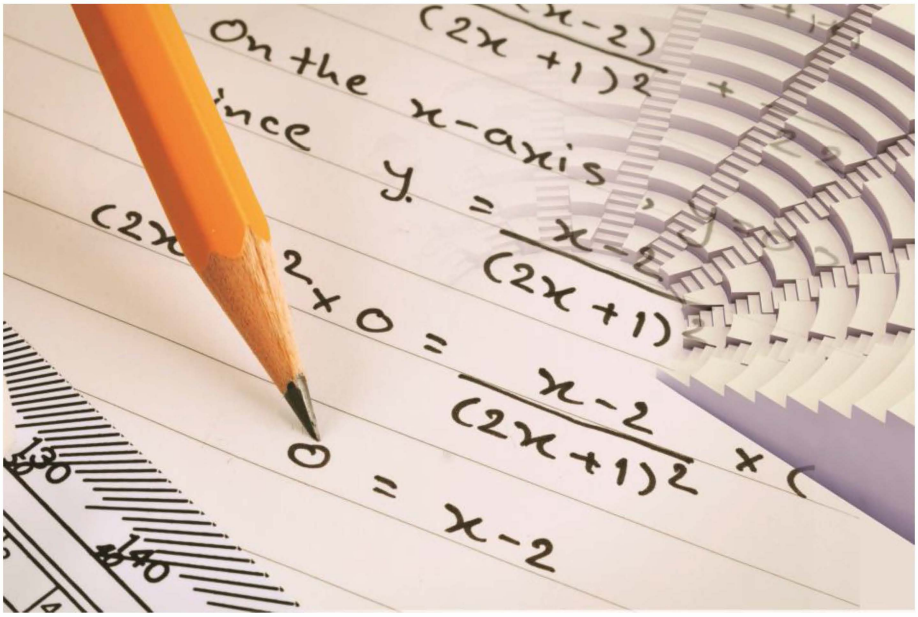
IN-DEPTH ANALYSIS
for the AFCO Committee

European Parliament

The Composition of the European Parliament



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CONSTITUTIONAL AFFAIRS

THE COMPOSITION OF THE EUROPEAN PARLIAMENT

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LINKING THE PERMANENT SYSTEM OF THE DISTRIBUTION OF SEATS IN THE EUROPEAN PARLIAMENT WITH THE DOUBLE-MAJORITY VOTING IN THE COUNCIL OF MINISTERS

Prof. Dr. Friedrich PUKELSHEIM and Prof. Geoffrey GRIMMETT

KEY FINDINGS

- This paper proposes the adoption of the **Cambridge Compromise** which is a transparent allocation method for determining the composition of the European Parliament (EP). The method is responsive to population changes and impartial to politics as well as objective, fair and durable. An alternative method is the Cambridge Compromise with power-adjusted populations, called **Power Compromise** for short. The latter is more flexible with respect to the 2014 allocation, but at the cost of some transparency, arising through the involvement of an additional power parameter.
- The EP composition must obey the operational principle of **degressive proportionality** whereby the Member States' representation ratios, that is, the population figure divided by the number of seats before rounding, are decreasing when passing from a more populous Member State to a less populous Member State.
- The same **population figures** should be used for the EP composition and for the qualified majority voting rule in the Council.
- The **Jagiellonian Compromise** is a qualified majority voting rule for the Council providing a more principled method than the current double-majority voting rule.
- **Our principal recommendation** is twofold: the adoption of the Cambridge Compromise, and the (independent) adoption of the Jagiellonian Compromise. While each of these two recommendations stands alone, their coordinated adoption as a pair will bring a balance to the dual structure of Parliament and Council.
- **Seat allocation tables for the 2019 EP** are shown for a Union including the UK. For a Union without the UK, three scenarios are adjoined: with 751 EP seats (maximum size), with 678 EP seats (without the 73 UK seats), and with 723 EP seats. The last is the smallest EP size for which the Power Compromise assigns to every Member State at least as many seats as in its 2014 allocation.

1. TWO PROPOSALS FOR THE ALLOCATION OF EP SEATS

The European Council Decision of 2013 establishing the composition of the EP states in its Art.4 that the decision shall be revised with the aim of establishing a system to allocate the seats between Member States in an objective, fair, durable and transparent way:[1]

> This Decision shall be revised sufficiently far in advance of the beginning of the 2019-2024 parliamentary term on the basis of an initiative of the European Parliament presented before the end of 2016 with the aim of establishing a system which in future will make it possible, before each fresh election to the European Parliament, to allocate the seats between Member States in an objective, fair, durable and transparent way, translating the principle of degressive proportionality

[1] OJ L 181, 29.6.2013, pp. 57–58 (www.uni-augsburg.de/bazi/OJ/2013L181p57.pdf).

as laid down in Article 1, taking account of any change in their number and demographic trends in their population, as duly ascertained thus respecting the overall balance of the institutional system as laid down in the Treaties.

We propose two allocation methods that satisfy the requirements well: the **Cambridge Compromise**,[2] and the **Power Compromise**.[3] The operational details of the two procedures are presented first. Thereafter follow assessments of the methods' merits from the viewpoint of primary and secondary Union law.

Cambridge compromise

The Cambridge Compromise may be paraphrased as follows:

- Every Member State is assigned a common number of *base seats*. The remaining seats are allocated proportionately to population figures, using the *divisor* method with upward rounding and subject to a maximum allocation. In the case of the current EP, the number of base seats is 5, so that the least populous Member State finishes with 6 seats, and the proportional allocation is capped in order to produce a maximum of 96 seats.

For instance, in Table 1 the Cambridge Compromise proceeds as follows:

- Every Member State is assigned 5 base seats, plus one seat per 846 000 citizens or part thereof, with a maximum cap of 96 seats.

The currently smallest State, Malta, ends with a final tally of 6 seats (with only 4 base seats, Malta would finish with 5 seats; with 6 base seats, it would finish with 7 seats). The initial assignment of 5 base seats to each of the 28 Member States utilises a total of 140 seats, leaving 611 seats for the proportional allocation.

The remaining 611 seats are allocated using the divisor method with upward rounding. The allocation key to be determined is the so-called divisor (846 000).[4] For example, when dividing the Austrian population 8 711 500 by 846 000, the resulting quotient is 10.3. This quotient is rounded upwards to obtain the number of proportionality seats (11). Thus Austria is allocated a total of 16 seats: 5 base seats plus 11 proportionality seats. A similar calculation is carried out for the other Member States. In the case of Germany, the quotient 5 + 97.003 = 102.003 exceeds the capping and is replaced by the 96 seat maximum.

Tables 1, 3, 5 illustrate the Cambridge Compromise, with 5 base seats in each case.

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[2] G.R. Grimmett / J.-F. Laslier / F. Pukelsheim / V. Ramírez González / R. Rose / W. Słomczyński / M. Zachariasen / K. Życzkowski: *The Allocation Between the EU Member States of the Seats in the European Parliament – Cambridge Compromise. Note.* European Parliament, Directorate-General for Internal Policies, Policy Department C: Citizen's Rights and Constitutional Affairs, PE 432.760, March 2011 (www.uni-augsburg.de/pukelsheim/2011f.pdf).

[3] G.R. Grimmett / K.-F. Oelbermann / F. Pukelsheim: A power-weighted variant of the EU27 Cambridge Compromise. *Mathematical Social Sciences* 63, 2012, pp. 136-140 (www.uni-augsburg.de/pukelsheim/2012a.pdf). – Power adjustments have been proposed in the literature since quite some time. See, for example: (1) H. Theil: The desired political entropy. *American Political Science Review* 63, 1969, pp. 521-525. (2) H. Theil / L. Schrage: The apportionment problem and the European Parliament. *European Economic Review* 9, 1977, pp. 247-263. (3) A. Moberg: The voting system in the Council of the European Union. The balance between large and small countries. *Scandinavian Political Studies* 21, 1998, pp. 347-365. Reprinted in: A. Moberg: *The Weight of Nations. Four Papers on the Institutional Negotiations in the EU 1996–2007*. Malmö, 2014 (www.uni-augsburg.de/ bazi/Moberg2014.pdf). (4) F. Arndt: Distribution of seats at the European Parliament – Democratic political equality, protection of diversity and the enlargement process. In: *The Emerging Constitutional Law of the European Union – German and Polish Perspectives, Beiträge zum ausländischen öffentlichen Recht und Völkerrecht* 163, 2003, pp. 93-115. (5) F. Arndt: Ausrechnen statt aushandeln: Rationalitätsgewinne durch ein formalisiertes Modell für die Bestimmung der Zusammensetzung des Europäischen Parlaments (with an English summary). *Zeitschrift für ausländisches öffentliches Recht und Völkerrecht – Heidelberg Journal of International Law* 68, 2008, pp. 247-279.

[4] An efficient way to determine the divisor is discussed, for example, in Sect. 4.6 of F. Pukelsheim: *Proportional Representation – Apportionment Methods and Their Applications. With a Foreword by Andrew Duff MEP.* Cham, 2014 (www.uni-augsburg.de/pukelsheim/2014a-FrontMatter.pdf).

Power Compromise

The Power Compromise is a variant of the Cambridge Compromise that refers not to original population figures, but to power-adjusted population units. It may be worded as follows:

- Every Member State is assigned a common number of *base seats*. The remaining seats are allocated proportionately to **adjusted population units** (that is, the population figures raised to a common *power*) using the *divisor* method with upward rounding. In the case of the current EP, the number of base seats, the power, and the divisor are determined so that the least populous Member State is allocated 6 seats, the most populous is allocated just 96 seats, and the size of the EP is 751.

For instance, in Table 2 the method proceeds as follows:

- Every Member State is assigned 5 base seats, plus one seat per 254 500 adjusted population units or part thereof, where the adjusted units are obtained by raising the population figures to the power 0.93.

The power 0.93 is determined so that the most populous Member State is allocated just 96 seats.[5] The divisor 254 500 is determined so that the 28 Member States altogether are allocated 751 seats. The base seat assignment in Table 2 is identical to that of Table 1, namely 5. The presence of these allocation keys is dictated by the goal to satisfy the requirements of primary and secondary Union law as discussed below.

Tables 2, 4, 6, 7 illustrate the Power Compromise. The number of base seats varies, with Tables 2 and 6 using 5 base seats, and Tables 4 and 7 using 4 base seats.

In all seven Tables, the number of seats remaining for proportional allocation depends on the numbers of base seats (4 or 5) and of Member States (28 or 27), and on the EP size under consideration (751 or 678 or 723 seats).

Assessment by primary Union law

Primary Union Law, as set forth in the Treaty on European Union (TEU), lays conditions upon possible allocation methods.[6] Of particular relevance are the following requirements which we rearrange and paraphrase to ease cross-referencing in this briefing.

1. Citizens are directly represented in the EP (Art. 10(2) TEU).
2. The EP shall be composed of representatives of the Union's citizens (Art. 14(2) TEU).
3. Representation of citizens shall be degressively proportional (Art. 14(2) TEU).
4. The size of the EP shall not exceed 751 seats (Art. 14(2) TEU).
5. Every Member State shall be allocated at least 6 seats (Art. 14(2) TEU).
6. Every Member State shall be allocated at most 96 seats (Art. 14(2) TEU).

There is a potential ambiguity in the term "Member State" over whether it refers to government or to people. When "Member State" is interpreted to mean "government", Art. 10(2) TEU decrees that the appropriate representative body is the European Council and the Council, rather than the EP. As far as the composition of the EP is concerned, the term "Member State" means people, that is, a Member State's citizenry.

The Cambridge Compromise complies perfectly well with requirements 1 and 2. The initial assignment of base seats to a Member State secures the representation of its citizenry as a whole. The subsequent proportional allocation of the remaining seats represents the

[5] Generally there are several powers that guarantee 96 seats for the most populous Member States. For the data in Table 2 five powers serve the purpose: 0.93, 0.932, 0.935, 0.937, 0.94. The smallest power (0.93) conforms best to the principle of degressive proportionality. Powers larger than 0.93 involve the transfer of a seat from a less populous Member State to a more populous Member State: for 0.932 from Lithuania to Poland, for 0.935 from Poland to United Kingdom, for 0.937 from the Netherlands to Poland, and for 0.94 from Portugal to Italy. The selection of the smallest power is expressed by saying that the most populous Member State realizes "just" 96 seats. An efficient algorithm to determine the power is described in Grimmett et al., note 3; see also Sect. 12.8 of Pukelsheim, note 4. The algorithm is implemented in the free software Bazi (www.uni-augsburg.de/bazi/).

[6] OJ C 326, 26.10.2012, pp. 13–45 (http://www.uni-augsburg.de/bazi/OJ/2012C326p13.pdf).

citizens as individuals. Degressive proportionality (requirement 3) will be dealt with in greater detail in Section 3 below. Requirements 4–6 are numerical restrictions which are clearly fulfilled.

In contrast, it is harder to fit the Power Compromise within the framework of requirements 1 and 2. Requirement 1 calls for a direct representation of citizens. At the stage of proportional allocation, the Power Compromise allocates the remaining seats in a manner proportional to "population units" which are a power of the population figures. That is, direct population figures are replaced by transformed quantities. While the invocation of a transformation signals a deviation from the principle of direct representation, it may be justified by the principle of degressive representation.

There is a tension between the principles of direct representation (requirement 1) and of degressive representation (requirement 3), each of which is stipulated by primary Union law. Requirement 1 supports an allocation proportional to population, whereas requirement 3 favours an allocation giving some priority to smaller states. The Cambridge Compromise may be viewed as prioritizing direct representation over degressivity. In contrast, the Power Compromise allows greater degressivity, but at some cost to direct representation.

The two methods yield seat allocations that become increasingly identical as the power parameter becomes closer to unity. They coincide when the power equals unity, and this could occur in the future. For instance, if in Table 2 the German population were to decline by five million to 77 064 489 (with other populations unchanged), the Power Compromise yields power 1 and is hence identical to the Cambridge Compromise. This possibility of future coincidence of the two methods mitigates the marginal disregard by the Power Compromise of the principle of direct representation.

Assessment by secondary Union law

The extended deliberations of the EP on its composition have led to detailed specifications that have found their way into Art. 1 of the 2013 European Council Decision mentioned above:

7. Any more populous Member State shall be allocated at least as many seats as any less populous Member State.
8. The least populous Member State shall be allocated 6 seats.
9. The most populous Member State shall be allocated 96 seats.
10. The principle of degressive proportionality shall require decreasing representation ratios when passing from a more populous Member State to a less populous Member State, where the representation ratio of a Member State is defined to be the ratio of its population figure relative to its number of seats before rounding.

These requirements are satisfied by the Cambridge Compromise as well as by the Power Compromise. Tables 1–7 include columns labelled "Repr. Ratio" in witness of degressive proportionality (requirement 10).

Requirement 9 insists on allocations that achieve the maximum of 96 seats. The requirement can be met with the current data, but it has the potential to breed conflict. For instance, if in Table 1 the German population were to be seven million fewer, namely 75 064 489 (with other population figures unchanged), the Cambridge Compromise would allocate 94 seats to Germany. A forced allocation of 96 seats would violate degressive proportionality (requirement 3). Council has reasoned that requirements 8 and 9 reflect as closely as possible the spectrum of populations-sizes of Member States, but this reasoning is invalid in general.

The Cambridge Compromise results in an increased bunching of Member States near the maximum of 96 seats (as permitted by requirement 9). While this does not threaten degressive proportionality, it disadvantages citizens of any capped Member State relative to

those of other large States. In contrast, the seat allocation of the Power Compromise is smoother across its entire range, and in particular at the upper end.

2. FURTHER PROPOSALS

In the literature one can find other proposals on how to determine the composition of the EP. The topic received renewed attention during the 2003 Convention on the Future of Europe. Since then the Treaty of Lisbon entered into force and the 2013 European Council decision decreed further details. The parts that are relevant for the composition of the EP are enunciated in requirements 1–10 above. Of course past literature could not anticipate these later requirements. Therefore one has to be careful when relating past sample allocations with current settings.

Some authors proposed to refer the allocation not only to population figures, but also to gross domestic product.[7] We believe that this reference base can no longer be upheld in view of requirements 1–2. The Members of the EP represent human beings, not economic performance.

Other approaches make use of adjusted population units as does the Power Compromise, but in a different fashion. Rather than raising a population figure N to a power c (in Table 2: $N^c = N^{0.93}$) they advocate other transformations. The parabolic method subtracts a multiple of the squared population: $N - cN^2$; the hyperbolic method subtracts a multiple of the inverse population $N - c/N$.[8] The coefficient c is contingent on the type of transformation chosen and must be calculated from the given population figures.

The challenge is not mathematical multitude, but constitutional adequacy. The more sophisticated the adjustment function, the harder is the proof of its closeness to the principles of primary and secondary Union law, and the more opaque is the method. Moreover the Cambridge Compromise and the Power Compromise yield seat allocations which for many data sets sandwich the allocations of other methods. For this reason we restrict our briefing to these two methods which can be firmly justified by the legal principles of the Union.

3. DEGRESSIVE PROPORTIONALITY

The oxymoron of "degressive proportionality" has a long tradition in the debates of the EP. One may have degressive representation, proportional representation, or progressive representation just as one may have degressive taxation, proportional taxation, or progressive taxation. "Degressive proportionality", however, is a paradoxical concept. The notion is presented as a manifestation of solidarity in a 2007 text adopted by the EP:[9]

- The more populous States agree to be under-represented in order to allow the less populous States to be represented better.

The 2007 resolution included an attempted specification of degressive proportionality, which has since been recognized as a potential contradiction. Meanwhile the abstract principle of degressive proportionality (requirement 3) has been given a concrete specification capable of practical implementation (requirement 10).

The implementation of degressive proportionality is challenging because the meanings of "citizens" in requirements 2 and 3 differ significantly even though both requirements appear

[7] C. Bertini / G. Gambarelli / I. Stach: Apportionment strategies for the European Parliament. *Homo Oeconomicus* 22, 2005, pp. 589-604.

[8] See, for example, V. Ramírez / A. Palomares / M.L. Márquez: Degressively proportional methods for the allotment of the European Parliament Seats amongst the EU Member States. In: B. Simeone / F. Pukelsheim (Editors): *Mathematics and Democracy – Recent Advances in Voting Systems and Collective Choice.* Berlin, 2006, pp. 205–220. Or W. Słomczyński / K. Życzkowski: Mathematical aspects of degressive proportionality. *Mathematical Social Sciences* 63, 2012, pp. 94-101. Note that the latter authors introduce in their equation (10) a "*base* + *power*" allocation method which is close to, but not identical with the Power Compromise of the present briefing.

[9] OJ C 227 E, 4.9.2008, pp. 132-138, no. 5 (http://www.uni-augsburg.de/bazi/OJ/2008C227Ep132.pdf).

in the same section of Art. 14 TEU. Reference to "Union citizens" (requirement 2) appears to place all citizens on an equal footing. However, the principle of degressive proportionality (requirement 3) discriminates the "citizens" by Member States. The citizens of more populous Member States agree to be under-represented in order to allow the citizens of less populous Member States to be represented better.

The Cambridge Compromise achieves degressive proportionality without distorting the meaning of "citizens" beyond the minimum. It does so in each of its two stages. The first stage of assigning base seats treats all Member States alike. This is extremely degressive since it neglects population figures entirely. The second stage of proportional allocation of the remaining seats embodies a mild form of degressivity through the use of *upward* rounding. Upward rounding is known to introduce a slight bias in favour of the less populous Member States.[10] This type of bias reinforces the effect of degressive proportionality.

In contrast the Power Compromise achieves degressive proportionality by interpreting the term "citizens" in a rather broad sense. The method replaces lucent population figures – which count concrete citizens – by arcane population units – which measure abstract units. In Table 2, Malta's population of 434 403 citizens is transformed to 175 082 population units. Does this mean that only forty percent of the citizenry is accounted for?[11] Or forty percent of each citizen? Neither interpretation seems profitable; the interim power-adjustments remain obscure. Their justification lies in the final result which thereby achieves a higher degree of degressivity.

4. POPULATION CRITERION

How does one determine the number of citizens in a Member State? Whom does one count? These questions are fundamental to requirements 1–3. They demand quick practical answers, while also inviting more principled reflections.

Available population figures are those decreed annually by Council Decision for the qualified majority voting (QMV) rule in the Council of Ministers. The figures for the calendar year 2017 provide the input data for our tables.[12] The corresponding columns are labelled "QMV2017".

Since the Council and the EP are constitutional organs of the European Union with joint governance responsibility, we are compelled to the recommendation that the two institutions employ the same population data.

We return to the questions above. Presumably everybody would endorse the aim that

- Every individual who qualifies as a "citizen" in the sense of requirements 1–3 shall be counted at least once and at most once. That is, he or she shall be counted exactly once.

This modest aim is challenging to achieve, considering that the data are gathered by a host of domestic statistical offices before being communicated to Eurostat. To this end it seems efficient and appropriate to continue to base all population figures on the internationally (UN) approved notion of "total resident population".

5. INTER-INSTITUTIONAL BALANCE

Population figures feature not only in the seat allocation of the EP, but also in the qualified majority voting rule of the Council. A group of Member States constitutes a qualified majority provided the group consists of at least 55 percent of all Member States (that is, at

[10] See Chap. 7 in Pukelsheim, note 4. – The counterpart of the divisor method with upward rounding is the divisor method with downward rounding. The latter is also known as the D'Hondt method. The D'Hondt method is biased in favour of stronger parties at the expense of weaker parties, which explains much of its widespread usage.

[11] 175 082 is 40.3 percent of 434 403.

[12] OJ L 348, 12.12.2016, pp. 27–29 (www.uni-augsburg.de/bazi/OJ/2016L348p27.pdf). See www.uni-augsburg.de/bazi/literature.html#QMV-Pop for links to the QMV population figures prior to 2017.

least 16 Member States out of 28) and the Member States in the group represent at least 65 percent of the Union's population. This decision rule is known as the **double-majority voting rule**.

There is an established mathematical framework for the evaluation of fairness within a qualified majority voting system. It focuses on two quantities: the **decision power of a Member State**, and the **indirect decision power of a Union citizen.**[13] The indirect decision power of a Union citizen is determined from the decision power of his or her Member State by dividing the latter by the square root of the Member State's population figure. It transpires that the double-majority voting rule leads to an uneven distribution of the indirect decision powers of the Union citizens. Citizens from middle-sized Member States have slightly less power than citizens from Member States that are smaller (due to the 55 percent clause) or larger (due to the 65 percent clause).[14]

The **Jagiellonian Compromise** is a qualified majority voting rule that awards all Union citizens an equal indirect decision power. It assigns to each Member State a voting weight that is defined to be the square root of its population figure. Furthermore it introduces a **quota**. The Jagiellonian quota is defined to be the average of the square root of the population total and the sum of the voting weights.[15] According to the Jagiellonian Compromise a group of Member States qualifies as a majority provided the sum of their voting weights meets or exceeds the quota.

In terms of conceptual analysis the Jagiellonian Compromise is unique in its transparency. It turns out that the decision power of a Member State is practically the same as its normalized voting weight, that is, its voting weight divided by the voting weight total.[16] As a consequence the indirect decision power of every citizen attains the same value. Numerically this cannot be but a tiny quantity in a Union of half a billion of citizens. The principal conclusion is conceptual:

- The Jagiellonian Compromise awards to all Union citizens the same and equal power to participate, indirectly via their governments, in Council's decisions.

Table 8 illustrates the application of the Jagiellonian Compromise to the current Council, Table 9 to a Council without UK.

6. RECOMMENDATIONS

We believe that the Union's institutions will be served best by the adoption of the Cambridge Compromise for the composition of the EP, and the Jagiellonian Compromise for the qualified majority voting rule in the Council. These are two independent recommendations.

There is added strength in the above recommendations when viewed as a pair. The Cambridge Compromise would transfer some of the representative weight from middle-sized Member States to smaller and larger Member States. The Jagiellonian Compromise would transfer some of the decision power from smaller and larger Member States to middle-sized

[13] In repeated decision-making systems the term "a priori decision power" denotes the probability that the participant's vote is critical for adopting an act. For large populations the indirect decision power of a citizen behaves proportionally to the quotient of the decision power of his or her Member State and the square root of the Member State's population figure. See, for example, equation (8) in D. Leech / H. Aziz: The double majority voting rule of the EU Reform Treaty as a democratic ideal for an enlarging Union: An appraisal using voting power analysis. In: M. Cichocki / K. Życzkowski (Editors): *Institutional Design and Voting Power in the European Union*. London, 2010, pp. 59–73. For the double-majority voting rule, the decision powers of the Member States were kindly provided by Dan Felsenthal (Jerusalem) and Dennis Leech (London).

[14] See also Figure 14.1 in F. Pukelsheim: Putting citizens first: Representation and power in the European Union. In Cichocki / Życzkowski, note 13, pp. 235–253 (www.uni-augsburg.de/pukelsheim/2010a.pdf).

[15] W. Słomczyński / K. Życzkowski: Penrose voting system and optimal quota. *Acta Physica Polonica* B 37, 2006, pp. 3133–3143 (chaos.if.uj.edu.pl/~karol/pdf/SZapp06.pdf).

[16] For the Jagiellonian Compromise, the Member States' decision powers were calculated using the program ipgenf on the website *Computer Algorithms for Voting Power Analysis* of Dennis Leech and Richard Leech (homepages.warwick.ac.uk/~ecaae/). However, this specific decision rule makes computers superfluous since a Member State's decision power is practically equal to the normalized voting weight (=voting weight / total of all voting weights). Sample calculation in Table 8 for Austria: 2 952 / 99 012 = 2.98 percent.

Member States. The transfer directions have a balancing effect, and thus the pair is in equilibrium. Each of these transfers is soundly rooted in the constitutional directive to put citizens first.

If the full Cambridge Compromise is viewed as being too insensitive to the current composition of the EP, the Power Compromise may be considered as an interim measure. If the latter were to be adopted for the 2019 allocation, our recommendation of the Jagiellonian Compromise would still stand.

We recommend that the adopted allocation method be firmly rooted in primary and secondary Union law, and that such consideration should receive prominent emphasis. Neither the Cambridge Compromise nor the Power Compromise requires a change to primary Union law. Moreover both methods are compatible with the establishment of a joint constituency, as supported by the EP in a 2015 vote.[17]

7. SEAT ALLOCATION TABLES FOR THE 2019 EP

Seven tables are presented to illustrate how the Cambridge Compromise (CC) and the Power Compromise (PC) apply to various scenarios.

Table 1 (CC-28-751) and Table 2 (PC-28-751) deal with the current Union of 28 Member States – that is, including the UK – and maintain the current EP size of 751 seats.

Table 3 (CC-27-751) and Table 4 (PC-27-751) deal with a Union of 27 Member States – that is, without the UK – and an EP of continuing size of 751 seats.

Table 5 (CC-27-678) and Table 6 (PC-27-678) are based on the assumption that, upon Brexit, the 73 UK seats are left vacant. This option reduces the EP size to 678 seats.

All tables include a final column exhibiting the differences between the proposed seat allocations and the 2014 status quo seats. These differences are sometimes appreciable. We emphasize that the 2014 allocation is a patchwork without systematic rationale, and that it fails to satisfy the principle of degressive proportionality. It is a challenging undertaking to achieve a progression towards a representative equilibrium among Union citizens. Once a start is made and the inherited unevenness is reduced, future re-allocations are solely reflective of natural population dynamics.

Of the six tables, only Table 4 (PC-27-751) does not imply any reduction in the number of seats of a Member State assigned to it in 2014. That is, no Member State has to relinquish any of its current seats. We note that a no-loss situation emerges also with fewer seats. The smallest EP size to achieve this effect, with the Power Compromise and with the 2017 population figures, is 723 seats.

Table 7 (PC-27-723) displays the allocation of 723 EP seats between the 27 Member States (without UK) that emerges when using the Power Compromise.

[17] European Parliament resolution of 11 November 2015 on the reform of the electoral law of the European Union (2015/2035(INL)) (www.uni-augsburg.de/bazi/OJ/20xxCyyEpzz.pdf).

Table 1: Cambridge Compromise, including UK

CC-28-751	QMV2017	Quotient	Seats	Repr.Ratio	2014	Diff.
Germany	82 064 489	~~5+97.003~~	96	854 838	96	0
France	66 661 621	5+78.8	84	795 520	74	10
United Kingdom	65 341 183	5+77.2	83	794 562	73	10
Italy	61 302 519	5+72.5	78	791 392	73	5
Spain	46 438 422	5+54.9	60	775 373	54	6
Poland	37 967 209	5+44.9	50	761 194	51	-1
Romania	19 759 968	5+23.4	29	696 830	32	-3
The Netherlands	17 235 349	5+20.4	26	679 286	26	0
Belgium	11 289 853	5+13.3	19	615 419	21	-2
Greece	10 793 526	5+12.8	18	607 802	21	-3
Czech Republic	10 445 783	5+12.3	18	602 157	21	-3
Portugal	10 341 330	5+12.2	18	600 410	21	-3
Sweden	9 998 000	5+11.8	17	594 483	21	-4
Hungary	9 830 485	5+11.6	17	591 487	20	-3
Austria	8 711 500	5+10.3	16	569 480	18	-2
Bulgaria	7 153 784	5+8.5	14	531 642	17	-3
Denmark	5 700 917	5+6.7	12	485 653	13	-1
Finland	5 465 408	5+6.5	12	476 899	13	-1
Slovakia	5 407 910	5+6.4	12	474 698	13	-1
Ireland	4 664 156	5+5.5	11	443 648	11	0
Croatia	4 190 669	5+4.95	10	421 024	11	-1
Lithuania	2 888 558	5+3.4	9	343 289	11	-2
Slovenia	2 064 188	5+2.4	8	277 447	8	0
Latvia	1 968 957	5+2.3	8	268 713	8	0
Estonia	1 315 944	5+1.6	7	200 739	6	1
Cyprus	848 319	5+1.003	7	141 322	6	1
Luxembourg	576 249	5+0.7	6	101 432	6	0
Malta	434 403	5+0.5	6	78 789	6	0
Sum (Divisor)	**510 860 699**	**(846 000)**	**751**	**–**	**751**	**±33**

Notes:

Cambridge Compromise:
Every Member State is assigned 5 base seats, plus one seat per 846 000 citizens or part thereof, with a maximum cap of 96 seats.

Allocation Keys:
There are two allocation keys: the number of base sets (5) and the divisor (846 000). They are determined so that so the least populous Member state is allocated 6 seats and the size of the EP is 751 seats.

Sample calculations for Malta:
Quotient = Base seats+(QMV2017/Divisor) = 5+0.5 = 5.5, rounded upwards to 6 seats
Representation Ratio = QMV2017/Quotient = 434 403 / (5 + 434 403 / 846 000) = 78 789

Maximum cap active only for Germany:
Quotient for Germany 5+97.003 = 102.003 is discarded and capped at 96 seats.

Verification of degressive proportionality:
Representation ratios decrease when passing from more populous to less populous Member States.

Column "Diff." exhibits deviations of proposed "Seats" from "2014" seats. Altogether 33 seats are transferred between middle-sized and larger or smaller Member States.

Table 2: Power Compromise, including UK

PC-28-751	QMV2017	Adjusted	Quotient	Seats	Repr.Ratio	2014	Diff.
Germany	82 064 489	22 917 350	5+90.05	96	863 396	96	0
France	66 661 621	18 888 808	5+74.2	80	841 482	74	6
United Kingdom	65 341 183	18 540 605	5+72.9	78	839 310	73	5
Italy	61 302 519	17 472 492	5+68.7	74	832 302	73	1
Spain	46 438 422	13 495 719	5+53.03	59	800 271	54	5
Poland	37 967 209	11 190 515	5+43.97	49	775 306	51	-2
Romania	19 759 968	6 096 509	5+23.95	29	682 441	32	-3
The Netherlands	17 235 349	5 368 719	5+21.1	27	660 481	26	1
Belgium	11 289 853	3 622 431	5+14.2	20	586 988	21	-1
Greece	10 793 526	3 474 097	5+13.7	19	578 720	21	-2
Czech Republic	10 445 783	3 369 885	5+13.2	19	572 648	21	-2
Portugal	10 341 330	3 338 536	5+13.1	19	570 776	21	-2
Sweden	9 998 000	3 235 335	5+12.7	18	564 460	21	-3
Hungary	9 830 485	3 184 892	5+12.5	18	561 283	20	-2
Austria	8 711 500	2 846 338	5+11.2	17	538 277	18	-1
Bulgaria	7 153 784	2 369 836	5+9.3	15	499 854	17	-2
Denmark	5 700 917	1 918 795	5+7.5	13	454 638	13	0
Finland	5 465 408	1 844 969	5+7.2	13	446 178	13	0
Slovakia	5 407 910	1 826 911	5+7.2	13	444 056	13	0
Ireland	4 664 156	1 592 058	5+6.3	12	414 384	11	1
Croatia	4 190 669	1 441 198	5+5.7	11	393 016	11	0
Lithuania	2 888 558	1 019 608	5+4.01	10	320 726	11	-1
Slovenia	2 064 188	745 962	5+2.9	8	260 265	8	0
Latvia	1 968 957	713 904	5+2.8	8	252 265	8	0
Estonia	1 315 944	490 784	5+1.9	7	189 934	6	1
Cyprus	848 319	326 257	5+1.3	7	135 041	6	1
Luxembourg	576 249	227 702	5+0.9	6	97 757	6	0
Malta	434 403	175 082	5+0.7	6	76 373	6	0
Sum (Keys)	**510 860 699**	**(0.93)**	**(254 500)**	**751**	**–**	**751**	**±21**

Notes:

Power Compromise:
Every Member State is assigned 5 base seats, plus one seat per 254 500 adjusted population units or part thereof, where the adjusted units are obtained by raising the population figures to the power 0.93.

Allocation Keys:
There are three allocation keys: the number of base seats (5), the power (0.93), and the divisor (254 500). They are determined so that the least populous Member State is allocated 6 seats, the most populous Member State is allocated just 96 seats, and the size of the EP is 751 seats.

Sample calculations for Malta:
Adjusted population units = $434\,403^{0.93} = 175\,082$
Quotient = Base seats+(Adjusted/Divisor) = 5+0.7 = 5.7, rounded upwards to 6 seats
Representation Ratio = QMV2017/Quotient = $434\,403 / (5 + 434\,403^{0.93} / 254\,500) = 76\,373$

Maximum cap is automatic since it is built into determination of power 0.93.

Verification of degressive proportionality:
Representation ratios decrease when passing from more populous to less populous Member States.

Column "Diff." exhibits deviations of proposed "Seats" from "2014" seats. Altogether 21 seats are transferred between middle-sized and larger or smaller Member States.

Table 3: Cambridge Compromise, without UK and with 751 EP seats

CC-27-751	QMV2017	Quotient	Seats	Repr.Ratio	2014	Diff.
Germany	82 064 489	~~5+116.7~~	96	854 838	96	0
France	66 661 621	~~5+94.8~~	96	694 392	74	22
Italy	61 302 519	5+87.2	93	665 235	73	20
Spain	46 438 422	5+66.02	72	653 879	54	18
Poland	37 967 209	5+53.98	59	643 766	51	8
Romania	19 759 968	5+28.1	34	597 121	32	2
The Netherlands	17 235 349	5+24.5	30	584 191	26	4
Belgium	11 289 853	5+16.1	22	536 325	21	1
Greece	10 793 526	5+15.3	21	530 530	21	0
Czech Republic	10 445 783	5+14.9	20	526 225	21	-1
Portugal	10 341 330	5+14.7	20	524 889	21	-1
Sweden	9 998 000	5+14.2	20	520 355	21	-1
Hungary	9 830 485	5+13.98	19	518 057	20	-1
Austria	8 711 500	5+12.4	18	501 097	18	0
Bulgaria	7 153 784	5+10.2	16	471 565	17	-1
Denmark	5 700 917	5+8.1	14	435 025	13	1
Finland	5 465 408	5+7.8	13	427 989	13	0
Slovakia	5 407 910	5+7.7	13	426 214	13	0
Ireland	4 664 156	5+6.6	12	401 015	11	1
Croatia	4 190 669	5+5.96	11	382 439	11	0
Lithuania	2 888 558	5+4.1	10	317 195	11	-1
Slovenia	2 064 188	5+2.9	8	260 151	8	0
Latvia	1 968 957	5+2.8	8	252 456	8	0
Estonia	1 315 944	5+1.9	7	191 526	6	1
Cyprus	848 319	5+1.2	7	136 693	6	1
Luxembourg	576 249	5+0.8	6	99 025	6	0
Malta	434 403	5+0.6	6	77 329	6	0
Sum (Divisor)	**445 519 516**	**(703 400)**	**751**	**–**	**678**	**79-6**

Notes:

Cambridge Compromise:
Every Member State is assigned 5 base seats, plus one seat per 703 400 citizens or part thereof, with a maximum cap of 96 seats.

Allocation Keys:
There are two allocation keys: the number of base sets (5) and the divisor (703 400). They are determined so that so the least populous Member state is allocated 6 seats and the size of the EP is 751 seats.

Sample calculations for Malta:
Quotient = Base seats+(QMV2017/Divisor) = 5+0.6 = 5.6, rounded upwards to 6 seats
Representation Ratio = QMV2017/Quotient = 434 403 / (5 + 434 403 / 703 400) = 77 329

Maximum cap active for Germany and France:
Quotient for Germany 5+116.7 = 121.7 is discarded and capped at 96 seats.
Quotient for France 5+94.8 =99.8 is discarded and capped at 96 seats.

Verification of degressive proportionality:
Representation ratios decrease when passing from more populous to less populous Member States.

Column "Diff." exhibits deviations of proposed "Seats" from "2014" seats. In addition to the reallocation of 73 former UK seats, 6 seats are transferred between middle-sized and larger or smaller Member States.

Table 4: Power Compromise, without UK and with 751 EP seats

PC-27-751	QMV2017	Adjusted	Quotient	Seats	Repr.Ratio	2014	Diff.
Germany	82 064 489	1 311 119	4+91.3	96	849 331	96	0
France	66 661 621	1 116 493	4+77.8	82	802 940	74	8
Italy	61 302 519	1 046 455	4+72.9	77	784 646	73	4
Spain	46 438 422	844 299	4+58.8	63	725 593	54	9
Poland	37 967 209	722 575	4+50.3	55	684 163	51	4
Romania	19 759 968	436 157	4+30.4	35	556 946	32	3
The Netherlands	17 235 349	392 421	4+27.3	32	531 580	26	6
Belgium	11 289 853	282 962	4+19.7	24	455 719	21	3
Greece	10 793 526	273 297	4+19.03	24	447 895	21	3
Czech Republic	10 445 783	266 466	4+18.6	23	442 225	21	2
Portugal	10 341 330	264 404	4+18.4	23	440 491	21	2
Sweden	9 998 000	257 592	4+17.9	22	434 679	21	1
Hungary	9 830 485	254 250	4+17.7	22	431 781	20	2
Austria	8 711 500	231 575	4+16.1	21	411 255	18	3
Bulgaria	7 153 784	198 864	4+13.8	18	378 570	17	1
Denmark	5 700 917	166 857	4+11.6	16	342 188	13	3
Finland	5 465 408	161 504	4+11.2	16	335 588	13	3
Slovakia	5 407 910	160 189	4+11.2	16	333 942	13	3
Ireland	4 664 156	142 877	4+9.9	14	311 267	11	3
Croatia	4 190 669	131 530	4+9.2	14	295 295	11	3
Lithuania	2 888 558	98 652	4+6.9	11	242 860	11	0
Slovenia	2 064 188	76 085	4+5.3	10	200 078	8	2
Latvia	1 968 957	73 358	4+5.1	10	194 440	8	2
Estonia	1 315 944	53 724	4+3.7	8	150 319	6	2
Cyprus	848 319	38 263	4+2.7	7	110 547	6	1
Luxembourg	576 249	28 376	4+1.98	6	82 522	6	0
Malta	434 403	22 808	4+1.6	6	65 880	6	0
Sum (Keys)	**445 519 516**	**(0.773)**	**(14 360)**	**751**	**–**	**678**	**73-0**

Notes:

Power Compromise:
Every Member State is assigned 4 base seats, plus one seat per 14 360 adjusted population units or part thereof, where the adjusted units are obtained by raising the population figures to the power 0.773.

Allocation Keys:
There are three allocation keys: the number of base seats (4), the power (0.773), and the divisor (14 360). They are determined so that the least populous Member State is allocated 6 seats, the most populous Member State is allocated just 96 seats, and the size of the EP is 751 seats.

Sample calculations for Malta:
Adjusted population units = $434\,403^{0.773} = 22\,808$
Quotient = Base seats+(Adjusted/Divisor) = 4+1.6 = 5.6, rounded upwards to 6 seats
Representation Ratio = QMV2017/Quotient = $434\,403 / (4 + 434\,403^{0.773} / 14\,360) = 65\,880$

Maximum cap is automatic since it is built into determination of power 0.773.

Verification of degressive proportionality:
Representation ratios decrease when passing from more populous to less populous Member States.

Column "Diff." exhibits deviations of proposed "Seats" from "2014" seats. With the reallocation of 73 former UK seats no Member State has to relinquish any of its 2014 seats.

Table 5: Cambridge Compromise, without UK and with 678 EP seats

CC-27-678	QMV2017	Quotient	Seats	Repr.Ratio	2014	Diff.
Germany	82 064 489	~~5+99.2~~	96	854 838	96	0
France	66 661 621	5+80.6	86	778 698	74	12
Italy	61 302 519	5+74.1	80	774 742	73	7
Spain	46 438 422	5+56.2	62	759 383	54	8
Poland	37 967 209	5+45.9	51	745 778	51	0
Romania	19 759 968	5+23.9	29	683 888	32	-3
The Netherlands	17 235 349	5+20.8	26	666 982	26	0
Belgium	11 289 853	5+13.7	19	605 303	21	-2
Greece	10 793 526	5+13.1	19	597 932	21	-2
Czech Republic	10 445 783	5+12.6	18	592 469	21	-3
Portugal	10 341 330	5+12.5	18	590 777	21	-3
Sweden	9 998 000	5+12.1	18	585 038	21	-3
Hungary	9 830 485	5+11.9	17	582 136	20	-3
Austria	8 711 500	5+10.5	16	560 807	18	-2
Bulgaria	7 153 784	5+8.7	14	524 076	17	-3
Denmark	5 700 917	5+6.9	12	479 331	13	-1
Finland	5 465 408	5+6.6	12	470 802	13	-1
Slovakia	5 407 910	5+6.5	12	468 656	13	-1
Ireland	4 664 156	5+5.6	11	438 367	11	0
Croatia	4 190 669	5+5.1	11	416 265	11	0
Lithuania	2 888 558	5+3.5	9	340 118	11	-2
Slovenia	2 064 188	5+2.5	8	275 372	8	0
Latvia	1 968 957	5+2.4	8	266 766	8	0
Estonia	1 315 944	5+1.6	7	199 651	6	1
Cyprus	848 319	5+1.03	7	140 782	6	1
Luxembourg	576 249	5+0.7	6	101 153	6	0
Malta	434 403	5+0.5	6	78 621	6	0
Sum (Divisor)	**445 519 516**	**(827 000)**	**678**	**–**	**678**	**±29**

Notes:

Cambridge Compromise:
Every Member State is assigned 5 base seats, plus one seat per 827 000 citizens or part thereof, with a maximum cap of 96 seats.

Allocation Keys:
There are two allocation keys: the number of base sets (5) and the divisor (827 000). They are determined so that so the least populous Member state is allocated 6 seats and the size of the EP is 678 seats.

Sample calculations for Malta:
Quotient = Base seats + (QMV2017/Divisor) = 5+0.5 = 5.5, rounded upwards to 6 seats
Representation Ratio = QMV2017/Quotient = 434 403 / (5 + 434 403 / 827 000) = 78 621

Maximum cap active only for Germany:
Quotient for Germany 5+99.2 = 104.2 is discarded and capped at 96 seats.

Verification of degressive proportionality:
Representation ratios decrease when passing from more populous to less populous Member States.

Column "Diff." exhibits deviations of proposed "Seats" from "2014" seats. Altogether 29 seats are transferred between middle-sized and larger or smaller Member States.

Table 6: Power Compromise, without UK and with 678 EP seats

PC-27-678	QMV2017	Adjusted	Quotient	Seats	Repr.Ratio	2014	Diff.
Germany	82 064 489	19 099 537	5+90.2	96	861 842	96	0
France	66 661 621	15 774 870	5+74.5	80	838 351	74	6
Italy	61 302 519	14 604 277	5+68.99	74	828 572	73	1
Spain	46 438 422	11 311 684	5+53.4	59	794 735	54	5
Poland	37 967 209	9 398 444	5+44.4	50	768 643	51	-1
Romania	19 759 968	5 153 750	5+24.3	30	673 377	32	-2
The Netherlands	17 235 349	4 544 713	5+21.5	27	651 184	26	1
Belgium	11 289 853	3 079 450	5+14.5	20	577 596	21	-1
Greece	10 793 526	2 954 679	5+13.96	19	569 371	21	-2
Czech Republic	10 445 783	2 866 987	5+13.5	19	563 337	21	-2
Portugal	10 341 330	2 840 601	5+13.4	19	561 478	21	-2
Sweden	9 998 000	2 753 722	5+13.01	19	555 208	21	-2
Hungary	9 830 485	2 711 246	5+12.8	18	552 057	20	-2
Austria	8 711 500	2 425 970	5+11.5	17	529 270	18	-1
Bulgaria	7 153 784	2 023 825	5+9.6	15	491 336	17	-2
Denmark	5 700 917	1 642 363	5+7.8	13	446 851	13	0
Finland	5 465 408	1 579 839	5+7.5	13	438 544	13	0
Slovakia	5 407 910	1 564 542	5+7.4	13	436 461	13	0
Ireland	4 664 156	1 365 436	5+6.4	12	407 355	11	1
Croatia	4 190 669	1 237 373	5+5.8	11	386 417	11	0
Lithuania	2 888 558	878 672	5+4.2	10	315 670	11	-1
Slovenia	2 064 188	645 014	5+3.05	9	256 522	8	1
Latvia	1 968 957	617 586	5+2.9	8	248 691	8	0
Estonia	1 315 944	426 283	5+2.01	8	187 627	6	2
Cyprus	848 319	284 626	5+1.3	7	133 710	6	1
Luxembourg	576 249	199 417	5+0.9	6	96 979	6	0
Malta	434 403	153 766	5+0.7	6	75 860	6	0
Sum (Keys)	**445 519 516**	**(0.92)**	**(211 700)**	**678**	**–**	**678**	**±18**

Notes:

Power Compromise:
Every Member State is assigned 5 base seats, plus one seat per 211 700 adjusted population units or part thereof, where the adjusted units are obtained by raising the population figures to the power 0.92.

Allocation Keys:
There are three allocation keys: the number of base seats (5), the power (0.92), and the divisor (211 700). They are determined so that the least populous Member state is allocated 6 seats, the most populous Member State is allocated just 96 seats, and the size of the EP is 678 seats.

Sample calculations for Malta:
Adjusted population units = $434\,403^{0.92} = 153\,766$
Quotient = Base seats+(Adjusted/Divisor) = 5+0.7 = 5.7, rounded upwards to 6 seats
Representation Ratio = QMV2017/Quotient = $434\,403 / (5 + 434\,403^{0.92} / 211\,700) = 75\,860$

Maximum cap is automatic since it is built into determination of power 0.92.

Verification of degressive proportionality:
Representation ratios decrease when passing from more populous to less populous Member States.

Column "Diff." exhibits deviations of proposed "Seats" from "2014" seats. Altogether 18 seats are transferred between middle-sized and larger or smaller Member States.

Table 7: Power Compromise, without UK and with 723 EP seats

PC-27-723	QMV2017	Adjusted	Quotient	Seats	Repr.Ratio	2014	Diff.
Germany	82 064 489	2 717 742	4+91.2	96	862 305	96	0
France	66 661 621	2 295 150	4+76.99	81	823 058	74	7
Italy	61 302 519	2 143 976	4+71.9	76	807 448	73	3
Spain	46 438 422	1 710 690	4+57.4	62	756 493	54	8
Poland	37 967 209	1 452 310	4+48.7	53	720 182	51	2
Romania	19 759 968	854 032	4+28.6	33	605 221	32	1
The Netherlands	17 235 349	764 204	4+25.6	30	581 571	26	4
Belgium	11 289 853	541 796	4+18.2	23	509 126	21	2
Greece	10 793 526	522 350	4+17.5	22	501 496	21	1
Czech Republic	10 445 783	508 627	4+17.1	22	495 947	21	1
Portugal	10 341 330	504 488	4+16.9	21	494 246	21	0
Sweden	9 998 000	490 828	4+16.5	21	488 536	21	0
Hungary	9 830 485	484 132	4+16.2	21	485 682	20	1
Austria	8 711 500	438 829	4+14.7	19	465 336	18	1
Bulgaria	7 153 784	373 885	4+12.5	17	432 455	17	0
Denmark	5 700 917	310 873	4+10.4	15	395 116	13	2
Finland	5 465 408	300 391	4+10.1	15	388 255	13	2
Slovakia	5 407 910	297 819	4+9.99	14	386 539	13	1
Ireland	4 664 156	264 066	4+8.9	13	362 735	11	2
Croatia	4 190 669	242 056	4+8.1	13	345 766	11	2
Lithuania	2 888 558	178 868	4+6.0003	11	288 848	11	0
Slovenia	2 064 188	136 110	4+4.6	9	240 977	8	1
Latvia	1 968 957	130 983	4+4.4	9	234 569	8	1
Estonia	1 315 944	94 393	4+3.2	8	183 625	6	2
Cyprus	848 319	66 057	4+2.2	7	136 475	6	1
Luxembourg	576 249	48 237	4+1.6	6	102 569	6	0
Malta	434 403	38 336	4+1.3	6	82 180	6	0
Sum (Keys)	**445 519 516**	**(0.813)**	**(29 810)**	**723**	**–**	**678**	**45-0**

Notes:

Power Compromise:
Every Member State is assigned 4 base seats, plus one seat per 29 810 adjusted population units or part thereof, where the adjusted units are obtained by raising the population figures to the power 0.813.

Allocation Keys:
There are three allocation keys: the number of base seats (4), the power (0.813), and the divisor (29 810). They are determined so that the least populous Member state is allocated 6 seats, the most populous Member State is allocated just 96 seats, and the size of the EP is 723 seats.

Sample calculations for Malta:
Adjusted population units = $434\ 403^{0.813} = 38\ 336$
Quotient = Base seats+(Adjusted/Divisor) = 4+1.3 = 5.3, rounded upwards to 6 seats
Representation Ratio = QMV2017/Quotient = $434\ 403 / (4 + 434\ 403^{0.813} / 29\ 810) = 82\ 180$

Maximum cap is automatic since it is built into determination of power 0.813.

Verification of degressive proportionality:
Representation ratios decrease when passing from more populous to less populous Member States.

Column "Diff." exhibits deviations of proposed "Seats" from "2014" seats. With the reallocation of 45 of the former UK seats no Member State has to relinquish any of its 2014 seats.

Table 8 : Jagiellonian Compromise, including UK

JagCom-28	QMV2017	Voting Weight	Decision Power	DM2017	Diff.
Germany	82 064 489	9 059	9.13	10.25	-1.12
France	66 661 621	8 165	8.25	8.44	-0.19
United Kingdom	65 341 183	8 083	8.17	8.29	-0.12
Italy	61 302 519	7 830	7.91	7.86	0.05
Spain	46 438 422	6 815	6.89	6.18	0.71
Poland	37 967 209	6 162	6.23	5.07	1.16
Romania	19 759 968	4 445	4.49	3.75	0.74
The Netherlands	17 235 349	4 152	4.19	3.49	0.70
Belgium	11 289 853	3 360	3.39	2.89	0.50
Greece	10 793 526	3 285	3.32	2.85	0.47
Czech Republic	10 445 783	3 232	3.26	2.81	0.45
Portugal	10 341 330	3 216	3.25	2.80	0.45
Sweden	9 998 000	3 162	3.19	2.77	0.42
Hungary	9 830 485	3 135	3.17	2.75	0.42
Austria	8 711 500	2 952	2.98	2.64	0.34
Bulgaria	7 153 784	2 675	2.70	2.48	0.22
Denmark	5 700 917	2 388	2.41	2.33	0.08
Finland	5 465 408	2 338	2.36	2.31	0.05
Slovakia	5 407 910	2 325	2.35	2.30	0.05
Ireland	4 664 156	2 160	2.18	2.23	-0.05
Croatia	4 190 669	2 047	2.07	2.18	-0.11
Lithuania	2 888 558	1 700	1.72	2.05	-0.33
Slovenia	2 064 188	1 437	1.45	1.97	-0.52
Latvia	1 968 957	1 403	1.42	1.96	-0.54
Estonia	1 315 944	1 147	1.16	1.89	-0.73
Cyprus	848 319	921	0.93	1.84	-0.91
Luxembourg	576 249	759	0.77	1.82	-1.05
Malta	434 403	659	0.66	1.80	-1.14
Sum	**510 860 699**	**99 012**	**100.00**	**100.00**	**±6.81**
Quota		**60 807**	**61.41**		

Notes:

Jagiellonian Compromise:
A group of Member States constitutes a qualified majority provided the sum of their voting weights meets or exceeds the quota 60 807.

System keys:
A Member State's voting weight is the square root of its population figure, rounded to the nearest whole number.
The quota 60 807 is the average of the square root of the population total ($510\ 860\ 699^{0.5} = 22\ 602.2$) and the sum of the voting weights (99 012), rounded upwards.

System merits:
The decision power of a Member State is identical to the percentage voting weight.
The indirect decision powers of all Union citizens are equal.

Column "Diff." exhibits the deviations of the "Decision Power" of the proposed Jagiellonian Compromise from the "DM2017" decision power of the double-majority voting rule in 2017. A total of 6.81 percent decision power is transferred between middle-sized and larger or smaller Member States.

Table 9 : Jagiellonian Compromise, without UK

JagCom-27	QMV2017	Voting Weight	Decision Power	DM2017	Diff.
Germany	82 064 489	9 059	9.94	11.98	-2.04
France	66 661 621	8 165	8.98	9.95	-0.97
Italy	61 302 519	7 830	8.62	9.18	-0.56
Spain	46 438 422	6 815	7.51	7.62	-0.11
Poland	37 967 209	6 162	6.79	6.49	0.30
Romania	19 759 968	4 445	4.89	4.00	0.89
The Netherlands	17 235 349	4 152	4.57	3.71	0.86
Belgium	11 289 853	3 360	3.70	3.02	0.68
Greece	10 793 526	3 285	3.61	2.96	0.65
Czech Republic	10 445 783	3 232	3.55	2.92	0.63
Portugal	10 341 330	3 216	3.54	2.91	0.63
Sweden	9 998 000	3 162	3.48	2.87	0.61
Hungary	9 830 485	3 135	3.45	2.85	0.60
Austria	8 711 500	2 952	3.25	2.71	0.54
Bulgaria	7 153 784	2 675	2.94	2.53	0.41
Denmark	5 700 917	2 388	2.62	2.35	0.27
Finland	5 465 408	2 338	2.57	2.33	0.24
Slovakia	5 407 910	2 325	2.56	2.32	0.24
Ireland	4 664 156	2 160	2.37	2.23	0.14
Croatia	4 190 669	2 047	2.25	2.17	0.08
Lithuania	2 888 558	1 700	1.87	2.02	-0.15
Slovenia	2 064 188	1 437	1.58	1.92	-0.34
Latvia	1 968 957	1 403	1.54	1.90	-0.36
Estonia	1 315 944	1 147	1.26	1.83	-0.57
Cyprus	848 319	921	1.01	1.77	-0.76
Luxembourg	576 249	759	0.83	1.74	-0.91
Malta	434 403	659	0.72	1.72	-1.00
Sum	**445 519 516**	**90 929**	**100.00**	**100.00**	**±7.77**
Quota		**56 019**	**61.61**		

Notes:

Jagiellonian Compromise:
A group of Member States constitutes a qualified majority provided the sum of their voting weights meets or exceeds the quota 56 019.

System keys:
A Member State's voting weight is the square root of its population figure, rounded to the nearest whole number.
The quota 56 019 is the average of the square root of the population total ($445\ 519\ 516^{0.5} = 21\ 107.3$) and the sum of the voting weights (90 929), rounded upwards.

System merits:
The decision power of a Member State is identical to the percentage voting weight.
The indirect decision powers of all Union citizens are equal.

Column "Diff." exhibits the deviations of the "Decision Power" of the proposed Jagiellonian Compromise from the "DM2017" decision power of the double-majority voting rule in 2017. A total of 7.77 percent decision power is transferred between middle-sized and larger or smaller Member States.

THE r-DP METHODS TO ALLOCATE THE EP SEATS TO MEMBER STATES

Prof. Victoriano RAMÍREZ GONZÁLEZ

KEY FINDINGS

- The concept of degressive proportionality, (henceforth DP), introduced by the Lamassoure-Severin report, has a very broad meaning. For example, allocating 26 seats for each of the 28 states fulfils the requirements of the Treaty of Lisbon. Even if the least populous country gets a minimum 6 seats and the most populous 96 seats, it is possible to set up many different allocations which can also give a wide range of results for the same state.
- This paper proposes limiting the DP concept because MEPs on some occasions vote on the basis of **national preferences** and on other occasions vote on the basis of **ideological affinity.** Depending on whether a greater influence of national preferences or ideology is desired, we can use one from a series of parametric methods. This paper proposes **allocating half of the seats** in the EP **in proportion to Member States' populations (**ideological affinity) **the other half in proportion to the square root of their populations (and** national preferences), rounding up with the Adams method and without minimum or maximum limitations. The minimum and maximum limitations of 6 and 96, respectively, are included later in the formulas.
- The current EP composition contains allocations to some states that **contradict DP,** as for example, when comparing the seats allocated to **Germany** with those allocated to **France**, **UK** or **Spain.** A new composition of the EP must reverse this situation, whereby either Germany's representation will have to decrease or the other Member States' increase (or both).
- On the other hand, **Hungary** has 21 MEPs and **Sweden** has 20, but Hungary has declined in population and Sweden has increased and surpassed Hungary, so these allocations cannot be maintained in the future because **they contradict the Lisbon Treaty.**
- Likewise, **Lithuania** which currently has a population of less than 3 million and **Ireland** with more than 4.5 million, both have 11 MEPs. This **unbalanced situation** is unreasonable.
- The formula to be adopted should not differ significantly from the current one, with the exception of the previously noted inconsistencies.

1. INTRODUCTION

From the very beginning of the EP's existence up until the present, the seat allocation for each EU Member State has always been determined through negotiations, so that after every Union enlargement, or when a state's population changes significantly, new negotiations are required.

The need for a formula to distribute seats has been frequently raised over the last decades. For example, some scholars have noted that *"there is neither a formula to determine the*

vote weight of each State on the Council of the EU nor a formula to calculate the number of seats in Parliament"[1].

The basic concept in degressive proportionality consists of assigning **fewer seats to larger states than their corresponding proportion**, in order to assign **more seats to smaller states**. Of course a country that is more populous than another cannot have fewer representatives.

In 2007, the then EP vice-president Jacek Saryusz-Wolski proposed a meeting in order to develop formulas for allocating EP seats and also to determine the members of the European Commission. The meeting was organised by Marek A. Cichocki and Karol Życzkowski in the Natolin European Centre in Warsaw. The contributions presented during that meeting were published in a book and, though at that time the concept of degressive proportionality was not rigorously defined, the book contains some papers that propose degressively proportional distributions which meet the established limitations. This book also contains other articles that recommend different voting systems for the Council of the European Union[2].

Also in 2007, two Members of the European Parliament Alain Lamassoure and Adrian Severin rigorously defined the term "degressive proportionality" which is included in a resolution of the EP[3]. Specifically, the meaning adopted was that the ratio between the population and the number of seats of each Member State must be larger for the more populous state, when any two states are compared.

However, this definition does not imply a particular method of seat allocation in the EP.

That is why, after the 2009 elections, Andrew Duff, MEP and member of the AFCO Committee of the EP, tried to obtain a formula to distribute EP seats among EU Member States. To this end, he promoted a meeting of electoral systems researchers at the University of Cambridge. The result of this meeting was a Report for the EU called the **Cambridge Compromise** (or CamCom)[4]. The Cambridge Compromise agrees on a somewhat more flexible definition of degressive proportionality (considering the number of seats before rounding, to obtain the ratio between population and seats). Finally, the Report provided a simple, transparent and durable formula for the distribution of seats in the EP.

The journal *Mathematical Social Science* echoed the difficulty of the problem and published a special issue[5], in which many researchers presented new proposals for allocating seats in the EP.

Many other articles have been written, before and after the cited events; however the EP's acceptance of a particular method is still pending. The composition of the EP for the period 2014-2019 had to be agreed by negotiation in which MEPs Gualtieri, R. and Trzaskowski, R. proposed a criterion for seat distribution: "*Nobody gains seats and nobody loses more than one*". [6] They also achieved a "*pragmatic solution*" for the 2014-2019 parliamentary term, which was finally approved although it does not fully comply with the Lisbon Treaty provisions.

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[1] Hosli, MO. and Machover, M (2004) The Nice Treaty and Voting Rules in the Council: A Reply to Moberg, Journal of Common Market Studies 42(3): 497–521.

[2] Słomczyński, W. and Życzkowski K., 2010. Jagiellonian Compromise: an alternative voting system for the Council of the European Union, in Cichocki MA and Życzkowski K. (eds) (2010) Design and Voting Power in the European Union. Farnham: Ashgate Publishing Limited, pp. 43-57.

[3] Lamassoure A. and Severin A. (2007): A proposal to amend the Treaty provisions concerning the composition of the European Parliament. Brussels: Draft Explanatory statement, 31 August 2007. Available at: http://www.europarl.europa.eu/sides/ getDoc.do?pubRef= //EP//NONSGML+REPORT+A6-2007-0351+0+DOC+PDF+V0//EN.

[4] Grimmett, G., Laslier, JF, Pukelsheim, F, Ramírez-González, V, Rose, R, Słomczyński, W, Zachariasen, M and Życzkowski, K. (2011): The allocation between the EU Member States of the seats in the European Parliament – Cambridge Compromise. European Parliament Studies, PE 432.760.

[5] Laslier JF (ed.) (2012) Special Issue around the Cambridge Compromise: Apportionment in Theory and Practice. Elsevier, (Mathematical Social Science, 63)

[6] Gualtieri, R. and Trzaskowski, R. (2013) Report on the composition of the European Parliament with view to the 2014 elections. Committee on Constitutional Affairs (2012/2309(INI))

In this briefing and after the introduction, section 2 describes how some assignments in the current composition of the EP are contrary to the idea of DP and how others are not reasonable. Section 3 provides arguments for removing constraints (maximum 96 seats and minimum 6 seats). Section 4 defends and justifies the concept of degressive proportionality in EP seat distribution. Section 5 proposes a parametric family of methods to obtain the composition of the EP. These methods are based on a more precise and restrictive definition of DP than the proposal in the "Cambridge Compromise". All methods belonging to this parametric family give Malta fewer than six seats. However, as it is necessary to respect the limits imposed by the Lisbon Treaty[7], we have made an allocation with the minimum and maximum requirements[9], to obtain a method, called **0.5-DPL**, which respects all constraints. The results of the proposed method and those obtained with the "Cambridge Compromise" method, the parabolic method and the power method, are compared with the current distribution and we can observe that the **0.5-DPL method gives a closer allotment to the current distribution**. The proposed method is applicable to any other scenario where there are changes in the states, in their population, in the minimum and maximum requirements or in the size of the EP. Thus, in section 6 we give an example of a possible composition of the EP after Brexit for which we have proposed an EP with 701 MEPs, maintaining the minimum 6 and maximum 96 for each state. We also include a table with the allocation of seats following a possible EU enlargement and, finally, in section 7 we present the main conclusions of this briefing

2. NON-DEGRESSIVITY AND OTHER FORMS OF ILLOGICAL BEHAVIOUR IN CURRENT EP COMPOSITION

Currently, the population/seat ratio for the five most populous countries is:

Germany:	82064489/96=**854838**	Italy:	61302519/73=**839761**
France:	66661621/74=900833	Spain:	46438422/54=859971
UK:	65341183/73=895085		

We can see that there is **inverse degressivity** when comparing **Germany** with **France**, the **UK** and **Spain**, since they all have a higher population/seat ratio than Germany although they are less populated. In particular **Spain** has almost 35 million fewer inhabitants than **Germany**. Thus, this situation has to be reversed either by diminishing the representation of Germany or by increasing the representation of the other countries (or both).

Another important example of **inverse degressivity** occurs when comparing **Romania** with **The Netherlands**. The Netherlands should receive more seats or Romania's representation should be decreased.

On the other hand, the population of Hungary has decreased while Sweden's has increased. Currently **Sweden has a larger population than Hungary**, but, in the EP, **Hungary has 21 MEPs and Sweden 20**. Therefore this situation cannot be maintained.

The new composition of the EP should also lead to another major change in the **unjustified allocation of seats for Lithuania**, as Lithuania has a population of less than 2.9 million and currently has the same number of seats as Ireland whose population exceeds 4.6 million. It would be reasonable for **Ireland to have about 3 seats more than Lithuania**.

Apart from these considerations, the allocation obtained with the formula that is adopted should give results close to present day ones for the rest of the Member States. The formula should work well even if the constraints 6 and 96 were to be replaced by other numbers or even if they were abolished altogether.

[7] Consolidated versions of the Treaty on European Union and the Treaty on the Functioning of the European Union (Treaty of Lisbon). Official Journal of the European Union. C306:1-388, 2007. Available at: http://europa.eu/lisbon_treaty/ full_text/index_en.htm

3. WHY DOES THE MINIMUM CORRESPOND TO 6 SEATS AND THE MAXIMUM TO 96?

The Lisbon Treaty stipulates that no state may obtain more than 96 seats or fewer than 6 seats. These maximum and minimum limitations were set in response to a political agreement. However, no justification of these values was given and, moreover, if they were justified it would be for a situation similar to that which existed in 2007. Also at that time, the EP had fewer powers than at present and it is now necessary to question these limitations. Moreover, the EU may be enlarged by accepting new States while other Member States may wish to leave the EU. In any event, the population of Member States may change from one election to the next, more so in some states than in others.

Therefore, the minimum and maximum limitations that the Lisbon Treaty imposes upon Member States concerning their number of seats in the composition of the EP are not guaranteed to last. Let us work on the assumption that a very small state, for example having about 100,000 inhabitants, were to join the EU: how could assigning 6 seats to this state be justified? If that were to happen, an MEP from this hypothetical small state would represent fewer than 16,500 inhabitants in the Union, whereas an MEP from Malta would represent 70,000 and a MEP from one of the most populous countries would represent about one million. Such differences would be difficult to justify logically.

Moreover, the same goes for the maximum limitation of 96 seats. Let us imagine that several of the most populated countries in the EU (but not Germany) decide to leave the EU. In that case, if the size of EP remained as 751, then what purpose would imposing a limit of 96 seats on Germany serve?

Initially, therefore, the proposed method shall not set maximum or minimum limits. At a later stage, the formula can include the current limitations (of the Lisbon Treaty) or other new limitations.

4. WHY MUST THE REPRESENTATION IN THE EUROPEAN PARLIAMENT NOT BE PROPORTIONAL?

The voting behaviour of MEPs does not always follow the same pattern. Sometimes MEPs join forces with their parliamentary group, while on other occasions they tend to vote more in accordance with their nationality. On other times, finally, they follow neither of the previous two patterns.

When MEPs vote according to their ideology, as in national parliaments, seat allocation proportional to the people who elect them is justified.

However, when MEPs vote according to their nationality the most reasonable distribution is obtained in proportion to the square root of states' population, because in this case a similar "citizen power" is obtained in proportion to the square root of their inhabitants (some related works: Barberá[8], Beisbart[9], Penrose[10], Moberg[11] and Cichocki and Życzkowski[12]).

Therefore, **a proportional allocation of EP seats among Member States would be unfair to the citizens of smaller states because their power of vote, when their MEPs vote en bloc, is much smaller than the power of the citizens of the most populous countries**.

[8] Barberá, S. and Jackson, M.O. (2006). "On the Weights of Nations: Assigning Voting Weights in a Heterogeneous Union." Journal of Polit. Econ. 114(2): 317 – 339.
[9] Beisbart, C. and Bovens, L. (2007). "Welfarist Evaluations of Decision Rules for Boards of Representatives." Soc. Choice and Welfare 29: 581 – 608.
[10] Penrose, Lionel S. 1946. "The Elementary Statistics of Majority Voting." J. Royal Statis. Soc. 109:53 – 57.
[11] Moberg, A. (2012) EP seats: the politics behind the math. Mathematical Social Scieces, 63, 78-84.
[12] Cichocki M. A. and Życzkowski K. (eds) (2010) Design and Voting Power in the European Union. Farnham: Ashgate Publishing Limited.

Moreover, **the seat allocation in the EP in proportion to the square root of the population of the states would give all EU citizens more equal power if all the representatives of each state were to vote en bloc, but that is not so in many cases and therefore it would also be unfair**.

The proportional allocation according to the number of inhabitants assigns more representatives to the most populous states than proportional distribution to the square root of inhabitants, and the opposite occurs with the least populous states.

Thus, the most populous countries are interested in allocations closest to proportional representation to inhabitants and the least populated countries are interested in an allocation closest to proportional representation to the square root of inhabitants.

Therefore, there is no justification for a set distribution in the EP in which:

1. *The most populous country obtains more seats than it would obtain with a proportional allocation to its inhabitants.*
2. *The least populous country receives more seats than it would obtain with a proportional allocation to the square root of the number of inhabitants.*

The perfect method will never exist, but perhaps **a method that combines proportional representation to population with proportional representation to the square root of the number of inhabitants** is one of the most suitable outcomes for determining the composition of the EP.

The question is: **what combination** between the two distributions should we use?

Thus, **the more influence** the distribution in proportion to the square root exercises, **the more degressive** it will be.

Therefore, we can define the **degree of DP**, ***r***, in terms of the **proportion of seats allocated to the states in proportion to the square root of their population**. This definition is clarified in the next section, which also contains the distribution tables corresponding to different degrees of DP.

5. THE FAMILY OF r-DP METHODS

As discussed above, a part *r* of the seats in the EP will be distributed in proportion to the square root of the population of each country and the other part (1-*r*) in proportion to its population.

Thus, *r* is a parameter that can take any value in the interval [0-1]. The corresponding method is denoted by *r*-DP and we shall say that this method has a degree of degressivity *r*. **The parametric r-DP method yields different sets of seat representation for states in function of their degree of degressivity**.

For example, *r* = 0.6 means to allocate 60% of seats in proportion to the square root of population and, as (1-*r*) = 0.4, 40% of seats in proportion to the population.

Specifically, once the degree of degressivity *r* is fixed, the representation of state i, with population p_i would be proportional to its adjusted quota $A(p_i)=q_i$:

$$A(p_i) = q_i = \frac{751 * r\sqrt{p_i}}{\sum_{i=1}^{n}\sqrt{p_i}} + \frac{751*(1-r)*p_i}{\sum_{i=1}^{n} p_i} \qquad (1)$$

Thus, according to the populations shown in Table 1, the adjusted quotas are:

$$A(p_i) = q_i = \frac{751 * r\sqrt{p_i}}{99010} + \frac{751*(1-r)*p_i}{510860699}$$

and if we choose a degree of DP r = 0.6 (for example), then the adjusted quota for France is

$$A(66661621) = q_2 = \frac{751*0.6\sqrt{66661621}}{99010} + \frac{751*0.4*66661621}{510860699} = 76.36$$

The rounding of fractions will be in accordance with the Adams method (divisors method with upward rounding[13]). So any Member State receives at least one seat.

Thus we obtain the value of *k* which verifies

$$\sum_{j=1}^{n} \lceil kq_i \rceil = 751 \qquad (2)$$

Where $\lceil x \rceil$ is the integer number which is greater or equal to *x*. The value of k is always somewhat less than 1, because the rounding of the fractions is upward.

And the allotment S_r is:

$$S_r = \left(\lceil kq_1 \rceil, \quad \lceil kq_2 \rceil, \quad ..., \quad \lceil kq_{28} \rceil \right) \qquad (3)$$

The question now is, **what value to choose for *r***? Maybe it should not be a value either close to zero or to one.

Degressivity of Sr

Each allotment S_r is degressively proportional as stated in the Cambridge Compromise

An important result is: $\forall r \in [0,1]$, the obtained distribution S_r is degressively proportional as indicated by the Cambridge Compromise, i.e. the ratios between populations and seats (before rounding) are decreasing as we move from more populated states to less populated ones.

$$\frac{q_i}{p_i} = \left(\frac{751*r\sqrt{p_i}}{\sum_{i=1}^{n}\sqrt{p_i}} + \frac{751*(1-r)*p_i}{\sum_{i=1}^{n} p_i} \right) / p_i = \frac{c_1}{\sqrt{p_i}} + c_2 \qquad (4)$$

When *r* is fixed, c_1 and c_2 are constants in (4). So the ratio p_i/q_i **is decreasing** when p_i decreases.

Some particular allocations obtained through different degrees of DP

Table 1 shows the results for various values of *r* ranging between 0 and 1. The last column shows the current allocation. The Adams method has been used for rounding.

[13] Pukelsheim, F. Proportional representation. Apportionment methods and their applications (2014), Springer.

Table 1: Composition of the EP with different degrees of DP and current allotment

Country	Population	$S_{0.}$=*Pro.*	$S_{0.40}$	$S_{0.50}$	$S_{0.60}$	S_1=PSR	Current
Germany	82064489	119	98	**93**	88	68	96
France	66661621	97	82	**79**	76	61	74
United Kingdom	65341183	95	81	**78**	74	61	73
Italy	61302519	89	77	**74**	71	59	73
Spain	46438422	68	61	**59**	58	51	54
Poland	37967209	55	52	**51**	50	46	51
Romania	19759968	29	31	**31**	32	34	32
The Netherlands	17235349	25	28	**28**	29	31	26
Belgium	11289853	17	20	**21**	22	26	21
Greece	10793526	16	20	**21**	21	25	21
Czech Rep.	10445783	16	19	**20**	21	25	21
Portugal	10341330	15	19	**20**	21	24	21
Sweden	9998000	15	19	**19**	20	24	20
Hungary	9830485	15	18	**19**	20	24	21
Austria	8711500	13	17	**18**	19	23	18
Bulgaria	7153784	11	15	**16**	17	20	17
Denmark	5700917	9	13	**13**	14	18	13
Finland	5465408	8	12	**13**	14	18	13
Slovakia	5407910	8	12	**13**	14	18	13
Ireland	4664156	7	11	**12**	13	17	11
Croatia	4190669	7	10	**11**	12	16	11
Lithuania	2888558	5	8	**9**	10	13	11
Slovenia	2064188	3	7	**7**	8	11	8
Latvia	1968957	3	6	**7**	8	11	8
Estonia	1315944	2	5	**6**	6	9	6
Cyprus	848319	2	4	**5**	5	7	6
Luxembourg	576249	1	3	**4**	4	6	6
Malta	434403	1	3	**3**	4	5	6
Total	510860699	751	751	751	751	751	751

Source: Figures concerning the population of each Member State in accordance with Council Decision (EU, Euratom) 2016/2353 of 8 December 2016 amending the Council's Rules of Procedure (http://eurlex.europa.eu/legal-content/EN/TXT/HTML/?uri=CELEX:32016D2353&from=EN)

Remarks 1

a. Obviously the S_0 and S_1 values, for proportional allocation to population and proportional allocation to the square root of the population (respectively), are disposable, because they are far removed from the results obtained by negotiations in the past. However, **values of r close to 0.5 lead to allocations which are very similar to the current ones** for all countries except those affected by the minimum limitation, **inverse degressivity** and **Lithuania** and **Hungary** as indicated in section 2.

b. In comparison with the last column, which contains the current allocation of seats, it can be observed that, when the distribution tables are calculated with the three values *r* close to 0.5, France, UK, Spain and The Netherlands are the only countries in each case which receive more seats than their current apportionment. In part this is quite logical, because the present distribution is contrary to degressive proportionality among the most populous states. For example France is the country in the EU whose MEPs represent most inhabitants, and in the case of Spain its MEPs represent more inhabitants than MEPs from Italy or Germany, even though Germany has almost double the population of Spain. By contrast, there are other countries in the three distribution tables receiving fewer seats than they actually have in the EP. They are the three least populated states (Malta, Luxembourg and Cyprus) together with Lithuania and Hungary.

c. Perhaps the choice r = 0.5 is the one which is the most consistent with current allocation. However, Scully et al. [14] states that political and ideological affiliations explain 60 per cent of the variance in the personal positions of MEPs on EU policy issues and the remaining 40 per cent is explained by national affiliations. Thus, 0.4 is another important value of r that must be considered.

On the other hand, Dniestrzański[15] introduces a measure of degressivity *MD(S)* for distribution S, as

$$MD(S) = \frac{\sum_{i=1}^{n} \left| s_i - \frac{H * p_i}{V} \right|}{\sum_{i=1}^{n} \left| \frac{H}{n} - \frac{H * p_i}{V} \right|}$$

Here, V is the total population of the European Union.

If we calculate the measure of degressivity $MD(S_r)$ for the five previous distributions we obtain the following results:

r	0	0.40	0.50	0.60	1
$MD(S_r)$	0.02	0.21	0.26	0.30	0.50

Therefore the degree of degressivity r established in this paper reproduces the same order as the degree of degressivity established by Dniestrzański, but the values are different.

Although the values $MD(S_r)$ in these examples, are approximately $r/2$, this is not always the case.

In fact, the rate of Dniestrzański is more appropriate for measuring disproportionality than measuring degressive proportionality, because the formula does not distinguish whether the represented countries are the most or the least populated ones.

The proposed method

In view of the negotiations that led to the composition of the EP for the 2014-2019 term, we consider that an appropriate method to distribute EP seats may be the one which has a degree of degressivity r = 0.50, the 0.5-DP method, (50% of the seats in proportion to the number of inhabitants and 50% in proportion to the square root of the number of inhabitants), whose distribution is obtained by applying the expressions (1)-(3) with r = 0.5 and k = 0.982. It is contained in the fifth column of Table 1.

The proposed method with the limitations of 6 and 96

To obtain the allotment with the proposed methods 0.5-DP while respecting the Treaty limitations of 6 and 96 seats, we must find a value of k such that

$$\sum_{j=1}^{n} \text{median}(6, \lceil kq_i \rceil, 96) = 751 \qquad (5)$$

We call it the **0.5-DPL method**. The corresponding allocation is obtained using k = 0.974.

In Table 2, below, column 2 shows the assignments before rounding, column 3 contains the rounded allotment and column 4the ratio of degressivity. Finally the degressivity for the current allotment is shown in the last column of Table 2.

[14] Scully, R., Hix, S. and Farrel, D. (2012) 'National or European parlamentarians? Evidence from a New Survey of the Members of the European Parliament'. *Journal of Common Market Studies*, Vol. 50, No. 4, pp. 670–683.
[15] Dniestrzański, P., (2014), "Proposal for measure of degressive proportionality". Elsevier, Procedia – Social and Behavioral Sciences 110, 140 – 147

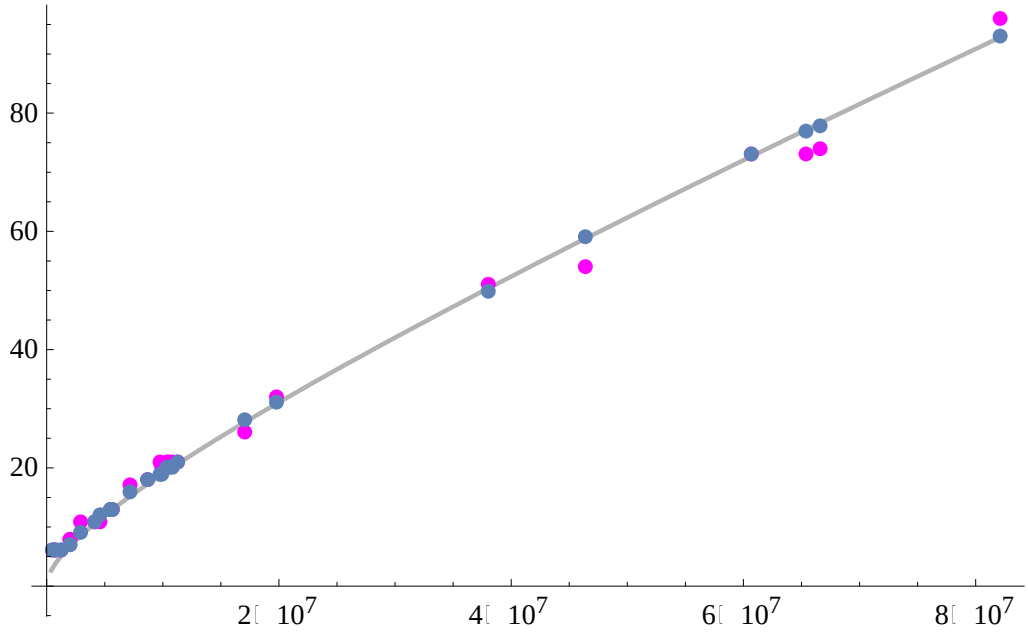
Table 2: Degressivity of the proposed and current methods

Country	*A*(p)	*0.5-DPL*	Pop./*A*(p)	Pop./Current
Germany	92.22	93	889877	**854838**
France	77.88	78	855953	900833
United Kingdom	76.64	77	852573	895085
Italy	72.81	73	841952	**839761**
Spain	58.42	59	794906	859971
Poland	49.94	50	760256	744455
Romania	30.57	31	646384	**617499**
The Netherlands	27.67	28	622889	662898
Belgium	20.49	21	550993	537612
Greece	19.86	20	543481	513977
Czech Republic	19.42	20	537888	**497418**
Portugal	19.28	20	536376	**492444**
Sweden	18.84	19	530679	499900
Hungary	18.62	19	527953	**468118**
Austria	17.14	18	508256	483972
Bulgaria	15.01	16	476919	**420811**
Denmark	12.90	13	441932	438532
Finland	12.55	13	435491	420416
Slovakia	12.46	13	434022	**415993**
Ireland	11.32	12	412028	424014
Croatia	10.56	11	396844	380970
Lithuania	8.35	9	345935	262596
Slovenia	6.79	7	304453	258024
Latvia	6.59	7	298780	246120
Estonia	5.18	6	219324	219324
Cyprus	4.01	6	141386	141386
Luxembourg	3.22	6	96042	96042
Malta	2.76	6	72400	72400
		751		

Source: Own elaboration. From Estonia down to Malta we have used as divisor A(p) the value 6.

In the last column, whenever a quotient appears in bold it is because there is at least one other quotient further down in the table that has a larger value (contrary to DP after rounding).

Graphically, the proposed (blue points on the curve) and current (purple) allotments are shown in the following graph.



The curve corresponds to the adjusted quotas. Therefore, the allocations with the 0.5-DPL method are on the curve, except those affected by the minimum 6.

The points distant from the curve correspond to the current distribution, mainly those that correspond to Germany, the UK, France and Spain.

6. THE COMPOSITION OF THE EP AFTER BREXIT

Size of the new EP in 2019

The UK's withdrawal from the EU frees up its **73 seats** and makes it easier to adopt a method which achieves DP, while also reducing EP size from the current size of 751 to 701, as is shown in this paper: some of the current UK seats can be used to correct the present inverse DP among the more populous countries to achieve DP. In such a situation, **few countries lose seats** and, more importantly**, the number of seats that would be lost is not significant**.

In any case, it might be politically prudent to reduce the size of the EP after the withdrawal of the UK so that any possible future incorporation of new states does not lead to loss of seats for current Member States.

This would also be useful, if a transnational list is established, which would not require a reduction in the allocation of Member States' seats.

A reduction of between 50 and 60 seats may be adequate to achieve both objectives. Specifically we will simulate results corresponding to a reduction of 50 seats, so that the EP's size is 701 seats.

Comparative allotments in 2019

Table 3 shows the results of our proposed method with limitations 6 and 96 in comparison with other important methods: the Power method (Po), the parabolic method (Pa)[16] and the Cambridge Compromise method (CC).

[16] Ramírez V., Palomares, A. Márquez, M. L. (2006), Degressively proportional methods for the allotment of the European Parliament seats amongst the EU Member States. Mathematics and Democracy, pp. 205-220. Berlin: Springer.

Table 3: Compositions of EP after Brexit with different methods

Country	$S_{0.5L}$.	Po.	Pa.	CC	Current
Germany	96	96	96	96	96
France	82	81	83	90	74
Italy	77	76	77	83	73
Spain	62	60	62	65	54
Poland	53	51	53	54	51
Romania	32	31-	31-	31-	32
The Netherlands	29	28	28	27	26
Belgium	22	21	21	20-	21
Greece	21	21	20-	19-	21
Czech Republic	21	20-	20-	19-	21
Portugal	21	20-	20-	19-	21
Sweden	20	20	19-	18-	20
Hungary	20-	19-	19-	18-	21
Austria	18	18	17-	17-	18
Bulgaria	16-	16-	15-	15-	17
Denmark	14	14	14	13	13
Finland	14	14	13	12-	13
Slovakia	13	14	13	12-	13
Ireland	12	13	12	11	11
Croatia	11	12	12	11	11
Lithuania	9-	10-	10-	9-	11
Slovenia	7-	9	9	8	8
Latvia	7-	9	9	8	8
Estonia	6	8	8	7	6
Cyprus	6	7	7	7	6
Luxembourg	6	7	7	6	6
Malta	6	6	6	6	6
Total	701	701	701	701	678

Remarks 2 :

a. All allotments shown in Table 3, except the one in force (Current), respect the degressive proportionality as in the Cambridge Compromise Report which has been established (before rounding).
b. The sum of the absolute differences between the seats allocated in the present-day distribution (Current) and that obtained with each of the other four methods is:

$$\sum_{i=1}^{28}\left|CC_i - Cur_i\right| = 63 \quad ; \quad \sum_{i=1}^{28}\left|Pa_i - Cur_i\right| = 45$$

$$\sum_{i=1}^{28}\left|Po_i - Cur_i\right| = 37 \quad ; \quad \sum_{i=1}^{28}\left|S_{0.5Li} - Cur_i\right| = 35$$

In the above differences, 23 seats come from the departure of the UK (701-678 = 23), which allows for the **correction of the inverse degressivity**, mainly between Germany vis-a-vis France and Spain. In this way, few states lose representation with respect to the 2014-2019 term.

Using the ***0.5-DPL* method** only **five states lose seats** (six seats in total): Hungary, Bulgaria, Slovenia and Latvia lose one seat each and Lithuania loses two seats.

Using the ***Power* method** only **six states lose seats** (seven seats in total): Romania, Czech Republic, Portugal, Bulgaria and Lithuania lose one seat each, and Hungary loses two seats.

Using the ***Parabolic* method** only **nine states lose seats** (eleven seats in total): Romania, Greece, Czech Republic, Portugal, Sweden, Austria and Lithuania lose one seat each, and Hungary and Bulgaria lose two seats each.

Using the ***Cambridge Compromise* method twelve states lose seats** (twenty seats in total).

So, in this sense we can say that the distribution $S_{0.5L}$, which has been obtained using the 0.5-DPL method, is the nearest to the current distribution

In addition, the 0.5-DPL method is the only one, of the four previous methods, which does not need limitations.

On the other hand, the entry into force of the Lisbon voting system in the Council from 1 November 2014 (that is the double majority, with 55% of the EU Member States representing 65% of the EU population), favours both the most populous states and the smaller states; so **the middle-size states must be compensated** through a fairer representation in the EP and the most favourable method for these states is the 0.5-DPL method.

Enlargements of the EU in the 2019-2024 term

If we use the 0.5-DPL method to obtain the composition of the EP with 701 seats in the 2019-2024 term, then the value of k in (5) is k=0.977. So the $S_{0.5L}$ apportionment can also be obtained by using the function

$$A(p) = 0.977\left(\frac{701*0.5\sqrt{p}}{90926.6} + \frac{701*0.5p}{445519516}\right) \qquad (6)$$

Because, in the denominators of (1), the sum of the square root of the populations is 90926.6 and the sum of the populations is 445519516.

If a new state joins the EU during the period 2019-2014, its seat allocation is obtained by replacing the value of p in (5) by its population, and rounding upwards. If the result is greater than 96, we must allocate it 96 seats, if the result does not reach 6, we must assign it 6 seats.

For example, if we assume that Turkey is admitted into the EU and that its population is 78214000, then its adjusted quota is

$$A(78214000) = 0.977\left(\frac{701*0.5\sqrt{78214000}}{90926.6} + \frac{701*0.5*78214000}{445519516}\right) = 93.42$$

And the rounding upwards of 93.42 gives 94 seats to Turkey.

However if Montenegro were the country that enters the EU, its adjusted quota is 3.44 and it receives 6 seats. Table 4 shows the results for several countries.

Table 4: Enlargements of the EU during the 2019-2024 term

Country	p=Population	*A*(p)	$S_{0.5L}$.
Turkey	78214000	93.42	94
Serbia	7103000	15.50	16
Bosnia and Herz.	3750000	10.18	11
Albania	2887000	8.61	9
F.Y.R.O.M.	2071000	7.01	8
Montenegro	620000	3.44	6

Source for the populations: https://en.wikipedia.org/wiki/List_of_European_countries_by_population

If all the six previously mentioned States joined to the EU during the period 2019-2024, the size of the EP would temporarily exceed the maximum of 751 seats by 94 seats.

In this case, for the period 2024-2029 there should be a readjustment to the 751 seats, and the results with the 0.5-DPL method would be those that appear in Table 5.

In Table 5 we consider four scenarios for this hypothetical enlargement of the EU:

a) The limitations are 6-96,

b) The limitations change to 5-96 (as CamCom suggests),

c) The limitations change to 4-96 and

d) Without limitations.

Table 5: Enlargements of the EU. 2024-2029 term. 0.5-DPL method

Country	6-96	5-96	4-96	No Limitations	Current
Germany	86	87	87	87	96
Turkey	82	83	84	84	-
France	72	73	73	74	74
Italy	68	68	69	69	73
Spain	54	54	55	55	54
Poland	47	47	47	47	51
Romania	29	29	29	29	32
The Netherlands	26	26	26	26	26
Belgium	19	19	20	20	21
Greece	19	19	19	19	21
Czech Rep.	18	18	19	19	21
Portugal	18	18	18	18	21
Sweden	18	18	18	18	20
Hungary	18	18	18	18	21
Austria	16	16	16	16	18
Bulgaria	14	14	14	14	17
Serbia	14	14	14	14	-
Denmark	12	12	12	13	13
Finland	12	12	12	12	13
Slovakia	12	12	12	12	13
Ireland	11	11	11	11	11
Croatia	10	10	10	10	11
Bosnia-Herz	9	9	10	10	-
Albania	8	8	8	8	-
Lithuania	8	8	8	8	11
F.Y.R.O.M.	7	7	7	7	-
Slovenia	7	7	7	7	8
Latvia	7	7	7	7	8
Estonia	6	5	5	5	6
Cyprus	6	5	4	4	6
Montenegro	6	5	4	4	-
Luxembourg	6	5	4	3	6
Malta	6	5	4	3	6
Total	751	751	751	751	678

7. CONCLUSIONS

A formula is required to assign seats in the European Parliament to EU Member States clearly and objectively.

At present, the distribution of EP seats among the 28 Member States diverges from the Lisbon Treaty provisions because it violates the principle of degressive proportionality in the sense proposed by Lamassoure and Severin, and also by the "*Cambridge Compromise*".

The withdrawal of UK from the EU is regrettable. But at least we may use some of the seats that UK releases to increase the representation of those Member States affected by inverse DP, without a significant reduction in the representation of many other Member States.

There are many allocations that conform to DP before rounding. In this paper we show a parametric family of methods that respect the principle of DP. These methods have a

scientific justification. Some methods differ from others in the degree of DP that they achieve.

We chose a method with an intermediate degree of degressivity, **the 0.5-DPL** method, which also has a **political justification**: the distribution of seats obtained by 0.5-DPL is close to the current one, with the exception of the countries affected by inverse DP or illogical allocation, as shown in section 2.

Regarding the size of the EP after Brexit, our recommendation is that it should be higher than 678 (= 751-73), because, in that way, we can use seats released by the UK to correct inverse DP. On the other hand, **the size of the EP should not exceed by much the number of 700** seats, since **this would do very detrimental to Germany**, as explained in section 3. In fact, this paper also includes a critical analysis of the minimum and maximum limits of 6 seats and 96 seats respectively for each Member State, and suggests removing them in a future Treaty revision. To compare our method with other known methods, we have taken the size of the EP to be 701 seats.

Then, the 0.5-DPL method allocates seats in the European Parliament, which in the case of the 2014-2019 parliamentary term would be closer to the current distribution than other methods such as the Cambridge Compromise, the parabolic and the power methods.

The 0.5-DPL method can be perfectly and succinctly described as follows: the quota for each Member State is calculated by distributing half of the seats in proportion to the square root of the population and the other half of the seats in proportion to the population of that state. The obtained quota is rounded to integers using the Adams method, with a minimum of 6 and a maximum of 96.

Therefore, the allocation with the 0.5-DPL method can be made with an elementary calculator. **This method is transparent, simple and durable**.

Finally, the **Jagiellonian Compromise** is a very good system to assign voting weight to each Member State in the Council. In addition, it is an easier method to be applied than the double–majority system. In the event that a country leaves the EU the size of the EP is reduced according to the number of seats of the outgoing country, and the weight of the votes in the Council must be recalculated.

DEGRESSIVE PROPORTIONALITY IN THE EUROPEAN UNION

Dr. Wojciech SŁOMCZYŃSKI AND Prof. Karol ŻYCZKOWSKI

KEY FINDINGS

- Allocating seats in the European Parliament (EP) according to a selected mathematical formula based on the populations of the Member States, allows us to avoid potential problems which may occur with any change in the number of Member States or with any considerable variation of their population.
- There exists a plethora of mathematical systems for the seats apportionment that agree with the bounds adopted by the Treaties and the rule of degressive proportionality. One of the simplest is the base + prop scheme, known also as the Cambridge Compromise.
- The Modified Cambridge Compromise (base + power scheme) is better suited in the case of the predicted exit of the United Kingdom from the EU than the original Cambridge Compromise, and results in the minimum transfer of seats in the EP, regardless of the size of the EP, with the rounding method adjusted to the size.
- Brexit provides a unique opportunity to implement a smooth transition to a new balanced allocation system in such a way that each Member State obtains at least the current number of seats in the EP. Such solutions exist also for an appropriately reduced size of the Parliament.
- The minimum size of the EP for which such a smooth solution exists in case of the Modified Cambridge Compromise is 721 (according to the current population data).
- Transition to one of the systems mentioned above will increase the share of representatives for a few of the largest Member States, and will reduce it for the medium-sized ones. Thus, to preserve the overall balance of power in the European Union, one should consider a simultaneous modification of the voting system in the Council of the European Union. For this purpose we recommend the degressive proportional system called the Jagiellonian Compromise that strengthens the voting power of the medium-sized states.

1. ALLOCATING SEATS IN THE EUROPEAN PARLIAMENT

According to the Treaty on European Union (EU) (in particular Article 14(2))[1] and the Council of the European Union Decision of 28 June 2013 establishing the composition of the European Parliament (EP)[2], the apportionment of seats in the EP **should be based on the principle of degressive proportionality** further technically specified in Article 1 of the Decision as follows:

- *the allocation of seats in the European Parliament **shall fully utilise the minimum and maximum numbers set** by the Treaty on European Union in order to reflect as closely as possible the sizes of the respective populations of Member States,*

[1] Consolidated version of the Treaty on European Union, OJ C 326, 26.10.2012.
[2] European Council Decision of 28 June 2013 establishing the composition of the European Parliament, OJ L 181, 29.6.2013, p. 57–58.

- *the ratio between the population and the number of seats of each Member State **before rounding to whole numbers** shall vary in relation to their respective populations in such a way that each Member of the European Parliament from a more populous Member State represents more citizens than each Member from a less populous Member State and, conversely, that the larger the population of a Member State, the greater its entitlement to a large number of seats*[3].

Since we have analysed this principle thoroughly in the paper entitled «Mathematical aspects of degressive proportionality» published four years ago[4], here we shall present only a brief résumé and refer the reader to the paper itself and the references therein for further details.

Degressive proportionality

The notion of **degressive proportionality**[5] plays a crucial role in the current apportionment scheme for the European Parliament. The principle of degressive proportionality enshrined in the Lisbon Treaty was probably borrowed from discussions on taxation rules, where the term appeared as early as the nineteenth century, when many countries introduced income tax for the first time in their history. It was already included in the debate on apportionment in the Parliament in the late 1980s[6], but at first it was a rather vague idea that gradually evolved into a formal legal (and mathematical) term in the Lamassoure & Severin report[7] adopted by the European Parliament in 2007.

The notion hardly appears in the constitutional solutions of the apportionment problem adopted either inside or outside the EU, where the proportional apportionment schemes seem to be prevalent. However, one can find several cases in political practice where **degressively proportional solutions** have been implemented, though not necessarily precisely defined and not necessarily under this name.

Firstly, many allocation systems that reserve a minimum number of seats in a political body, for all subunits represented, usually fail to be proportional, and so, some amount of degressive proportionality seems to be a natural solution in this case. The most famous example that comes to mind here is the **Electoral College** that formally elects the President and Vice President of the United States of America, where each state is allocated as many electors as it has Senators (equal base) and Representatives (proportional representation, with at least one seat per state) in the United States Congress. The idea of combining these two approaches to the apportionment problem was first put forward by one of the Founding Fathers of the United States and future American President, James Madison[8] in 1788.

Secondly, we can find at least two examples from European political practice: the apportionment of seats both in the **upper house of the German Parliament**[9] (*Bundesrat*), and in the **electoral body comprising the members of the twelve Provincial Councils** (*Provinciale Staten*) that elects the Senate (*Eerste Kamer*) of the Dutch Parliament, that are also *de facto* though not *de jure* degressively proportional.

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3 Cegiełka, K., 2013, Composition of the European Parliament in the 2009-2014 term. Didactics of Mathematics 9, 25-34.

4 Słomczyński, W., Życzkowski, K., 2012, Mathematical aspects of degressive proportionality. Mathematical Social Sciences 63, 94–101.

5 Pukelsheim, F., 2014, Proportional Representation – Apportionment Methods and Their Applications. With a Foreword by Andrew Duff MEP. Springer, Cham (CH), p. 166.

6 However, the concept does not appear in Balinski, ML. Young, HP. 1982, Fair representation in the European Parliament. Journal of Common Market Studies 20, 361-373.

7 Lamassoure, A. - Severin, A., 2007, EP Resolution on 'Proposal to amend the Treaty provisions concerning the composition of the European Parliament' adopted on 11 October 2007 (INI/2007/2169).

8 Madison, J., 1788, The conformity of the plan to republican principles. The Independent Journal, January 18, Federalist Paper No. 39.

9 Allen, TJ., Taagepera, R., 2016, Seat allocation in federal second chambers. CSD Working Papers.

Thirdly, the distribution of **votes in the Council of the European Union** from the very beginning of the European Communities until quite recent times, when the system of 'double majority' was introduced, has reflected the principle of degressive proportionality[10].

There have also been suggestions in academic literature to apply this general principle to the apportionment process in some other parliamentary or quasi-parliamentary bodies, such as the projected Parliamentary Assembly of the United Nations[11].

Degressively proportional apportionment – an algorithm

There is a fundamental difference between **proportional** and **degressive proportional** apportionment. While the former is a precisely defined mathematical concept, where only the **rounding procedure** gives us some freedom of manoeuvre, the latter **does not provide us with a single solution, but instead offers an infinite number of options from which to choose.**

How to cope with such a plethora of options in a systematic way? One of the major mathematical approaches to the problem of degressively proportional apportionment in the European Parliament can be described by the following **general scheme**:

1. One has to choose a concrete characterisation of the size of a given Member State *i* by a number p_i (for example, equal to the total number of its inhabitants, citizens or voters), which we call here **population**, and precisely define by which means the required data should be collected and how often it should be updated. Then, one needs to transform these numbers by an **allocation function *A*** belonging to a given family (**allocation scheme**) indexed by some parameter ***d***, whose range of variability is determined by the requirement that the function fulfils constraints imposed by the Treaties: is **non-decreasing** and **degressively proportional**.
2. Additionally, the allocation function *A* has to satisfy certain **boundary conditions**: $A(p) = m$ and $A(P) = M$, where the population of the smallest and the largest state equals, respectively, p and P, with the smallest and the largest number of seats predetermined as, respectively, m and M. (In the case of the EP these quantities are explicitly determined by the Treaty and the Decision: ***m* = 6** and ***M* = 96**.)
3. To assign integer number of seats for each Member State one has to round the values of the allocation function, e.g. using one of three standard **rounding methods** (*upward*, *downward* or *to the nearest integer*).
4. Finally, one has to choose the **parameter *d*** in such a way that the sum of the seat numbers of all Member States equals the projected **number of seats in the EP** (***S***), solving (if possible) in *d* the equation:

$$\sum_{i=1}^{N} [A_d(p_i)] = S\,,$$

where N stands for the number of Member States, p_i for the population of the i-th state ($i = 1, \ldots, N$), and $[\,\cdot\,]$ denotes the rounded number.

Thus to prepare a degressive proportional apportionment of seats for the EP we have to set **three variables**:

a. the **number of seats in the EP** – ***S***;
b. the **allocation scheme** – ***A***;
c. the **rounding method** – **[·]**.

Knowing a), b) and c), we can choose the appropriate parameter *d* and, consequently, a concrete allocation function resulting in the distribution of seats in the EP associated with the given variables. Though usually there is a whole interval of parameters satisfying this

[10] Best, E., 2004, What is really at stake in the debate over votes? EIPAScope 1, p. 14-23.
[11] Bummel, A., 2010, The composition of a parliamentary assembly at the United Nations. Background Paper No. 3. Berlin: Committee for a Democratic UN; see also von Bogdandy, A., 2012, The European Lesson for International Democracy: The Significance of Articles 9–12 EU Treaty for International Organizations. The European Journal of International Law 23, 315-334.

requirement, nonetheless, in a generic case, the **distribution of seats established in this way is unique.** This technique bears a resemblance to **divisor methods** in the proportional apportionment problem used first by Thomas Jefferson[12] in 1792.

Please note that in the Lamassoure and Severin definition of degressive proportionality it was postulated that this property holds for the number of seats **after rounding** the values of the allocation function to whole numbers. However, one can show that there exist such distributions of population that there is no solution to the apportionment problem satisfying such defined degressive proportionality[13]. Consequently, Grimmett *et al.* recommended[14] to weaken this condition and to amend the definition of degressive proportionality assuming that the property holds for the number of seats **before rounding.**

Proposed forms of allocating schemes

In our papers published several years ago[15] we gathered together and analysed **seven natural allocation schemes**, i.e., seven one-parameter families of allocation functions[16], and studied their properties with the implementation to the apportionment for the EP under three rounding procedures **(downward, to the nearest integer, upward):**

- **base + prop**,
- **piecewise linear**,
- **quadratic (parabolic)**,
- **base + power**,
- **homographic**,
- **linear + hyperbolic**,
- **min-max proportional.**

All seven families mentioned above share a common element: the **linear** (**affine**) **allocation function**. This is undoubtedly the simplest allocation function one can imagine. However, under present circumstances, it would lead to a **smaller** parliament than the current one, but its size can serve as an indicator to estimate **how many seats we can allocate freely** besides the linear (or, more precisely, affine) distribution.

Note that all these solutions have been already discussed in the academic literature. The **base + prop** scheme, which seems to be the simplest of all these methods, was first analysed by Pukelsheim[17] and became the basis for the proposal, called 'Cambridge Compromise', elaborated in January 2011 by a group of mathematicians and political

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12 Balinski, ML., Young, HP., 1978, The Jefferson method of apportionment. SIAM Review 20, 278–284.

13 Ramírez-González, V., 2010, Degressive proportionality. Composition of the European Parliament. The parabolic method. In: Cichocki, M., Życzkowski, K. (Eds.), Institutional Design and Voting Power in the European Union. Ashgate, London (called further [CŻ10]), p. 215–234.

14 Grimmett, G., Laslier, J.-F., Pukelsheim, F., Ramírez-González, V., Rose, R., Słomczyński, W., Zachariasen, M., Życzkowski, K., 2011, The allocation between the EU member states of the seats in the European Parliament. Cambridge Compromise. European Parliament Studies, PE 432.760.

15 Słomczyński, W., Życzkowski, K., 2010, On bounds for the allocation of seats in the European Parliament. In: [CŻ10], p. 269-281; Słomczyński, Życzkowski, 2012, *op. cit.*; see also Cegiełka, K., Łyko, J., 2014, Application of Hamilton's and divisor methods to degressively proportional allocation functions. Procedia - Social and Behavioral Sciences 110, 103–112.

16 The detailed formulae can be found in Słomczyński & Życzkowski 2012, *op. cit.* Here, we would like to recall only one of them:

$$A_d(t) = (M(t^d - p^d) + m(P^d - t^d))/(P^d - p^d)$$

for the *base + power* allocation function with the exponent d, where m and M denote the number of seats for the smallest and the largest Member State, with population p and P, respectively, and $p \le t \le P$. This formula (combined with the appropriate rounding) was used for computing the numbers in Tables 1 and 2 (columns: MCC and L), with $m = 6$, $M = 96$, $p = 434\,403$ and $P = 82\,064\,489$, where t stands for the population of a given country. The calculation of the exponent d is based on the algorithm described in details in the preceding subsection. The values of exponents are given in the tables.

17 Pukelsheim, F., 2007, A Parliament of degressive representativeness? Preprint No. 015/2007, Institut für Mathematik, Universität Augsburg.

scientists[18], and discussed later by the Committee on Constitutional Affairs (AFCO) of the EP. The **piecewise linear** scheme was proposed for the first time by the authors of this briefing[19] and, independently, by Ramírez González *et al.*[20] under the name of the Linear Spline Method. On the other hand, the **quadratic (parabolic)** scheme was advocated by Ramírez González and his co-workers in a series of papers[21]. The **base + power** scheme has been studied by many authors from Ramírez González *et al.*[22] to Grimmett *et al*.[23], although it can be traced to the paper of Theil and Schrage[24] from 1977. Note that a similar method was proposed for solving the taxation problem as early as the nineteenth century by the Dutch economist Cohen-Stuart[25]. The **homographic** scheme functions introduced by the authors[26], were also studied under the name of projective quotas by Serafini[27]. The **linear + hyperbolic** scheme was used both in the apportionment problem for the EP[28], as well as in the tax schedule proposed by the Swedish economist Cassel at the beginning of the twentieth century[29]. Finally, the **proportional** apportionment method **with minimum and maximum requirements** was considered by Balinski and Young[30]. Moreover, the **linear** allocation function was studied under the name of base + strict prop by Kellerman[31].

We have observed that all these solutions are quite similar (with the notable exception of *min-max proportional*), which is a consequence of the fact that our choice is limited by two factors: the predetermined shape of the graph of an allocation function, and the fact that the **vast majority of seats are, in a sense, distributed in advance.** However, one can observe that the results for the *parabolic*, *base + power*, and *homographic* allocation schemes **lead to quite similar apportionments,** whereas the choice of the *base + prop* scheme is **advantageous for large countries,** and the *piecewise linear* and *linear + hyperbolic* schemes seems to be **beneficial for small countries**[32].

In 2011 the authors of this briefing joined the group of mathematicians and political scientists endorsing the so-called **'Cambridge Compromise'**[33]. This allocation system, was selected mainly because of its obvious simplicity[34]. However, this solution has been criticised for being 'not degressively proportional enough' and departing too much from the

[18] Grimmett *et al.* 2011, *op. cit.*
[19] Słomczyński & Życzkowski 2012, *op. cit.*
[20] Ramírez-González, V., Martínez Aroza, A., Márquez García, M., 2012, Spline methods for degressive proportionality in the composition of the European Parliament. Mathematical Social Sciences 63, 114-120.
[21] Ramírez González, V., 2004, Some guidelines for an electoral European system. In: Workshop on Institutions and Voting Rules in the European Constitution, Seville, 10–12 December; Ramírez-González, V., Palomares Bautista, A., Márquez García, M., 2006, Degressively proportional methods for the allotment of the European Parliament seats amongst the EU member states. In: Simeone, B., Pukelsheim, F. (Eds.), Mathematics and Democracy. Recent Advances in Voting Systems and Collective Choice. Springer, Berlin, p. 205–220.
[22] Ramírez-González *et al.* 2006, *op. cit.*
[23] Grimmett, G., Oelbermann, K.-F., Pukelsheim, F., 2012, A power-weighted variant of the EU27 Cambridge Compromise. Mathematical Social Sciences 63, 136–140; see also Pukelsheim 2014, *op. cit.,* p. 170., where the method is called the *downgraded-population variant* of the Cambridge Compromise.
[24] Theil, H., Schrage, L., 1977, The apportionment problem and the European Parliament. European Economic Review 9, 247–263.
[25] Cohen-Stuart, AJ., 1889, Bijdrage tot de Theorie der Progressieve Inkomstenbelasting. Martinus Nijhoff, Den Haag.
[26] Słomczyński & Życzkowski 2012, *op. cit.*
[27] Serafini, P., 2012, Allocation of the EU Parliament seats via integer linear programming and revised quotas. Mathematical Social Sciences 63, 107–113.
[28] Słomczyński & Życzkowski 2010, *op. cit.*
[29] Cassel, KG., 1901, The theory of progressive taxation. The Economic Journal 11, 481–491.
[30] Balinski, ML., Young, HP., 2001. Fair Representation. Meeting the Ideal of One Man, One Vote, second ed. Brookings Institution Press, Washington, p. 133.
[31] Kellermann, T., 2012, The minimum-based procedure: A principled way to allocate seats in the European Parliament. Mathematical Social Sciences 63, 102-106.
[32] Using the measure of degressive proportionality introduced in: Dniestrzański, P., 2014, Proposal for measure of degressive proportionality. Procedia - Social and Behavioral Sciences 110, 140–147, one can check that it is possible to order considered schemes from the least to the most degressively proportional: *base + prop* < *homographic* ≈ *parabolic* < *base + power* < *piecewise linear* ≈ *linear + hyperbolic*. A similar hierarchy can be reproduced by computing the relative entropy of the population distribution with respect to the seats distribution, measuring in this way the deviation form proportionality, see Lauwers, L., Van Puyenbroeck, T., 2008, Minimally Disproportional Representation: Generalized Entropy and Stolarsky Mean-Divisor Methods of Apportionment. HUB research paper, Brussel.
[33] Pukelsheim 2014, *op. cit.,* p. 168-174.
[34] Grimmett *et al.* 2011, *op. cit.*

status quo by Moberg[35]. In 2012 a solution very similar to the **base + power** scheme was considered by Grimmett *et al.* as a step along a continuous transition from the negotiated *status quo* composition to the constitutionally principled Cambridge Compromise[36]. The crucial point in these discussions seems to be **the meaning of the term 'degressive proportionality'.** Is it only a less perfect form of (pure) proportionality, as it was actually suggested by some authors or is it a **separate notion that requires distinct (and new) mathematical and political solutions,** as Moberg claims? Personally, we incline towards the latter suggestion.

2. RECOMMENDED SOLUTIONS FOR THE EUROPEAN PARLIAMENT

As 'it is unclear whether the UK's 73 seats will be lost or reallocated'[37], we have analysed **six** possible choices for the size of the EP:

- 751 - with the UK;
- 751 - without the UK;
- 678 = 751 – 73 - without the UK;
- Optimal size - without the UK;
- Minimum size - with the UK;
- Minimum size - without the UK,

along with **seven** allocation schemes and **three** rounding methods, which overall results in **seventy seven different allocations.** Analysing all these solutions, we primarily take into account Article 4 of the Decision that requires *establishing a system which in future will make it possible, before each fresh election to the European Parliament, to allocate the seats between Member States in an **objective**, **fair**, **durable** and **transparent** way, **translating the principle of degressive proportionality** as laid down in Article 1, taking account of any change in their number and demographic trends in their population, as duly ascertained thus **respecting the overall balance of the institutional system** as laid down in the Treaties*[38].

As all analysed schemes 'translate the (mathematical) principle of degressive proportionality' into the political realm, and their mathematical form guarantees that the resulted apportionment would be indeed objective, fair, durable and transparent, we have looked for the solutions that **change the status quo as little as possible**[39] trying (in order):

- to minimize the number of seats transferred;
- to minimize the number of Member States losing seats;
- to maximize the number of Member States gaining seats.

Such solutions would lead to a relatively **smooth transition** from the current apportionment into a new one. We call them **balanced solutions.** Note that **some**

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35 Moberg, A., 2012, EP seats: the politics behind the math. Mathematical Social Sciences 63, 78–84.

36 Grimmett *et al.* 2012, *op. cit.*

37 Patel, O., Reh, C., 2016, Brexit: The Consequences for the EU's Political System. UCL Constitution Unit Briefing Paper 2.

38 European Council Decision of 28 June 2013 establishing the composition of the European Parliament, *op. cit.*

39 Note that this fact seems to be crucial for practical implementation of a given apportionment scheme. For instance, in opinion of the authors of *Report on the composition of the European Parliament with a view to the 2014 elections.* A7-0041/2013, the rapporteurs, Gualtieri and Trzaskowski, both base + prop and parabolic schemes deviated too strongly from the *status quo*. As they wrote: *The implementation [of the 'Cambridge compromise' as the most 'proportional' mechanism respecting degressive proportionality] would trigger a traumatic reallocation of seats, with heavy losses for medium-sized and small Member States and huge increases for larger ones. Furthermore, failure to abolish the 96 upper limit would discriminate against Germany among the large Member States, introducing a steep rise in the population/seats ratio between France and Germany. Among the various possible mathematical formulae for implementing the principle of degressive proportionality the 'parabolic' method is one of the most degressive. It could, in the longer term, be used as a benchmark in the absence of a treaty change, but the redistribution which this model entails would be too drastic to be politically sustainable in a single step. Cf.* note 32.

transfer of seats is inevitable in this case, firstly, because of **demographic changes**[40] and, secondly, since the present apportionment in the EP is, in a sense, **erratic and irregular as a result of some historical bargaining**[41], rather than objective considerations. To be more specific, one can say, with some degree of unavoidable inaccuracy, that there are three groups of Member States for which the result of projected changes will be, relatively or absolutely (depending on the future size of the EP)

- **positive:** France, the United Kingdom (if applicable), Spain, Estonia;
- **neutral:** Germany, Italy, Poland, the Netherlands, Denmark, Finland, Slovakia, Ireland, Croatia, Slovenia, Latvia, Cyprus, Luxembourg, Malta;
- **negative:** Romania, Belgium, Greece, the Czech Republic, Portugal, Sweden, Hungary, Austria, Bulgaria, Lithuania.

The results of our considerations are outlined below, divided according to two main criteria - the size of the EP and the **presence of the British MEPs** (see Tab. 1 and Tab. 2 for details):

EP 751 (with the UK)

We are still convinced that the 'Cambridge Compromise' gives here the simplest acceptable solution. However, a **balanced solution** in this case is given by the **base + power scheme with the rounding downwards**.

EP 751 (without the UK)

Assume that the size of the EP remains unchanged after Brexit and all British seats are distributed among other Member States. Then the 'Cambridge Compromise' produces a solution with a substantial transfer of seats, especially to a few large states, while the balanced solution is given by the **base + power scheme rounding upwards.**

EP 678 (without the UK)

Assume that the size of the EP is reduced by the number of British seats. Here again the 'Cambridge Compromise' produces a solution with a substantial transfer of seats, while the balanced solution is given by the **base + power scheme with rounding downwards.**

EP Optimal size (without the UK)

Assume that the size of the EP is reduced by a smaller number than the number of British seats. We have been looking for the smallest size of the Parliament with no Member State losing seats. Several options are possible here with the **balanced solution** given in this case by the **base + power scheme with rounding to the nearest integer** and the size of the EP equal to **721.**

EP Minimum size (with or without the UK)

Assume that the simplest allocation function is chosen, i.e., **linear with the rounding to the nearest integer**. The resulting size of the EP would be either 718 (with the UK) or 640 (without the UK). Although probably politically hard to implement, this solution shows (approximately) how many seats can be in fact freely allocated in both situations: only 751 – 718 = **33** or 678 – 640 = **38**, respectively.

An additional argument for the base + power scheme

The base + power scheme has an additional property called **super-proportionality**[42]. To illustrate this property consider two pairs of Member States: Romania/France and Belgium/Poland, with the similar population quotient (approx. 29.7%), and another such configuration: Sweden/Romania and Finland/Greece (approx. 50.6%). Note that in all these

[40] E.g., the population of Sweden is currently larger than that of Hungary, yet Hungary has one more seat in the European Parliament.
[41] Balinski & Young 1982, *op. cit.*
[42] See Słomczyński & Życzkowski 2012, *op. cit.* for a precise definition of this notion.

cases the **seat quotient must be larger than the population quotient** because of degressive proportionality. However, in both cases (assuming the balanced solution and the current size of the EP: 28 states and 751 seats) the seat quotient **is greater for the 'smaller' pair than for the 'larger' one**, as we get 31.25% for Romania/France whereas 40% for Belgium/Poland, as well as 60% for Sweden/Romania whereas 68.42% for Finland/Greece. In other words, a super-proportional method leads to the following property of an allocation system (before rounding): **The smaller a pair of states is, the larger the gain in seats of the smaller member in the pair over the larger one**. Hence, if an allocation function is super-proportional, then the degressive proportionality acts more strongly for smaller states, and so such functions are, in a sense, **more degressively proportional** than others. Thus, this is in fact a kind of **degressive–degressive proportionality.**

Final recommendation for the European Parliament apportionment

We recommend the adoption of the **Modified Cambridge Compromise** (MCC), i.e., base + power system as the solution that **minimizes the transfer of seats** and, at the same time, fulfils all constitutional requirements and expresses **the principle of degressive proportionality** more accurately than other solutions considered. **The specific form of rounding** in the system should depend on the projective size of the EP and concrete population data, and should be chosen to further minimize the transfer of seats. Having at our disposal two extreme solutions: maintaining the current size of the Parliament (751) or reducing the size by all British seats (to 678), we advocate, however, for an **intermediate solution**. Assuming that no Member State should lose any seat during the transition procedure, and simultaneously trying to minimize the size of the Parliament, we arrive at the **'optimal'** number of **721** representatives

Table 1: Alternative proposals for the EP apportionment depending on the EP size (with the UK). Here (CC) stands for the *Cambridge Compromise*, (MCC) for the *base + power* scheme, (L) for the *linear* allocation, (d) and (n) for the rounding, resp., *downwards* and *to the nearest integer*. *Total transfer of seats* is the sum of losses and gains in the number of seats. The *exponents* are computed for the *base + power* schemes.

Member State	Population	Status quo	EP+UK 751 (CC)	EP+UK 751 (MCC) (d)	EP+UK Minimum (L) (n)
Germany	82 064 489	96	96	96	96
France	66 661 621	74	84	80	79
United Kingdom	65 341 183	73	83	79	78
Italy	61 302 519	73	78	74	73
Spain	46 438 422	54	60	59	57
Poland	37 967 209	51	50	50	47
Romania	19 759 968	32	29	30	27
The Netherlands	17 235 349	26	26	27	25
Belgium	11 289 853	21	19	20	18
Greece	10 793 526	21	18	19	17
Czech Republic	10 445 783	21	18	19	17
Portugal	10 341 330	21	18	19	17
Sweden	9 998 000	20	17	18	17
Hungary	9 830 485	21	17	18	16
Austria	8 711 500	18	16	17	15
Bulgaria	7 153 784	17	14	15	13
Denmark	5 700 917	13	12	13	12
Finland	5 465 408	13	12	13	12
Slovakia	5 407 910	13	12	12	11
Ireland	4 664 156	11	11	12	11
Croatia	4 190 669	11	10	11	10
Lithuania	2 888 558	11	9	9	9
Slovenia	2 064 188	8	8	8	8
Latvia	1 968 957	8	8	8	8
Estonia	1 315 944	6	7	7	7
Cyprus	848 319	6	7	6	6
Luxembourg	576 249	6	6	6	6
Malta	434 403	6	6	6	6
UE-27	510 860 699	751	751	751	718
Total transfer of seats		0	66	42	56
Exponent		-	-	0.903	1

Population data based on the Council Decision 2016/2353 of 8 December 2016 amending the Council's Rules of Procedure. (See endnote 16 for the formula used to compute the MCC and the linear apportionments.)

Table 2: Alternative proposals for the EP apportionment depending on the EP size (without the UK). Here (MCC) stands for the base + power scheme, (L) for the linear allocation, and (d), (n), and (u) for the rounding, resp., downwards, to the nearest integer, and upwards. Total transfer of seats is the sum of losses and gains in the number of seats. The exponents are computed for the base + power schemes.

Member State	Population	Status quo	EP-UK 751 (MCC) (u)	EP-UK Optimal (MCC) (n)	EP-UK 678 (MCC) (d)	EP-UK Minimum (L) (n)
Germany	82 064 489	*96*	96	**96**	96	96
France	66 661 621	*74*	83	**82**	80	79
United Kingdom	-	*73*	-	**-**	-	-
Italy	61 302 519	*73*	78	**76**	75	73
Spain	46 438 422	*54*	63	**62**	60	57
Poland	37 967 209	*51*	55	**53**	51	47
Romania	19 759 968	*32*	35	**33**	30	27
The Netherlands	17 235 349	*26*	31	**30**	27	25
Belgium	11 289 853	*21*	24	**22**	20	18
Greece	10 793 526	*21*	23	**22**	20	17
Czech Republic	10 445 783	*21*	23	**21**	19	17
Portugal	10 341 330	*21*	23	**21**	19	17
Sweden	9 998 000	*20*	22	**21**	19	17
Hungary	9 830 485	*21*	22	**21**	18	16
Austria	8 711 500	*18*	20	**19**	17	15
Bulgaria	7 153 784	*17*	18	**17**	15	13
Denmark	5 700 917	*13*	16	**15**	13	12
Finland	5 465 408	*13*	16	**14**	13	12
Slovakia	5 407 910	*13*	16	**14**	13	11
Ireland	4 664 156	*11*	14	**13**	12	11
Croatia	4 190 669	*11*	14	**13**	11	10
Lithuania	2 888 558	*11*	11	**11**	9	9
Slovenia	2 064 188	*8*	10	**9**	8	8
Latvia	1 968 957	*8*	10	**9**	8	8
Estonia	1 315 944	*6*	8	**8**	7	7
Cyprus	848 319	*6*	7	**7**	6	6
Luxembourg	576 249	*6*	7	**6**	6	6
Malta	434 403	*6*	6	**6**	6	6
UE-27	445 519 516	*751*	751	**721**	678	640
Total transfer of seats		0	73	43	34	56
Exponent		-	0.796	0.829	0.886	1

Population data based on the Council Decision 2016/2353 of 8 December 2016 amending the Council's Rules of Procedure. (See endnote 16 for the formula used to compute the MCC and the linear apportionments.)

THE SYSTEM OF VOTING IN THE COUNCIL OF THE EU

Adopting one of the mathematically motivated, fair and objective systems of allocation of seats in the EP recommended above, will lead to a certain transfer of power in the European Union. In particular, the largest Member States (with the exception of Germany) will increase their number of representatives in the Parliament. Therefore, to preserve the overall **balance of power** in the Union, it is well justified to consider a **simultaneous suitable modification of the existing voting system in the Council.**

The current solution, adopted in December 2007 in Lisbon, is based on the principle of **'double majority'**: a decision of the Council is taken if it is supported by a coalition, which:

a. is formed by at least **55%** of the Member States,
b. represents at least **65%** of the population of the Union.

Additionally, a decision is adopted if the supporting coalition consists of all but three (or fewer) countries even if it represents less than 65% of the population of the Union[43].

The case of **Brexit** creates an urgent need to discuss and reconsider these rules[44]. A detailed analysis by Moberg shows that the current system of the 'double majority' **is not really double**, as the population criterion (b) plays a dominant role[45]. As noted by several authors[46], the existing system is biased in favour of the most and the least populated countries. In particular, the voting power of a typical citizen in these states, measured by the Banzhaf-Penrose index, is larger than the power of a medium-sized state citizen. These disadvantages of any 'double majority' voting system were noted by Lionel Penrose[47] as long ago as in 1952. Working on the problem of voting power, Penrose formulated his **square root law** and proposed an objective voting system, in which the **voting weights are proportional to the square root of the population** for each state. A voting system based on the Penrose law was first proposed for the Council of Ministers by Laruelle and Widgrén[48] in 1996 and, independently, by Felsenthal and Machover[49] in 1997.

However, to construct any **weighted voting system** one has to choose not only the **voting weights**, but also to fix the **quota (threshold)** of the **qualified majority**, which plays a crucial role in the system[50]. In the past, the quotas in the voting systems for the Council had been established subjectively in a bargaining procedure, without an objective justification. A new solution to the problem relates the value of the quota to an **optimization procedure: the optimal quota** is set in such a way that the voting power of every citizen in each Member State is approximately equal[51]. Such a solution is known in the literature as the **Jagiellonian Compromise**[52] and its advantages have been

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43 Consolidated version of the Treaty on European Union, OJ C 326, 26.10.2012, Article 16(4).

44 Macháček, V., Hrtúsová, T., 2016, Brexit and the functioning of European institutions. Special analysis, EU Office/Knowledge Centre - Česká spořitelna; Kalcik, R., Wolff, GB., 2017, *Is Brexit an opportunity to reform the European Parliament? Bruegel Policy Contribution 2*.

45 Moberg, A., 2010, Is the double majority really double? The voting rules in the Lisbon Treaty. In: [CŻ10], p. 19-34.

46 See [CŻ10]: Kirsch, W., The distribution of power in the Council of Ministers of the European Union, p. 93-107; Pukelsheim, F., Putting citizens first: Representation and power in the European Union, p. 235-253; Słomczyński & Życzkowski 2010, *op. cit.*

47 Penrose, LS., 1952, On the Objective Study of Crowd Behaviour. H.K. Lewis & Co, London.

48 Laruelle, A., Widgrén, M., 1996, Is the Allocation of Voting Power Among the EU States Fair? CEPR Discussion Paper No. 1402.

49 Felsenthal, DS., Machover, M., 1997, The weighted voting rule in the EU's Council of Ministers, 1958–95: Intentions and outcomes. Electoral Studies 16, 33–47.

50 Machover, M., 2010, Penrose's square root rule and the EU Council of the Ministers: Significance of the quota. In: [CŻ10], p. 35-42.

51 Słomczyński, W., Życzkowski, K., 2004, Voting in the European Union: The Square Root System of Penrose and a Critical Point. Preprint cond-mat.0405396; Życzkowski, K., Słomczyński, W., Zastawniak, T., 2006, Physics for fairer voting. Physics World 19, 35-37; Słomczyński, W., Życzkowski, K., 2007, From a toy model to the double square root voting system. Homo Oeconomicus 24, 381-399.

52 Słomczyński, W., Życzkowski, K., 2010, Jagiellonian Compromise - an alternative voting system for the Council of the European Union. In: [CŻ10], p. 43-57; Słomczyński, W., Życzkowski, K., 2014, Square root voting system, optimal threshold and π. In: Fara, R., Leech, D., Salles, M. (Eds.), Voting Power and Procedures. Springer, Berlin, p. 127–146.

acknowledged by several experts[53]. Its name is related to the fact that it can be considered as an **objective and fair compromise** between the older **Nice voting system,** in which the largest Member States suffer a relatively small voting power, and the current **'double majority' voting system**, where they seem to have too much power.

The **Jagiellonian Compromise** is a voting system for the **Council of Ministers** consisting of a **single criterion** only determined by the following two rules:

1. Each Member State is attributed the **voting weight proportional to the square root of its population**;
2. The decision of the Council is taken if the sum of the weights of members of a coalition supporting it exceeds the quota equal to the **arithmetical mean of the sum of the weights and the square root of the total population of the Union**.

The quota for the qualified majority is considerably larger than 50% for any size of the voting body of a practical interest. Thus, the voting system is **moderately conservative**, as it should be. Furthermore, it is **transparent**: the voting power of each Member State, measured by the Banzhaf-Penrose index, is, up to a high accuracy, proportional to its voting weight. As a crucial advantage of the system one can emphasize its **extendibility**: if the number of Member States or their populations change, all one needs to do is to set the voting weights according to the rule (1), and adjust the quota according to the rule (2).

Currently, the value of the **optimal quota** equals (approximately) **61.4%**; while after Brexit it would change to **61.6%**. Implementing a new voting system in the Council based on the Jagiellonian Compromise would contribute to an increase in the *a priori* voting power of the medium-sized members of the EU. In a sense, this step would compensate for the losses incurred by these states due to the allocation of seats according to the (Modified) Cambridge Compromise and will contribute to **preservation of the current overall balance of power in the European Union**.

Final recommendation for the voting system in the Council

*We recommend the adoption of the **Jagiellonian Compromise** as the degressively proportional solution for the voting system in the Council of Ministers, counterbalancing the effects of the new apportionment of seats in the European Parliament.*

[53] Pukelsheim 2014, *op. cit.,* p. 174-176.

Abstract
The workshop, organized by the Policy Department for Citizens' Rights and Constitutional Affairs upon request by the AFCO Committee, provides insight in the issue of apportionment of seats in the European Parliament, one of the most politically sensitive decisions with direct impact on citizens' representation in the European Parliament. The workshop examined methods for the apportionment of seats in Parliament that would satisfy the requirements of the principle of degressive proportionality and that would be able to automatically adjust to the changing demographic picture in the Union as well as be able to accommodate changes in the number of EU Member States.

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