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## Disorder in Physical Systems

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A volume in honour of  
**John M. Hammersley**  
on the occasion of his  
70th birthday

Edited by  
G.R. Grimmett and D.J.A. Welsh



# Preface

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On 21 March 1990 John Hammersley celebrates his seventieth birthday. A number of his colleagues and friends wish to pay tribute on this occasion to a mathematician whose exceptional inventiveness has greatly enriched mathematical science.

The breadth and versatility of Hammersley's interests are remarkable, doubly so in an age of increased specialisation. In a range of highly individual papers on a variety of topics, he has theorised, and posed (and solved) problems, thereby laying the foundations for many subjects currently under study. By his evident love for mathematics and an affinity for the hard problem, he has been an inspiration to many.

If one must single out one particular area where Hammersley's contribution has proved especially vital, it would probably be the study of random processes in space. He was a pioneer in this field of recognised importance, a field abounding in apparently simple questions whose resolutions usually require new ideas and methods. This area is not just a mathematician's playground, but is of fundamental importance for the understanding of physical phenomena. The principal theme of this volume reflects various aspects of Hammersley's work in the area, including disordered media, subadditivity, numerical methods, and the like.

The authors of these papers join with those unable to contribute in wishing John Hammersley many further years of fruitful mathematical activity.

August 1989

G.R. Grimmett  
D.J.A. Welsh



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