J. Berestycki, N. Berestycki, J. Schweinsberg, *Probab. Th. Rel. Fields.*
16. Critical branching Brownian motion with absorption: survival probability, J. Berestycki, N. Berestycki, J. Schweinsberg

17. Three theorems in discrete random geometry, G. Grimmett. *Probability Surveys* 8 (2011) 403–441
18. A small-time coupling between Lambda-coalescents and branching processes, J. Berestycki, N. Berestycki, V. Limic, *Annals of Applied Probability* 24 (2014) 449–475.
19. The genealogy of branching Brownian motion with absorption, J. Berestycki, N. Berestycki, J. Schweinsberg, *Annals of Probability* 41 (2013) 527–618
20. Percolation since Saint-Flour, G. Grimmett, H. Kesten, in *Percolation Theory at Saint-Flour*, Springer, 2012, pages ix–xxvii
21. Cycle structure of the interchange process and representation theory, N. Berestycki, G. Kozma, *Bull. Soc. Math. France.*
22. Galton–Watson trees with vanishing martingale limit, N. Berestycki, N. Gantert, P. Moerters, N. Sidorova, *J. Stat. Phys.* 155 (2014) 737–762.
23. Critical temperature of periodic Ising models, Z. Li, *Communications in Mathematical Physics* 315 (2012) 337–381.
24. Spectral curve of periodic Fisher graphs, Z. Li
25. Bond percolation on isoradial graphs, G. Grimmett, I. Manolescu, *Probability Theory and Related Fields* 159 (2014) 273–327.
26. Asymptotic sampling formulae for Lambda-coalescents, J. Berestycki, N. Berestycki, V. Limic, *Ann. Inst. H. Poincaré B*
27. 1–2 model, dimers, and clusters, Z. Li, *Electronic Journal of Probability* 19 (2014) Paper 48.
28. Large scale behaviour of the spatial Lambda–Fleming–Viot process, N. Berestycki, A. M. Etheridge, A. Veber, *Ann. Inst. H. Poincaré B* 49 (2013) 374–401
29. Hastings-Levitov aggregation in the small-particle limit, J. Norris, A. Turner, *Commun. Math. Phys.* (2012) 316, 809–841
30. Weak convergence of the localized disturbance flow to the coalescing Brownian flow, J. Norris, A. Turner, *Annals of Probability*
31. Universality for bond percolation in two dimensions, G. Grimmett, I. Manolescu, *Annals of Probability* 41 (2013) 3261–3283.
32. Inhomogeneous bond percolation on square, triangular, and hexagonal lattices, G. Grimmett, I. Manolescu, *Annals of Probability* 41 (2013) 2990–3025.
33. Cluster detection in networks using percolation, G. Grimmett, E. Arias-Castro, *Bernoulli* 19 (2013) 676–719

4.2. **Seminars.** The weekly probability seminar has been lively as always. Details of events may be found at

http://talks.cam.ac.uk/show/archive/9938.

4.3. **Visitors.** Cambridge Probability has received a number of visitors in 2012–13, for short and longer periods. The following individuals are connected directly to RaG.

- Håkan Hedenmalm, November 2013
- Christophe Garban, May 2014
- Serge Cohen, June 2014

The following have visited with non-RaG support.

- Jason Miller
- Romain Allez
- Thierry Lévy, October 2014
- Noam Berger, November 2014
- Yuval Peres, April 2014
- Rémi Rhodes, May 2014
- Vincent Vargas, May 2014

4.4. **Visits by members of RaG.** Members of RaG have made numerous visits to other institutions, and have participated in numerous conferences and workshops. Listed here are visits made by research fellows.

4.5. **Research visits.**

- Aug 2013: Invariant subspaces of the shift operator, Centre de Recherches Mathématiques, Montréal, Canada [Sola]
- Dec 2013: University of Bristol, UK [Dumaz]
- Dec 2013: Berlin/Oxford meeting "rough paths", WIAS, Berlin [Dumaz]
- Mar 2014: Southeastern Analysis Meeting; Clemson, South Carolina, USA [Sola]
- Mar 2014: AMS Sectional Meeting, Knoxville, Tennessee, USA [Sola]
- Mar 2014: Baylor University; Waco, Texas, USA [Sola]
- Mar 2014: University of Tennessee; Knoxville, USA [Sola]
- Apr 2014: Probability, Analysis and Dynamics in Bristol, UK [Dumaz]
- Apr 2014: Probability, Analysis and Dynamics; Bristol [Sola]
- Apr 2014: Berlin, TU [Dumaz]
- Jun–Jul 2014: Berlin, TU [Dumaz]
- Jun 2014: School and Workshop on Random Interacting Systems, Bath [Li]
- Jul 2014: Two-Dimensional Random Critical Models MAC2 Workshop, Paris [Li]
- Aug 2014: IdeaLab 2014: Program for Early Career Researchers, ICERM, Brown University [Li]

5. Future activities

Amongst our immediate targets are the following.

- Planning is underway for the 6 month programme at the Isaac Newton Institute that will take place in the first half of 2015, with N. Berestycki as principal organizer.
- The search for and appointment of postdoctoral fellows within RaG.
- Planning for the next day of industrial outreach.

Statistical Laboratory, Centre for Mathematical Science, University of Cambridge, Wilberforce Road, Cambridge CB3 0WB